DISTORTION OF CRABBED BUNCH DUE TO ELECTRON CLOUD WITH GLOBAL CRAB

L. Wang, Y. Cai and T. Raubenheimer, SLAC, CA 94025, USA

Abstract

In order to improve the luminosity, crab cavities have been installed in the KEKB HER and LER. Since there is only one crab cavity in each ring, the crab cavity generates a horizontally titled bunch oscillating around the whole ring. The electron cloud in LER (positron beam) may distort the crabbed bunch and cause the luminosity drop. This paper briefly estimates the distortion of positron bunch due to the electron cloud with global crab.

INTRODUCTION

The KEKB originally adopted the finite angle crossing scheme, which can cause beam instability and limit the luminosity. One crab cavity has been installed in each ring in order to make head-on beam-beam collision [1]. There are some difficulties to achieve high luminosity at high bunch current with crab cavity [2]. Many factors, such as beam size, horizontal offset and lifetime, have been studied. Since bunches are titled in the whole ring, the electron cloud in the positron ring (LER) may distort the bunch. This paper briefly investigates the possible distortion of positron bunch due to electron cloud. A detail study can be found in [3]. We use KEKB as an example and Table 1 shows the main parameters used in this paper.

Table I: Main parameters of the KEKB LER beam and electron cloud used in this paper

Voltage of crab cavity	V	1.4MV
Frequency of Crab cavity	f_{RF}	509MHz
Beam energy	E	3.5GeV
Circumference	C	3016m
Transverse tune	Q_x, Q_y	45.506/43.57
Phase advance between Crab cavity and IP	$\Delta\phi_{x,\ Crab\_IP}$	10.25×2π
Distance between Crab cavity and IP	S_{Crab_IP}	683.5m
Half crossing angle at IP	$\theta_{x,IP}$	11mrad
Betatron function at crab cavity	$\beta_{x,\ crab}$	45m
Betatron function at IP	β_x^*	1.5m
Horizontal emittance	ε_x	17.7nm
Vertical emittance	ε_y	0.266nm
Average beam size	σ_x, σ_y	0.42/ 0.06mm
Half bunch length	$\hat{z}$	14 mm
Bunch intensity	N	7.5×10^{10}
Electron cloud density	ρ_e	1.0×10^{12} m^{-3}
Pinch factor	f_p	10

CRABBED BUNCH

In order to generate a betatron-tune independent tiled bunch at interaction point (IP), the required phase advance between the crab cavity and IP satisfies

$$\Delta\phi_{x,crab\_IP} = n\pi + \pi/2 \qquad (1)$$

or, equivalently

$$\Delta\phi_{x,crab\_IP} = 2\pi Q_x - n\pi - \pi/2 \qquad (2)$$

The effect of crab cavity on the beam's orbit depends on the working phase of the cavity. When the crab cavity works at 90^o (bunch center receiving maximum kick), it generates a dipole kick to the bunch center and causes a closed orbit in the ring

$$X(s) \approx \frac{\sqrt{\beta_x(s)\beta_{x,crab}}}{2\sin(\pi Q_x)}\frac{eV}{E}\cos(\pi Q_x - |\phi_x(s) - \phi_{x,crab}|) \cdot \qquad (3)$$

When the cavity works at zero phase (bunch center receiving zero kick), which is the normal working phase, there is crabbed bunch along the whole ring. The half crossing angle of titled bunch at IP is given by

$$\theta_{x,IP} = \frac{\pi eVf_{RF}\sqrt{\beta_x^*\beta_{x,crab}}}{Ec} . \qquad (4)$$

The tilted angle of the crabbed bunch along the whole ring is

$$\theta_x(s) = \theta_{x,IP}\frac{\sqrt{\beta_x(s)/\beta_x^*}}{\sin(\pi Q_x)}\cos(\pi Q_x - |\phi_x(s) - \phi_{x,crab}|) \cdot \qquad (5)$$

Figure 1 shows the tilted angle in the LER ring. There is a small tiled angle at crab cavity because Q_x is close to half integer. From Eq. (5), a larger β_x^* can reduce the tilted angle of the crabbed bunch. Comparing Eq. (3) and (5), the closed orbit at 90^o and the tilted angle at 0^o differs by a constant number.

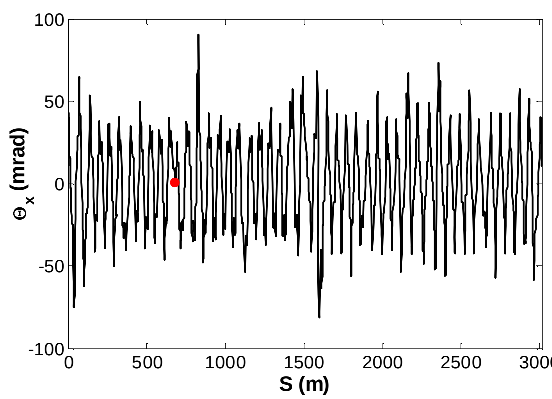

Figure 1: The titled angle of the crabbed bunch with realistic optics. It starts from the IP, and the crab cavity location is marked with the red dot.

DISTORION FORCE AND SHAPE

The wake field of electron cloud per unit length is

$$W(z) = P(z)\frac{\omega_e}{c}\frac{2\hat{z}}{N}\frac{\lambda_e}{\sigma_x(\sigma_y+\sigma_x)}e^{-\frac{\omega_e z}{2Qc}}\sin(\frac{\omega_e z}{c}) \qquad (6)$$

Where $\lambda_e = 2\pi\rho_e\sigma_x^2$, ρ_e is the electron density near bunch. The exponential decay of the wake is due to the nonlinear effect of the electron cloud. *P(z)* is the

*Work supported by the U.S. Department of Energy under contract DE-AC02-76SF00515
email address: wanglf@slac.stanford.edu

enhancement factor due to beam pinch effect and ω_e is the electron's bouncing frequency

$$\omega_e = c\left(\frac{Nr_e}{\hat{z}(\sigma_x+\sigma_y)\sigma_x}\right)^{1/2}. \tag{7}$$

At location *s*, a bunch has a tilted angle $\theta_x(s)$. The transverse kick received by a test charge at position *z* due to the preceding particles is [4]

$$\Delta x'(z) = \frac{Nr_e}{\gamma}\int_{-\infty}^{z} dz'\rho(z')x(z')W(z-z')\cdot \tag{8}$$

A uniform bunch is considered

$$\rho(z) = \begin{cases} \dfrac{1}{2\hat{z}} & \text{if } 2\hat{z} > z > 0 \\ 0 & otherwise \end{cases}. \tag{9}$$

Three wake models are studied:

(1)a constant wake function (without considering the effects of beam pinch and electron oscillation around the bunch)

$$W_I(z) = W_0 = \frac{\omega_e}{c}\frac{\hat{z}}{N}\frac{4\pi\rho_e\sigma_x}{(\sigma_y+\sigma_x)}; \tag{10}$$

(2)wake with considering the electron's oscillation

$$W_{II}(z) = W_0\sin\left(\frac{\omega_e z}{c}\right); \tag{11}$$

(3)Due to the beam pinch effect, the density of the electron cloud near the bunch increases from the bunch head to tail. To simplify the calculation, we assume *P(z)* linearly increase with *z* and there is a maximum factor of f_p at the bunch tail

$$P(z) = f_p\frac{z}{2\hat{z}}. \tag{12}$$

Then the wake function is

$$W_{III}(z) = W_0\frac{f_p z}{2\hat{z}}\sin\left(\frac{\omega_e z}{c}\right) \tag{13}$$

Using equations (8), (10-11) and (13), the kick force can be expressed as

$$\Delta x'(s,z) = \frac{Nr_e W_0\hat{z}\theta_x(s)}{\gamma} f_p F_z(z) \tag{14}$$

Where $F_s(s)$ is the distortion shape factor

$$F_z(z) = \begin{cases} \left(\dfrac{z}{2\hat{z}}\right)^2 & \text{(Mode I)} \\ -2\dfrac{z}{2\hat{z}}\alpha\cos(\omega_e z/c) + 2\alpha^2\sin(\omega_e z/c) & \text{(Mode II)} \\ -2\left(\dfrac{z}{2\hat{z}}\right)^2\alpha\cos(\omega_e z/c) + 4\dfrac{z}{2\hat{z}}\alpha^2\sin(\omega_e z/c) & \\ \quad + 4\alpha^3\cos(\omega_e z/c) - 4\alpha^3 & \text{(Mode III)} \end{cases}. \tag{15}$$

Where $\alpha = c/(2\omega_e\hat{z})$, which is the inverse of the electron oscillation number within one bunch length. f_p=*1* for model I and II. F_z has a maximum 1 at the bunch tail for the constant wake model. It represents the shape of the distorted bunch as late shown. Figure 2 shows distortion factor *Fz* for different wake models. The constant wake model causes a larger distortion than the model II. There is similar distortion shape for Model II and III, but note that the factor f_p, which is about 10 for KEKB, is not included in the plot. Therefore, there likely is a largest distortion when the beam pinch effect (Model III) is included. It is interesting that the distortion monotonously increases with *z* and bunch intensity *N* when beam is weak, for instance, α >0.5, (there is a smaller α for a higher intensity bunch) and the distorted bunch has a banana shape. But when beam becomes strong enough (α <0.2), the distortion starts to oscillate along the bunch and there is a snake shape of bunch. Therefore, the shape of the distorted bunch varies with bunch current and beam emittance. With the realistic optics, the calculated α ranges from 0.15 to 0.3 in most of the ring. Hence the distorted bunch may have a shape similar as the pink line in Figure 2. For LHC beam, α ≈0.6, hence the distorted bunch will have a banana shape.

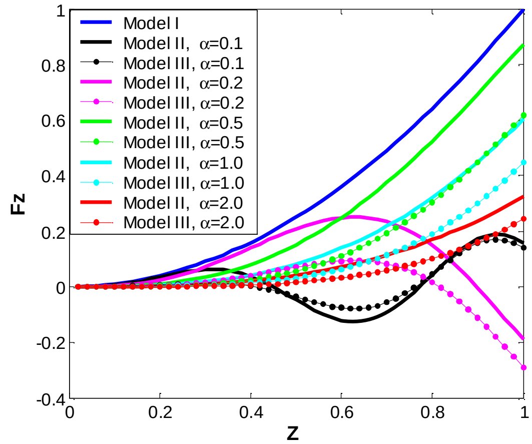

Figure 2: Distortion along the bunch for different wake models and beam strength factor α. The horizontal axis z is normalized by the bunch length.

BUNCH DISTORTION

In our first model, we assume the electron cloud is uniformly distributed along the ring. The COD due to the electron cloud is:

$$\Delta x(s,z) = \frac{\sqrt{\beta_x(s)}}{2\sin(\pi Q_x)}\int_0^C ds'\sqrt{\beta_x(s')}\cos(\pi Q_x - |\phi_x(s)-\phi_x(s')|)\Delta x'(s',z) \tag{16}$$

To simplify the calculations, a constant beta function is assumed, then the COD becomes

$$\Delta x(s,z) = A(s)F_s(s)F_z(z) \tag{17}$$

Where *A(s)* gives the amplitude of the distortion

$$A(s) = \frac{\beta_x(s)\theta_{x,IP}}{4\sin^2(\pi Q_x)\sqrt{\beta_x^*}}\frac{Nr_e W_0 C\hat{z}f_p}{\gamma}\sqrt{\overline{\beta}_x}, \tag{18}$$

The average A=0.57mm. $F_s(s)$ expresses the betatron phase effect of the electron cloud distribution in the ring

$$\begin{aligned} F_s(s) = {} & \frac{C-|s-s_{crab}|}{C}\cos(\phi_x(s)-\phi_{x,crab}) + \\ & \frac{|s-s_{crab}|}{C}\cos(2\pi Q_x - |\phi_x(s)-\phi_{x,crab}|) + \\ & \frac{2\overline{\beta}_x}{C}\sin(\pi Q_x)\cos(\pi Q_x - |\phi_x(s)-\phi_{x,crab}|) \end{aligned}. \tag{19}$$

From the above equation, we can get the betatron phase factor at the crab cavity as

$$F_s(s_{crab}) = 1 + \sin(2\pi Q_x)\overline{\beta}_x / C \,. \quad (20)$$

Since $\overline{\beta}_x << C$ for a large ring like KEKB, therefore, $F_s(s_{crab})$ is close to 1.

If $\Delta\phi_{x,\ Crab\_IP}$ different from Eq. (1) by $\Delta\phi$, the phase factor at IP is

$$F_s(s_{IP}) = \pm\left[\frac{C - S_{crab}}{C}\sin(\Delta\phi) + \frac{S_{crab}}{C}\sin(2\pi Q_x - \Delta\phi) + \frac{2\overline{\beta}_x}{C}\sin(\pi Q_x)\sin(\pi Q_x - \Delta\phi)\right] \quad (21)$$

(+, *if n is odd; - if n is even*)

For a larger ring, the third term is negligible. The 1st term is zero with a perfect condition Eq.(1). Therefore, the distortion at IP is proportional to $\sin(2\pi Q_x)$. $|F_s(s_{IP})|$ has a minimum when Q_x is close to half integer.

A Q_x close to half integer is chosen in KEKB in order to get a high luminosity

$$Q_x \approx m + 0.5 \,. \quad (22)$$

Substituting Eqs. (1) and (22) into Eqs.(20-21), then

$$F_s(s_{crab}) = 1 \quad \text{and} \quad F_s(s_{IP}) = \pm 2\overline{\beta}_x / C \quad (23)$$

The calculated $|F_s(s)|$ along the whole ring has a maximum about 1.0 at the crab cavity and minimum at the symmetrical position of the crab cavity $(S = C - S_{Crab\_IP})$. Very luckily, there is a small $F_s(s_{IP})$ of 0.0118 because of the Eqs. (1) and (22). If Eq.(1) is not satisfied ($\Delta\phi \neq 0$), the 1st term in Eq. (21) will be larger than the 3rd one if $\Delta\phi \geq 4^o$. For LHC, Q_x=64.28, F_s at IP is large since $\sin(2\pi Q_x) \approx 1$.

In the above estimation, a uniform electron cloud is assumed. Now let's assume the electron cloud locates at some specific locations, the *COD* at *IP* due to the electron cloud at these specific locations is

$$\Delta x(0,z) = \frac{\sqrt{\beta_x^*}\theta_{x,IP}}{2\sin^2(\pi Q_x)}\frac{N r_e W_0 \hat{z} f_p F(z)}{\gamma} \sum_i \sqrt{\beta_x(s_i)} L_i \cos(\pi Q_x - \phi_x(s_i))\cos(\pi Q_x - |\phi_x(s_i) - \phi_{x,crab}|) \quad (24)$$

Where L_i the length of each section. Using Eqs.(1) and (22), it can be simplified as

$$\Delta x(0,z) = \pm\frac{\sqrt{\beta_x^*}\theta_{x,IP}}{4}\frac{N r_e W_0 \hat{z} f_p F(z)}{\gamma}\sum_i \sqrt{\beta_x(s_i)} L_i \sin[2\phi_x(s_i)] \,. \quad (25)$$

Since there are many electron cloud sections in the ring, the average effects on *COD* at IP should be small due to the cancellation effect (sinusoidal term in Eq.24). Therefore, the distortion at IP is likely small due to the conditions Eqs.(1) and (22). In another words, there is a small $F_s(0)$ when Q_x is close to half integer, even with a non-uniform distributed electron cloud.

The COD with different electron distributions in the ring has been calculated. There is a similar overall distribution of COD along the ring for a uniformly and random distributed electron-cloud. The COD distribution does change a lot when the electron cloud is far from uniform or random distribution. But there is always a small COD at IP, which indicates that the COD at IP is always small no matter how the electron cloud is distributed along the ring. This agrees with Eq. (25).

Figure 3 shows the COD along the ring due to a uniform distributed electron cloud with the constant betatron function model and a realistic optics. The overall shape agrees well except some fluctuations due to the variation of the betatron function with realistic optics. The distortion depends on the location in the ring. The COD at IP is 2.4 μm, which is about 1.5% of the bam size at IP (σ_x*=0.163*mm*). The change of luminosity due to this offset is small [1]. Note that the COD with constant betatron function (red line in Fig. 3) is proportional to $F_s(s)$ (Eq.17).

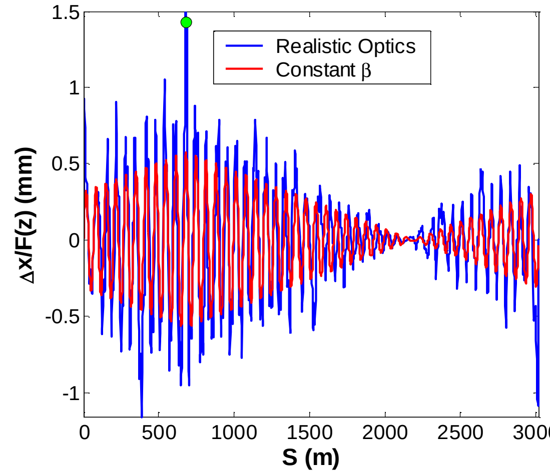

Figure 3: COD due to the electron cloud with a betatron function model and the realistic optics. The location of crab cavity is marked with green dot. f_p=10 is used.

SUMMARY

The shape of the distorted bunch due to electron cloud depends on the bunch line density and beam size. It can be a banana shape with a weak beam (α>0.5) or a snake shape with a strong beam (α~0.1).

The half integer betatron tune in KEKB and the specific phase advance between the crab cavity and IP causes a small distortion at IP due to the cancellation effect from many electron cloud sections. There is no clear distortion observed in the KEKB experiment [5], which probably can be explained by the negligible distortion according to the calculation here. However, if the betatron tune is not close to the half integer, like LHC case, a global crab correction may be more problematic.

ACKNOWLEDGEMENTS

We thank Dr. H. Fukuma for helpful discussions.

REFERENCES

[1] T. Abe, et. al., in the proceedings of PAC07, TUPAN045.
[2] Y. Funakoshi, et. al., in the proceedings of PAC07, THPAN037.
[3] L. Wang and T. Raubenheimer,SLAC-PUB-13242.
[4] A. W. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators(Wiley,New York, 1993).
[5] H. Ikeda, et. al., in the proceedings of PAC07, FRPMN035.