SPATIAL AUTOCORRELATION FOR TRANSVERSE BEAM QUALITY CHARACTERIZATION*

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Abstract

Low emittance beams are required for high brightness beams applications. Contributions to emittance degradations come from electromagnetic fields' non-linearities which can be reduced using a transversally and longitudinally uniform beam. For these reasons the evaluation of the beam quality is a very important task. Concerning the transverse analysis the spatial autocorrelation parameter has been introduced: it gives an evaluation of how beam non-uniformity is distributed. The paper describes the spatial autocorrelation concept and applies it to the evaluation of a laser beam for high brightness beam applications. Moreover the paper shows the spatial autocorrelation evolution along a photo-injector as an additional tool for beam dynamics studies.

INTRODUCTION

Concepts such as mean, variance and standard deviation can be used to evaluate uniformity of a set of data distributed on a surface, as in the case of the transverse spot of an electron beam or of the laser itself.

The mean describes the central value of the data. In the case of a laser or of a beam cross section analysis, a matrix of pixel of certain intensity is given (each pixel representing the electrons or photons charge). For a matrix of elements the mean is obviously calculated as:

$$<a>=\frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{M}a_{ij} \qquad (1)$$

where N and M are the matrix dimensions, $T=NM$ is the number of pixel involved, and a_{ij} the matrix element representing the generic sample, that is the pixel intensity.

The variance represents the distance from the central value, that is the spread, and for a *2D* matrix it is calculated as follows:

$$\mathrm{var}(a)=\frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{M}(a_{ij}-<a>)^2 \qquad (2)$$

The obtained quantity is always positive so that the standard deviation can be defined as:

$$\sigma_a=\sqrt{\mathrm{var}(a)} \qquad (3)$$

which is a quantity whose dimensions are comparable to the mean. The argument $(a_{ij}-<a>)$ defines a new matrix where every element represents the distance from the mean and $(a_{ij}-<a>)^2$ is the variance matrix. It is obtained as a distance squared so that bigger differences are emphasized respect to the smaller ones.

For a perfectly uniformly charged beam cross section, normalized to the higher sample, $<a>=1$ and $\sigma_a=0$. Of course more the cross section is non-uniform more the standard deviation will be far from the ideal values. The above parameters, describe non-uniformity without describing the way non-uniformity is distributed. It has been shown [2] the importance of the distribution of the non-uniformity because it can give different results concerning the emittance degradation.

Spatial correlation describes such a property [1]. It is necessary to introduce the covariance for a matrix point to define the spatial correlation. The quantity covariance answers the question whether a sample and its neighbour are at the same time different or not from the mean and it's defined as:

$$\mathrm{cov}(a,h)=\frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{M}(a_{ij}-<a>)\cdot(a_{ijh}-<a>) \qquad (4)$$

where a_{ijh} is the mean of the samples localized around the main sample a_{ij}. The argument $(a_{ij}-<a>)(a_{ijh}-<a>)$ is called the covariance matrix.

The samples can be taken in different ways depending also from the distance h from a_{ij} as represented in Figure 1:

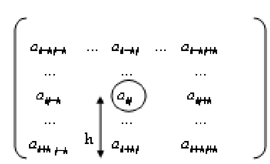

Figure 1: The a_{ij} is the generic sample whose variance is compared with the other samples' variance at a certain distance h..

As can be easly seen it results:

$$a_{ijh}=\frac{1}{(2h+1)^2-1}\left[\sum_{l=-h}^{h}\sum_{m=-h}^{h}a_{i+l\ j+m}-a_{ij}\right] \qquad (5)$$

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which is the mean of the samples around a_{ij}.

The distance h and the matrix dimensions N, M define the resolution of the spatial autocorrelation investigation. The index Λ, which describes the spatial correlation, can be defined as the the covariance normalized to the standard deviation σ_a squared:

$$\Lambda(a,h) = \frac{\mathrm{cov}(a,h)}{\sigma_a^2} \qquad (6)$$

which is a quantity whose value is between *-1* and *1*. The minus sign simply means most samples are lower than the mean.

Meaning of the spatial autocorrelation index

The spatial autocorrelation meaning is shown in the following examples.

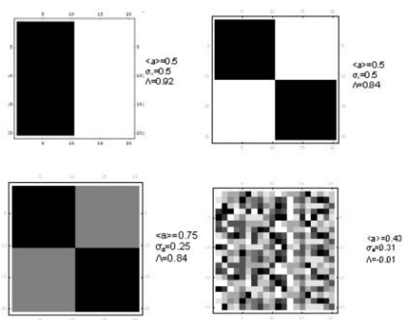

Figure 2: Examples of the spatial autocorrelation and standard deviation calculation.

The results underline that, given the same mean and the same standard deviation, the spatial correlation maybe different: the matrix on the left hand side has a unique big spot whilst the matrix on the right hand side has two distributed spots of intensity, resulting in more distributed spots; in the first case it gives $\Lambda=0.92$ whilst in the second case, as expected, the spatial correlation is smaller ($\Lambda=0.84$).

It's worth noting that, as represented on the left hand side (bottom) of Figure 2, the mean can be enhanced keeping the same spots distribution: in this case spatial correlation remains unchanged no matter of the intensity of the distribution.

Finally spatial correlation decreases as the spots of intensity become more random. This is shown in Figure 2 where on the right hand side (bottom) is depicted a completely random distribution of samples.

A plot of the spatial autocorrelation as a function of the distance h is called correlogram of a given spot. The spot and its correlogram are represented in Figure 3.

Two points P_1 and P_2 are correlated if they are placed at a certain distance. This distance can be evaluated arbitrarily fixing the Λ to be less than a certain value and it coincides with the mean distance of the non homogeneity.

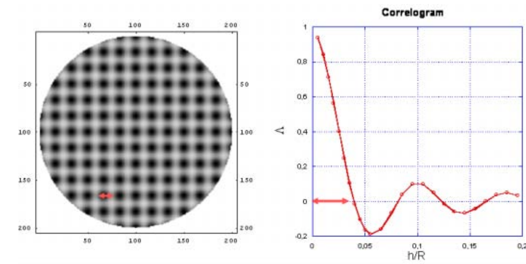

Figure 3: A theoretical laser spot with distributed non-uniformities and the corresponding correlogram.

SPATIAL AUTOCORRELATION AND BEAM DYNAMICS STUDIES

Theoretical laser spot analysis

Concerning the evaluation of the beam quality, it is clear from the previous examples that a well-behaving beam has a low standard deviation and spatial correlation. This property has been verified studying the effects of beam charge in-homogeneities on the emittance[2].

The charge distribution extracted from the cathode has been modelled as a sine and cosine function having a frequency n and a charge intensity δ. The latter will be presented in details here. Figure 4 shows the matrix representing a non-uniform beam as the frequency n increases. The generic matrix element is represented by the following function:

$$\rho(i,j) = \rho_0(1+\delta\cos k_n i)(1+\delta\cos k_n j) \qquad (7)$$

where $k_n=2\pi n/r_p$ and r_p is the beam radius.

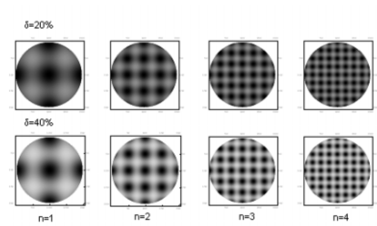

Figure 4: Matrix representation of Eq.7 showing non-uniform distribution versus n and for different δ ($\delta=40\%$ and $\delta=20\%$). In this case $\rho_0=1$, $r_p=100$..

Table 1 and Table 2 show the obtained spatial correlation for different frequency n and the standard deviation for different intensity of non uniformity δ.

Table 1: Λ for different frequency n of Eq. (7)

n	1	2	3	4
Λ	0.92	0.71	0.45	0.18

Table 2: σ for different charge intensity δ of Eq. (7)

δ	10%	20%	30%	40%
σ	0.08	0.14	0.18	0.21

Such distributions have been analyzed, concerning the emittance degradation with the Parmela code where the accelerator machine set up is the one used for the SPARC project [3]. The results are depicted in the Figure 5 where the emittance growth is represented as a function of the spatial autocorrelation Λ for different value of the standard deviation.

Real laser spot analysis

A program devoted to the calculation of spatial correlation has been built using the Mathematica software. Briefly the algorithm reads the image, coming directly from a camera acquisition, and it changes it in a matrix whose elements represent the intensity of the pixels. The bias is eliminated and the threshold is chosen making the mean of the pixels around the barycentre of the filtered distribution and lowering it of a percentage: in this way the beam boundary are established. Note that the spatial autocorrelation depends very little from the threshold.

Thus a good laser image, medium and bad obtained at SPARC have been analyzed and are shown in the ε-Λ plot of Figure 5.

It's worth noting the brilliance measured at SPARC with the good laser spot is better than that obtained with the bad laser spot [4].

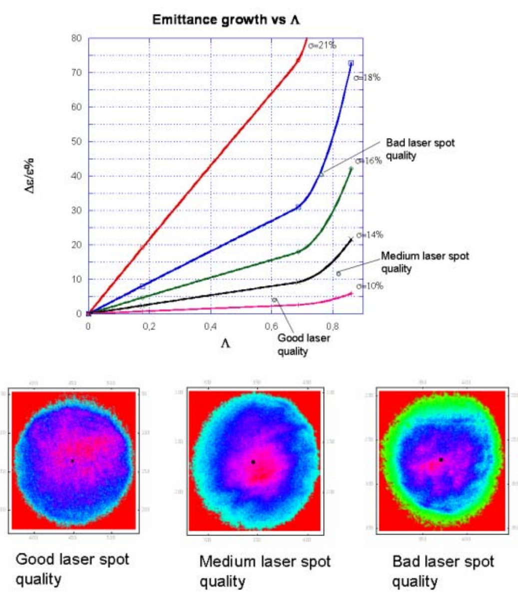

Figure 5: ε-Λ curve: emittance growth percent versus the spatial autocorrelation for different values of the standard deviation σ. In the plot the position of a good, medium, bad laser spot are reported..

Autocorrelation evolution along a photoinjector

As a conclusion it is reported in Figure 6 the evolution of the autocorrelation Λ along the SPARC photo-injector. A uniform beam evolution is first considered, transported with the Parmela code. In the waist the beam shows a wave breaking due to space charge non-linearities enhanced by the solenoid focusing force; thus the autocorrelation increases.

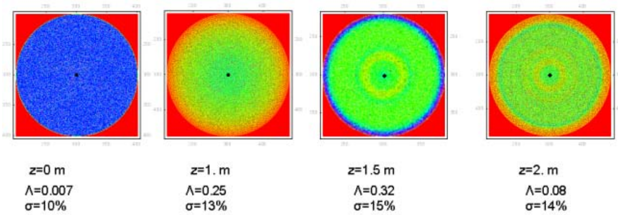

Figure 6: Evolution of a uniform laser beam along the photoinjector (z=1.5 m) is the waist.

Figure 7 shows a real beam behaviour along the photo-injector: the autocorrelation index demonstrates that non homogeneities don't spread along the photo-injector, on the contrary they appear again in the waist of the beam (solenoid focusing force) modified by space charge non linearities.

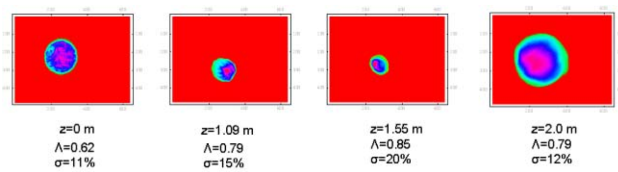

Figure 7: Evolution of a real laser beam along the photoinjector (z=1.5 m) is the waist.

CONCLUSIONS

The spatial autocorrelation index and the standard deviation describe the transverse quality of a laser beam. The latter describe the uniformity and the former how non uniformity is distributed. The knowledge of both allows to place the laser spot on the ε-Λ curve thus giving an idea of the corresponding emittance growth.

As a conclusion the evolution of the autocorrelation of a uniform laser spot and a real laser spot along the photo-injector demonstrates that non homogeneities don't spread along the photo-injector. On the contrary non homogeneities, modified by space charge, appear again in the waist of the beam due to solenoid focusing force.

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