A FORMULA FOR THE ELECTRON CLOUD MAP COEFFICIENT IN THE PRESENCE OF A MAGNETIC FIELD

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Abstract

The bunch-to-bunch evolution of the electron cloud density can be modeled using a cubic map. The map approach has been proved reliable for RHIC [1] and LHC [2]. The coefficients that parameterize the map may be obtained by fitting from time consuming numerical simulations. In this communication we derive a simple approximate formula for the linear coefficient in the electron cloud density map, along the lines laid in [3], in the presence of a dipole magnetic field, and compare the result to numerical simulations, for the LHC.

INTRODUCTION

The build-up of a quasi-stationary electron cloud through beam induced multipacting processes can be accurately modeled using sophisticated computer simulation codes like PEI , POSINST, and ECLOUD.

In [1] it was shown that the evolution of the electron cloud density from one bunch passage to the next can be described using a cubic map of the form:

$$\rho_{m+1} = a\rho_m + b\rho_m + c\rho_m \quad (1)$$

where ρ is the average electron cloud density between successive bunches, and the coefficients a, b and c can be extrapolated from simulations, and are functions of the beam parameters and of the beam pipe features. Simulations based on the above map are orders of magnitude faster than those based on particle-tracking codes. An analytic expression for the linear coefficient a in (1), valid for weak clouds, has been derived from first principles in [3], for the straight sections of RHIC. In this paper we obtain an analytic expression for a in the presence of a dipole magnetic field, with specific reference to a toy model of LHC.

We assume N_m electrons in the cloud, uniformly distributed across the (transverse) section of the beam pipe, sketched in Fig. 1, at the arrival of bunch m. We compute the average energy gain $\bar{E}_g$ of these electrons assumed initially at rest due to the passage of bunch-m, and the (average) energy-dependent wall-to-wall flight times in the strong magnetic field limit.

Next, following [3], we compute the total number N_{m+1} of electrons in the cloud at the arrival of bunch-$(m+1)$, by tracing appropriately the build up of the high-energy (back-scattering) and low-energy (secondary emission) electrons produced by successive collisions at the beam pipe wall. The ratio N_{m+1}/N_m gives the linear coefficient a, and the result is compared to numerical simulations obtained using ECLOUD[4], for the case of an LHC-like dipole.

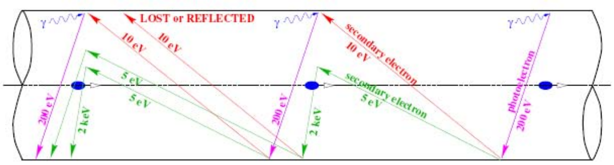

Figure 1: Schematic view of the evolution of an electron cloud between successive bunch passages. Courtesy F. Ruggiero

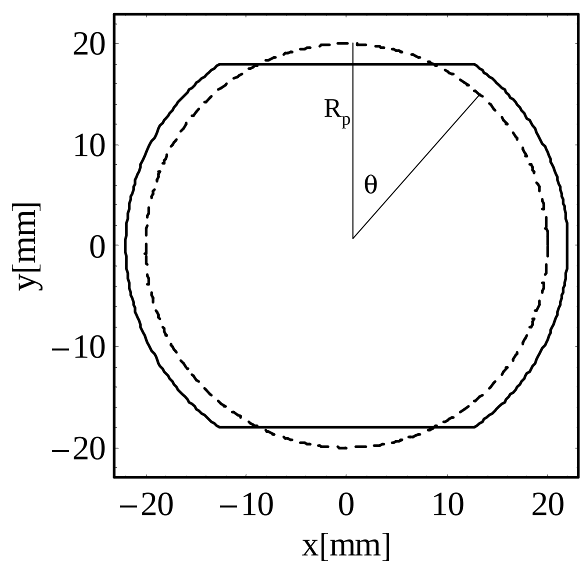

Figure 2: LHC beam pipe cross-section geometry, actual (solid line) and approximate (dashed line).

ELECTRON DYNAMICS

The actual cross section of the LHC beam-screen, is shown in Fig. 2, together with the (approximate) circular one used here. In the limit of a large y-directed magnetic field we may consider only the (transverse) vertical motion of the electrons (the cyclotron radius of the particle elical trajectories is very small compared to the transverse beam-pipe radius R_p). The same approximation is also allowed in electron-cloud simulation codes. The wall-to-wall flight-time for an electron with energy E originating at (R_p, θ) is accordingly

$$t_f(E,\theta) = \frac{2R_p \cos\theta}{\sqrt{2E/m_e}} \qquad (2)$$

where m_e is the electron mass, and θ is the polar angle defined in Fig.2.

The energy gained by an electron at (r,θ) after the passage of a bunch can be computed under the kick-approximation [5] as follows:

$$\Delta E(r,\theta) = 2m_e c^2 \frac{N_b^2 r_e^2 \cos^2\theta}{r^2}, \qquad (3)$$

where N_b is the number of electrons in each bunch, and r_e is the electron classical radius.

The average energy in a population of electrons uniformly distributed across the (transverse) section of the beam pipe, can be written as:

$$\bar{E}_g = \frac{1}{\pi R_p^2} \int_0^{2\pi} \int_{\sigma_r}^{R_p} \Delta E(r,\theta) r dr d\theta, \qquad (4)$$

where we neglect the contribution from electrons trapped inside the beam core, by setting the lower radial integration limit at the effective (transverse) beam radius σ_r.

The secondary emission yield (SEY) includes the contribution of both the *true secondary* electrons, and the *reflected* ones, denoted as $\delta_t(E)$ and $\delta_r(E)$ respectively; *rediffused* electrons are usually neglected.

We shall assume that the *reflected* electrons have exactly the same average energy $\bar{E}_g$ as the incident ones, whereas the true secondary electrons are emitted with an energy $E_s \ll \bar{E}_g$ (typically, $E_s \approx 5eV$ [6]).

The total secondary emission yield and its partial contributions from reflection and true secondary emission [6], [7] are shown in Fig.3.

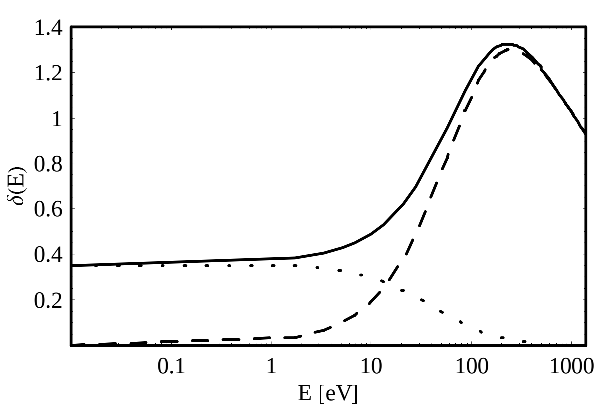

Figure 3: Secondary electron yield as a function of electron energy. The contributions of the secondary (dashed line), and backscattered electrons (dotted line) is also shown.

LINEAR MAP COEFFICIENT

To compute the linear term in (1) we follow Iriso and Peggs [3]. We denote by N_m the number of electrons uniformly distributed across the (transverse) section of the beam pipe just before bunch-m passes. After the passage of bunch-m, these electrons acquire an average energy $\bar{E}_g$. When these electrons hit the chamber wall, $N_m \delta_r$ *reflected* electrons with energy $\bar{E}_g$ emerge, and $N_m \delta_t$ *secondary* electrons, with energy $E_s \ll \bar{E}_g$ are created. Before the arrival of bunch-$(m+1)$, the reflected electrons travel across the beam pipe and undergo an average number n of collisions with the chamber wall given by:

$$n = \left\lfloor \frac{t_{bb}}{\bar{t}_f} \right\rfloor - 1, \qquad (5)$$

where t_{bb} is the time interval between successive bunches, and

$$\bar{t}_f(E) = \frac{4R_p}{\pi\sqrt{2E/m_e}} \qquad (6)$$

is the angular average of $t_f(E,\theta)$.

The total number of reflected electrons with energy $\bar{E}_g$ at the passage of bunch-$(m+1)$ will be accordingly:

$$N_{m+1}^{(ref)} = N_m \delta_r^n(\bar{E}_g). \qquad (7)$$

The secondary electrons originated upon each collision at the chamber wall, on the other hand, upon further collisions with the chamber wall, will produce *secondary* as well as *reflected* electrons all having the *same* low energy E_s.

The total number of secondary (low-energy) electrons at the passage of bunch-$(m+1)$ will be accordingly given by

$$N_{m+1}^{(sec)} = N_m \delta_t(\bar{E}_g) \sum_{p=1}^{n} \delta_r^{p-1}(\bar{E}_g) \delta_s^{k_p}(E_s), \qquad (8)$$

where $\delta_s = \delta_r + \delta_t$ and

$$k_p = \left\lfloor \frac{t_{bb} - p\bar{t}_f(\bar{E}_g)}{\bar{t}_f(E_s)} \right\rfloor, \qquad (9)$$

is the number of collisions undergone by the low-energy electrons originated after p wall-collisions of the high-energy population.

The *total* number of electrons at the passage of bunch-$(m+1)$ will be the sum of (7) and (8), viz.:

$$N_{m+1} = N_m \left(\delta_r^n(\bar{E}_g) + \delta_t(\bar{E}_g) \sum_{p=1}^{n} \delta_r^{p-1}(\bar{E}_g) \delta_s^{k_p}(E_s) \right), \qquad (10)$$

whereby the coefficient of the linear term in the map (1) can be written in closed form as follows:

$$a = \frac{N_{m+1}}{N_m} = \delta_r^n(\bar{E}_g) + \delta_t(\bar{E}_g)\delta_s^{\eta}(E_s) \cdot \cdot \frac{\delta_s^{n\eta}(E_s) - \delta_r^n(\bar{E}_g)}{\delta_s^{\eta}(E_s) - \delta_r(\bar{E}_g)}, \qquad (11)$$

where $\eta = \bar{t}_f(\bar{E}_g)/\bar{t}_f(E_s) = (E_s/E_g)^{1/2} \ll 1$. In fig. 4 the coefficient a is displayed as a function of the bunch spacing t_{bb} for different values of the maximum total SEY δ_{max}, using the machine parameters listed in Table I. For this same set of parameters, equation (11) is compared to ECLOUD based simulations in fig. 5.

Table 1: Parameters used for ECLOUD simulations.

parameter	units	value
beam particle energy	GeV	7000
bunch spacing	m	7.48
bunch length	m	0.075
number of bunches N_b	-	72
no. of particles per bunch	$N/10^{10}$	8 to 14
bending field B	T	8.4
length of bending magnet	m	1
vacuum screen half height	m	0.018
vacuum screen half width	m	0.022
circumference	m	27000
primary photo-emission yield	-	$7.98 \cdot 10^{-4}$
maximum SEY δ_{max}	-	1.3 to 1.7
energy for max. SEY E_{max}	eV	237.125
energy width for secondary e^-	eV	1.8
energy of secondary e^- E_s	eV	5

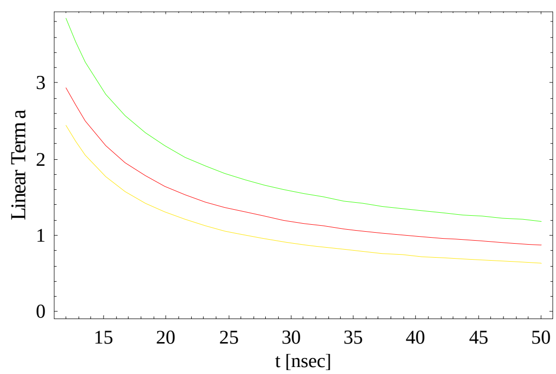

Figure 4: Approximate linear map coefficient a as a function of bunch spacing, for $\delta_{max} = 1.3$ (yellow), $\delta_{max} = 1.5$, (red) $\delta_{max} = 1.7$ (green).

CONCLUSIONS

An approximate formula has been derived for the linear coefficient in the map (1) describing the bunch-to-bunch evolution of the electron cloud density with a dipole magnetic field. The results are in acceptable agreement with numerical simulations obtained from ECLOUD, for an LHC-like dipole. Quick and dirty estimates of the safe regions in the machine parameter space where electron cloud. A more complicated (and more accurate) result is obtained by tracing separately electrons originating at different θ's, and averaging the final θ-dependent linear coefficient. This more general case will be discussed elsewhere.

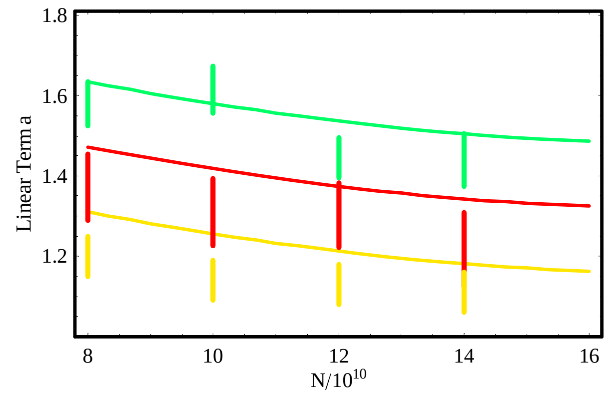

Figure 5: Comparison of the linear map coefficient a derived using ECLOUD simulations (bars) and using the analytic calculation (lines with the same color), as a function of the bunch population N for $\delta_{max} = 1.3$ (yellow), $\delta_{max} = 1.5$, (red) $\delta_{max} = 1.7$ (green).

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