BEAM TRANSVERSE SIZE EFFECTS ON THE TRANSITION RADIATION ENERGY SPECTRUM

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Abstract

A theoretical model for the transition radiation emission by a relativistic electron bunch is here presented. Such a model, based on an extension of the virtual quanta method to the case of high density charged beams, predicts the existence of beam transverse size effects on the short wavelength part of the transition radiation energy spectrum. The relevance of such effects to the transition radiation based beam diagnostics of an electron linear accelerator is discussed. The physical consistency of the proposed theoretical model for the transition radiation emission is demonstrated on the basis of the constraints imposed by the temporal causality and Huygens-Fresnel principles. Further arguments in favour of such a thesis, which concern the relativistic nature of the radiative mechanism, are discussed. A possible experiment, devoted to a crosscheck of the theoretical results in an electron linear accelerator, is also proposed.

INTRODUCTION

A relativistic charge passing through a dielectric interface generates an instantaneous and broad-band radiation emission, the so called transition radiation (TR) [1, 2]. In an electron linear accelerator [3], measurements of the beam transverse size and energy can be performed by imaging the Optical Transition Radiation (OTR) light spot at the radiator surface or by mapping the OTR angular distribution, respectively. OTR screens, typically made of a thin metallic coated dielectric substrate, can be suitably manufactured in order to respond as an ideal conductor in the visible optical region. Moreover, they can be suitably dimensioned so that possible diffractive cut-off due to the finite transverse dimension of the screen can be neglected even in case of ultra-relativistic electron beams. Given the above experimental conditions whose validity is also assumed in the following, no spectral alterations due to the dielectric and geometric features of the radiator are expected to be observed in the OTR spectrum. In particular, no beam-transverse-size dependent alterations of the OTR energy spectrum are expected to be observed. In fact, according to a well known theoretical model, the temporal incoherent part of the radiation energy spectrum by a N electron bunch simply reduces to the linear addition of the contributions from the N single electrons that are implicitly supposed to behave as isolated and individually radiating particles: any information about the reciprocal distances between the electrons and hence the distribution of the particle density in the transverse plane is indeed absent in the temporal incoherent part of the radiation energy spectrum. Possible effects of the beam finite transverse size on the OTR energy spectrum can be anyway reasonably expected to be observed. TR shares indeed a common wave nature with other electromagnetic radiative mechanisms such as the synchrotron radiation in an electron storage ring or the photon bremsstrahlung in a positron-electron collider whose beam-transverse-size dependent variations of Brilliance or Luminosity are commonly observed. Furthermore, kinematics of TR and photon bremsstrahlung by head-on collision of electron and positron beams are very similar. TR emission can be indeed view as the photon bremsstrahlung emission originated by a relativistic charge in collision with the charge induced conduction electron on the metallic screen at rest in the laboratory frame of reference. Therefore, the question if the beam finite transverse size may play a role in determining the temporal incoherent part of the TR energy spectrum can be reasonably raised. According to a recently proposed theoretical model, beam transverse size effects on the OTR energy spectrum by an a N electron beam can be predicted [4, 5]. Based on the virtual quanta method newly formulated in terms of the bunch particle density instead of the single individual electron, such a theoretical model foresees that a short wavelength diffractive cut-off is operated in the radiation energy spectrum by the finite transverse size of the electron beam [4, 5]. In particular, for fixed values of the beam energy and current, the number and the angular distribution of the transition photons emitted at a given wavelength - for instance, in the visible optical region - are expected to increase and broaden, respectively, with the decrease of the beam transverse size for fixed values of the beam energy and current [4, 5, 6, 7]. Several arguments in favour of the theoretical model describing such a possible phenomenon can be raised. In the present context, the attention will be mainly focused on the arguments dealing with the constraints imposed by the temporal causality and the Huygens-Fresnel principles.

TR ENERGY SPECTRUM

The case of N bunched electrons - in a rectilinear and uniform motion with a common velocity $\vec{w}$ - impinging at a normal angle of incidence upon an ideal conductor surface - placed on the plane $z = 0$ - will be here considered. On the radiator surface, a boundary condition constraints to a zero value the resultant of the virtual quanta and the charge induced electric fields with respect to the radiator surface. After the passage of the charge through the metallic surface, the charge induced conduction electrons relax to the equilibrium generating an electromagnetic field

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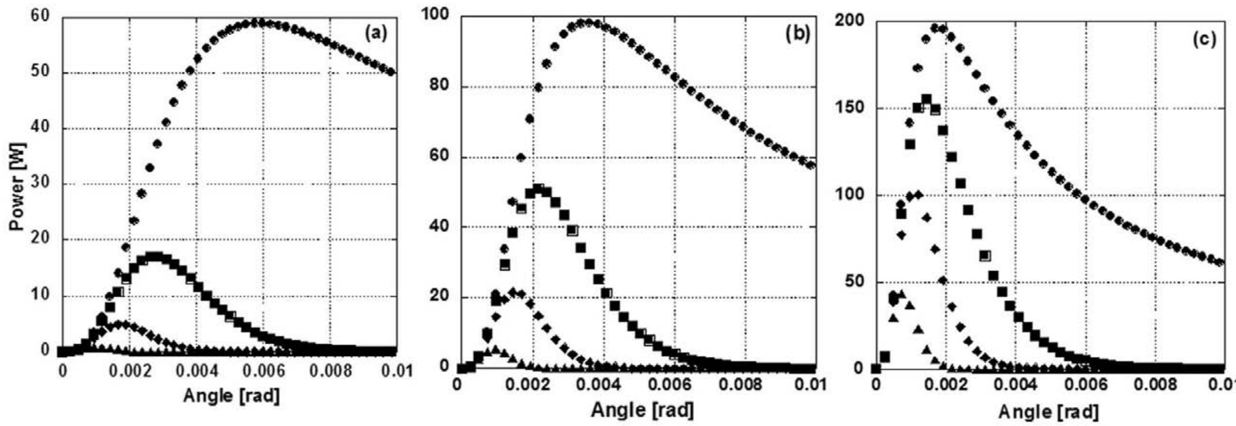

Figure 1: Polar angle distribution of the transition radiation power radiated in the wavelength band $0.4 - 0.7\ \mu m$ by a 1nC and $10\ ps$ long electron bunch with a cylindrical distribution ($\sigma_x = \sigma_y = \sigma$) for different values of the beam energy (E=150 (a), 250 (b), 500 (c) MeV). In each frame, the radiation power expected at each energy is calculated for different values of the beam transverse size (σ= 100 (▲), 50 (♦), 25 (■) and 0 (●) μm)

whose transverse component detaches from the radiator surface as a radiation field in order to maintain it equipotential. TR emission results thus from the coupling of the virtual quanta field and the current of the charge induced conduction electrons. In the framework of the Huygens-Fresnel principle [2], such an interaction can be described as the scattering of the virtual quanta field from a metallic screen. At a distance R from the radiator surface, TR field can be calculated on the basis of the Helmholtz-Kirchhoff integral theorem that, under the far field approximation, simply reduces to the Fourier transformation of the virtual quanta field with respect to the coordinates $\vec{\rho} = (x, y)$ of the radiator surface S:

$$E^{tr}_{x,y}(\vec{\kappa}, \omega) = \sum_{j=1}^{N} H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})\, e^{-i(\omega/w)z_{0j}} = \qquad (1)$$

$$= \frac{iek}{2Rw\pi^2} \sum_{j=1}^{N} e^{-i(\omega/w)z_{0j}} \int_S d\vec{\rho} d\vec{\tau} \frac{\tau_{x,y} e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2 + \alpha^2} e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}}.$$

where $\alpha = \frac{\omega}{w\gamma}$, $k = \omega/c$, $\vec{\kappa} = (k_x, k_y)$ and $[\vec{\rho}_{0j} = (x_{0j}, y_{0j}), z_{0j}]$ $j = 1, .., N$ are the electron coordinates. The Lorentz covariance of the formal expression of the TR field given in Eq.(1) can be also demonstrated [8]. In previous equation, the single electron field amplitude $H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})$ is a function of the electron Lorentz factor γ via the α factor, the electron transverse coordinates $\vec{\rho}_{0j}$ and the size and shape of the radiator surface S. Whether the radiator surface is finite ($S < \infty$) or infinite ($S = \infty$), $\vec{\rho}_{0j}$ is expected to play the same role in determining $H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})$. In particular, the circumstance that, in the case $S = \infty$, the $\vec{\rho}_{0j}$ dependent term may play the role of a relative phase factor of the single electron field amplitude while, in the case $S < \infty$, it may play - together with the γ-dependent term of the integrand - the role of an intrinsic constituent of $H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})$ is physically contradictory and must be formally excluded. In order that the same formal role played by $\vec{\rho}_{0j}$ in defining the single electron field amplitude $H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})$ be preserved independently of the particular value and shape of the radiator surface S, the single electron field amplitude $H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})$ will be in the following implicitly represented in terms of its own integral expression - see Eq.(1) - that can be considered as the integral definition of a special and, in general, unknown function. In conclusion, according to the formal expression of the TR field given in Eq.(1), the TR field is composed of the linear addition of N single electron field amplitudes $H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})$ - intrinsically dependent on the electron transverse coordinates $\vec{\rho}_{0j}$ - whose relative phase only depend on the electron longitudinal coordinate z_{0j}. The amplitude and relative phase structure of the TR field conforms to the constraints imposed by the temporal causality and Huygens-Fresnel principles [5]. The radiation field is indeed calculated under the hypothesis that the electrons of the bunch hit with a common velocity $\vec{w}$ the radiator surface at a normal angle of incidence. Therefore, if a temporal causality relation exists between the temporal sequence of the particle collision with the radiator and the temporal duration and shape of the resulting transition light pulse, the relative phase of the single electron amplitudes of the TR field must be only a function of the electron longitudinal coordinates z_{0j}. Concerning the TR wave nature and the intrinsic dependence of the single electron field amplitude on the electron transverse coordinates $\vec{\rho}_{0j}$, the following considerations can be done. The features of a TR wave front strongly depends on the field strength. It is well known indeed that the transverse extension of the harmonic component of the virtual quanta field by a single electron linearly increase with the Lorentz γ factor of the electron, which in turn is responsible for the single electron field amplitude, i.e., the field strength. In case of an electron beam, the TR field strength is expected to be also a function of the distribution of the electron transverse coordinates: the higher the transverse particle density the more intense the strength of the virtual quanta field that is scattered by the metallic screen. The electron transverse coordinates $\vec{\rho}_{0j}$ can be reasonably expected to play the same role as the Lorentz γ factor in determining the strength of the single electron TR field amplitude: the higher the electron transverse density the higher the TR field strength and the higher the Luminosity or Brilliance of the TR source. Moreover, similarly to what can be commonly observed in an electron storage ring where, for a fixed value of the beam energy, the angular distribution of the synchrotron radiation at a short wavelength mostly concentrates along the beam axis with respect to a longer wavelength by reason of a diffractive effect due to the finite transverse size of the electron beam, also in the TR case the shorter the wavelength with respect to beam transverse size the narrower the expected angular distribution of the TR intensity. On the basis of Eq.(1), the formula of the TR energy spectrum by an electron beam under the limit of a continuous distribution function of the electron coordinates reads:

$$\frac{d^2I}{d\Omega d\omega} = \frac{d^2I^{\sigma}}{d\Omega d\omega}[N + N(N-1)F(\omega)] \qquad (2)$$

$F(\omega)$ being the *longitudinal form factor* of the bunch and

$$\frac{d^2I^{\sigma}}{d\Omega d\omega} \propto \sum_{\mu=x,y}\left|\int_S d\vec{\rho}\int d\vec{\tau}\frac{\tau_\mu \rho_x(\tau_x)\rho_y(\tau_y)}{\tau^2+\alpha^2}e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}}\right|^2 \qquad (3)$$

where $\rho_x(\tau_x)$ and $\rho_y(\tau_y)$ are the Fourier transforms of the distribution function of the particle density along the x-axis and y-axis of the laboratory frame of reference. In the particular case of an infinite radiating surface ($S = \infty$) and a gaussian beam, Eq.(3) explicitly reads:

$$\frac{d^2I^{\sigma}}{d\Omega d\omega} = \frac{(e\beta)^2}{\pi^2 c}\frac{sin^2\theta e^{-k^2 sin^2\theta(\sigma_x^2 cos^2\phi + \sigma_y^2 sin^2\phi)}}{(1-\beta^2 cos^2\theta)^2}. \qquad (4)$$

EXPECTED EXPERIMENTAL RESULTS

For an $1\ nC$ and $10\ ps$ long electron beam, the angular distribution of the OTR intensity integrated in the wavelength region 0.4-$0.7\ \mu m$ has been calculated - on the basis of Eqs.(2,4) under condition $F(\omega) = 0$ - as a function of the beam transverse size for different value of the beam energy [7], see Fig(1). In order to observe the beam finite transverse size effects on the angular distribution of the OTR intensity, a standard diagnostic station equipped with an optical beam line composed of pass-band filters in the visible, an achromatic convergent lens and a detector (CCD camera and a photomultiplier/power-meter) is suitable. Goal of the experiment is to image - as a function of the beam transverse size - the OTR light spot at the radiator surface on the CCD camera by means of an achromatic lens and/or to map in the detector plane the angular distribution of the radiation intensity by placing the CCD camera in the focal plane of the achromatic convergent lens as well as to measure the integrated radiation intensity in the observed wavelength region by using a power-meter or a photomultiplier [6, 7]. A successful experimental check of the beam-transverse-size effect would open fruitful perspectives for the development of dedicated OTR based beam diagnostic techniques. In fact (1), even at a very short wavelength, information about both the beam energy and transverse size can be estimated by means of a spectroscopic analysis of the angular distribution of the TR intensity; (2) measurements of the beam energy and transverse size are not independent as long as the ratio of the beam transverse size to the observed wavelength is not negligible; (3) the shorter the beam transverse size, the higher the number of TR-photons emitted at a given wavelength and the wider the broadening of their angular distribution. Consequently, as the beam transverse size approaches the spatial resolutions of a given CCD camera, high precision measurements of the beam transverse size can be still performed by mapping the angular distribution of the OTR intensity instead of imaging the OTR light spot.

CONCLUSIONS

Beam transverse size effects are usually expected to affect the spectral features of a radiation source even at a very short wavelength as commonly observed, for instance, in a measurement of the synchrotron radiation Luminosity in an electron storage ring or the photon bremsstrahlung Brilliance in an electron-positron collider. In TR as well, beam transverse size effects can be predicted to be observed even at a short wavelength, in particular, in the visible optical region. The observation of such a phenomenon would open new perspectives in the electron beam accelerator diagnostics. The possibility to estimate both the beam energy and transverse size by measuring the angular distribution of the radiation intensity in the short wavelength region can be indeed envisaged as well as the possibility to perform beam transverse size measurements with a high spatial resolution taking advantage of the beam-transverse-size induced increase of the number of the TR photons and broadening of their angular distribution in the visible optical region.

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