ULTRA LOW EMITTANCE LIGHT SOURCES*

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Abstract

This paper outlines the special issues for reaching sub-nm emittance in a storage ring. Effects of damping wigglers, intra-beam scattering and lifetime issues, dynamic aperture optimization, control of optics, and their interrelations are covered in some detail. The unique choices for the NSLS-II are given as one example.

WHAT'S KNOWN

The first dedicated third generation light sources were commissioned in the early 80s, i.e., they have been optimized for over 20 years. Basically:

- The horizontal emittance (isomagnetic lattice) is given by

$$\varepsilon_x\,[\text{nm·rad}] = 7.84\times10^3 \cdot \frac{(E[\text{GeV}])^2 F}{J_x N_b^3}$$

where N_b is the number of dipoles, $J_x + J_z = 3$, and $F \geq 1$. No dipole gradients => $J_x \sim 1$.

- Generalized Chasman-Green lattices: DBA, TBA, QBA, 7-BA [1].
- Effective emittance => chromatic cells.
- Increasing N_b reduces ε_x but also reduces the peak dispersion, which makes the chromatic correction less effective => "chromaticity wall" [2].
- Damping wigglers (DWs): damping rings and conversion of HEP accelerators [3-4].
- Mini-Gap Undulators (MGUs), Three-Pole-Wigglers (TPWs) inside the DBA [5].

WHAT'S NEW

The NSLS-II design is conservative, i.e., it is based on well known techniques, but the approach is also novel because it combines these in a unique way:

- Use of damping wigglers to reduce horizontal emittance and as high flux X-ray sources => achromatic cells and weak dipoles.
- Medium energy ring (3 GeV) with ~30 DBA cells.
- Vertical orbit stability requirements.
- Generalized higher order achromat.

GLOBAL OPTIMIZATION

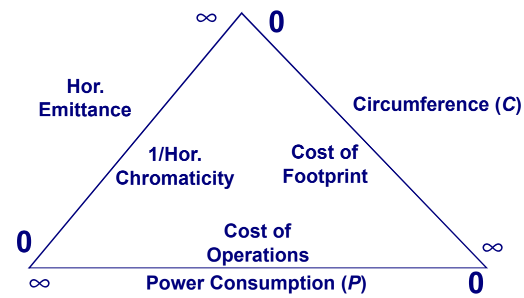
The (natural) horizontal emittance originates from the equilibrium:

$$\text{damping} \leftrightarrow \text{diffusion}$$

of three different processes: radiation damping, quantum fluctuations, and Intrabeam Scattering (IBS). One can show that (fundamental limit is IBS):

$$\varepsilon_x \sim \frac{1}{R^2 \cdot P}$$

where R is the bend radius, and P the radiated power. The design of a synchrotron light source is essentially a matter of balancing the conflicting trade-offs (optimized for Insertion Device (ID) beam lines) [6]:



The main lattice parameters are summarized in Tab. 1, where values particular for the NSLS-II have been highlighted.

Table 1: NSLS-II Lattice Parameters

Energy (E_0)	3 GeV
Circumference (C)	791.5 m
Beam Current (I_b)	500 mA
Bending Radius (R)	25.0 m
Dipole Energy Loss (U_0)	286.5 keV
Emittance: $(\varepsilon_x, \varepsilon_y)$ bare/w. 8 DWs	(2.1, 0.01)/(0.6,0.01) nm·rad
Momentum Compaction	0.00037
RMS Energy Spread: bare/w. 8 DW	0.05/0.1%
Working Point (ν_x, ν_y)	(32.4,16.3)
Chromaticity (ξ_x, ξ_y)	(-100, -42)
Peak Dispersion $(\hat{\eta}_x)$	0.45 m
Beta Function (β_x, β_y): long/short straight	(18, 3)/(3, 3) m

CHALLENGES

Given the design goals and approach, challenges related to non-linear dynamics issues are:

- Medium energy: control of Touschek lifetime and momentum aperture.
- 30 DBA cells: control of tune footprint.
- Control of impact of DWs and IDs -> include leading order nonlinear effects from DWs in the Dynamic Aperture (DA) optimizations.
- Optics requirements for IDs and top-up injection are contradictory: introduce alternating straights with high- and low horizontal beta functions <-> reduced symmetry (30 -> 15).

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- DBA: momentum dependence of optics functions => sufficient number of chromatic sextupole families.

There are also technical challenges:

- Weak dipoles: introduce TPWs (adjacent to the dipoles) => control of peak beta functions and horizontal dispersion.
- Vertical orbit stability: sub micron => pushing the state-of-the-arts [7-8].

INTRABEAM SCATTERING (IBS)

The governing equations for the horizontal emittance and momentum spread σ_δ are

$$\varepsilon_x = \varepsilon_x^{\mathrm{SR}} + \varepsilon_x^{\mathrm{IBS}} = \tau_x\left(E^{\mathrm{SR}}\right)\left\langle \mathcal{H}\cdot\left(D_\delta^{\mathrm{SR}}(R) + D_\delta^{\mathrm{IBS}}\right)\right\rangle,$$

$$\sigma_\delta^2 = \tau_\delta\left(E^{\mathrm{SR}}\right)\left(D_\delta^{\mathrm{SR}}(R) + D_\delta^{\mathrm{IBS}}\right)$$

where

$$\delta \equiv \frac{E-E_0}{E_0}, \quad \mathcal{H} \equiv \tilde{\eta}^{\mathrm{T}}\tilde{\eta}, \quad \bar{\eta} \equiv \begin{bmatrix}\eta_x \\ \eta'_x\end{bmatrix} \quad \tilde{\eta} \equiv A^{-1}\bar{\eta},$$

$$A^{-1} = \begin{bmatrix} 1/\sqrt{\beta_x} & 0 \\ \alpha_x/\sqrt{\beta_x} & \sqrt{\beta_x}\end{bmatrix}$$

and τ is the damping time, D the diffusion coefficient, E_0 the nominal energy, β the beta function, and η the dispersion. Increasing the bend radius reduces the contribution from the quantum fluctuations, whereas increasing the total radiated power (by damping wigglers) reduces the damping time. However, since the IBS is independent of the bend radius, it is the limiting factor, see Fig. 1.

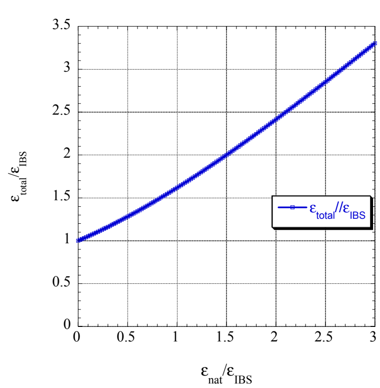

Figure 1: Relative IBS Contribution to the Horizontal Emittance.

TOUSCHEK LIFETIME

The Tousheck loss rate is the product of the cross secion (Møller scattering) and the phase-space density [9]

$$\frac{1}{\tau_{1/2}} = \frac{r_e^2 c_0 N_e}{8\pi\gamma^3\sigma_s}\frac{1}{C}\oint_C \frac{F\left[\left(\dfrac{\delta(s)}{\gamma\sigma_{x'}(s)}\right)^2\right]1}{\sigma_x(s)\sigma_z(s)\sigma_{x'}(s)\hat{\delta}(s)}ds$$

where

$$F(x) = \frac{1}{2}\int_0^1\left[\frac{2}{u} - \ln\left(\frac{1}{u}\right) - 2\right]e^{-x/u}du$$

and r_e is the classical electron radius, N_e the number of electrons per bunch, σ_s the rms bunch length, C the circumference, $\sigma_x(s), \sigma_y(s)$ the horizontal- and vertical rms beam size, $\sigma_{x'}(s)$ the horizontal rms beam divergence, and $\hat{\delta}_s(s)$ the momentum acceptance. Naively, one may think that reduction of emittance will lead to reduction of the life time. Actually, for a fixed energy acceptance, there is a threshold after which reduction of emittance causes exponential increase of the life time, see Fig. 2. The reason is that transverse momentum become so small that it is insufficient to kick particles outside the momentum acceptance [13]. However, due to its strong momentum aperture dependence, the latter is crucial parameter for current stability and the injection system, see Fig. 3.

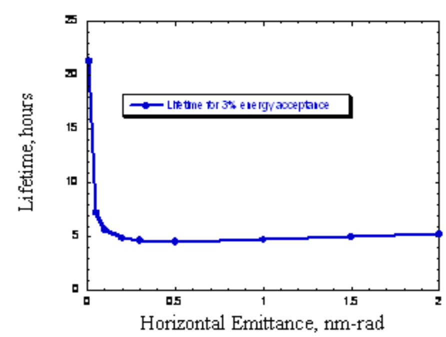

Figure 2: Touschek Lifetime vs. Horizontal Emittance.

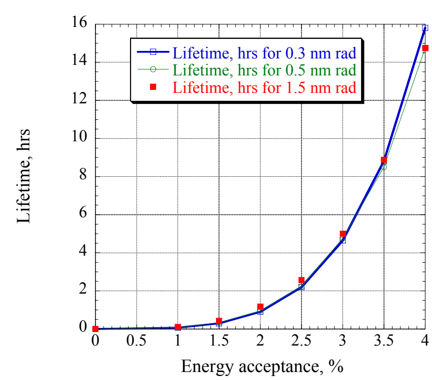

Figure 3: Touschek Lifetime vs. Momentum Aperture.

DAMPING WIGGLERS

The horizontal emittance essentially scales with the total radiated power; see Fig. 4, but the momentum spread is increased (due to the intrinsic dispersion)

$$\frac{\varepsilon_{\mathrm{w}}}{\varepsilon_0} \approx \frac{U_0}{U_0 + U_{\mathrm{w}}}, \quad \frac{\delta_{\mathrm{w}}}{\delta_0} = \sqrt{\frac{1 + \dfrac{8}{3\pi}\dfrac{B_{\mathrm{w}}}{B_0}\dfrac{U_{\mathrm{w}}}{U_0}}{1 + \dfrac{U_{\mathrm{w}}}{U_0}}}$$

Therefore, care is required to optimize brightness. In particular, the peak field should not be too strong. Clearly, the damping wigglers can be made more effective by increasing the bend radius, see Fig. 5.

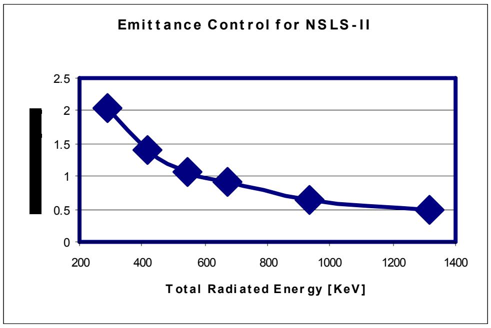

Figure 4: Emittance Reduction for: 0, 1, 2, 3, 5, and 8 DWs (with 1.8 T peak field and 7 m length).

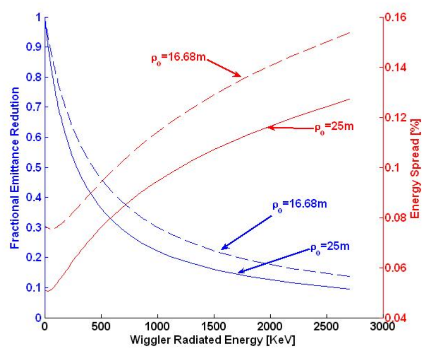

Figure 5: Emittance Reduction and Momentum Spread Increase for Two Different Values of the Bend Radius.

OPTICS DESIGN GUIDELINES

The traditional approach, i.e., to first design the linear optics and then attempt to control the DA is inadequate for high performance lattices; e.g. [10-11]. For a streamlined approach, the nonlinear effects must be considered from the start [12]. In particular, the following guidelines have been provided (the numbers have evolved with time) [13]:

- horizontal chromaticity per cell, $\xi_x \le 3.5$,
- horizontal peak dispersion $0.3\,\text{m} \le \eta_x \le 0.5$,

and

	Hor and Ver Dynamic Acceptance [mm·mrad]	Hor DA [mm]	δ [%]
Bare Lattice (2.5 D.O.F.)	~25	±20	±2.5
"Real" Lattice (3 D.O.F.)	~20	±15	±2.5

The DBAs have ~6 constraints:

- linear achromat ($\eta_x = \eta'_x = 0$ at the entrance),
- small emittance ($\min(\mathcal{H}) \Rightarrow (\alpha_x, \beta_x)$ fixed at the entrance),
- and symmetric ($\alpha_{x,y} = 0$ at the center).

Similarly, the long- and short matching sections have 10 constraints:

- symmetric ($\alpha_{x,y} = 0$ at the center),
- $\beta_{x,y} = 0$ at the center,
- and the cell tune $\nu_{x,y}^{\text{cell}}$.

On the other hand, the lattice has only 8 quadrupole families -> the linear optics design is a nontrivial task.

GENERALIZED HIGHER ORDER ACHROMAT

While the "chromaticity wall" has been avoided, the lattice is a strongly focusing, medium energy lattice. And, due to the large number of cells, the contribution to amplitude dependent tune shift and residual nonlinear chromaticity per cell is tighter than for existing compact lattices. To control the nonlinear effects from the sextupoles a generalized higher order achromat has been implemented:

- Introduce two chromatic sextupole families and choose the cell tune for N super cells $\mathcal{M}$ such that

$$\mathcal{M} = \mathcal{M}_1 \mathcal{M}_2 \ldots \mathcal{M}_N\,, \quad \mathcal{M}_k = \mathcal{A}^{-1} e^{:h:} \mathcal{R}\mathcal{A}, \quad \mathcal{R}^N = \begin{bmatrix} I & 0 \\ 0 & \pm I \end{bmatrix}$$

  In particular, so that resonances up to 4th order

$$n_x \nu_x + n_y \nu_y = n, \quad |n_x| + |n_y| \le 4$$

  are cancelled (by symmetry), see Fig. 6.
- Control the residual amplitude dependent tune shift and free up the choice of working point by adding geometric sextupoles [12].
- Control the residual nonlinear chromaticity by adding chromatic multipoles as needed.
- Optimize dynamic- and momentum aperture (from tracking) by joint minimization of the driving terms and variation of the cell tune.

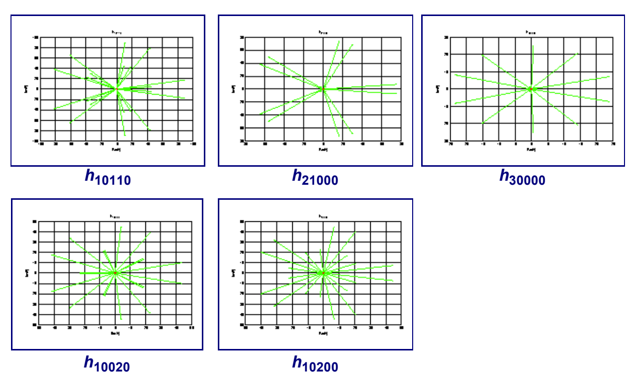

Figure 6: A 5-Cell Second Order Achromat (with 2 chromatic families).

LINEAR OPTICS AND DA OPTIMIZATION

The optics for a half cell is shown in Fig. 6. The dynamic- and momentum aperture are optimized by a joint optimization of the driving terms and the cell tune:

- An 11×11 grid of working points is obtained that meets the optics requirements.
- For each working point, the driving terms are minimized and a weighted average of the normalized dynamic- and momentum aperture ($DA/\sqrt{\beta_x \beta_y}$) is computed by tracking, see Fig. 7.
- A suitable working point is then selected and analyzed further. To actually achieve the predicted Touschek life time, it is crucial that leading order resonances are not being crossed by the damped betatron oscillations after a Touschek event, see Figs. 8-10.

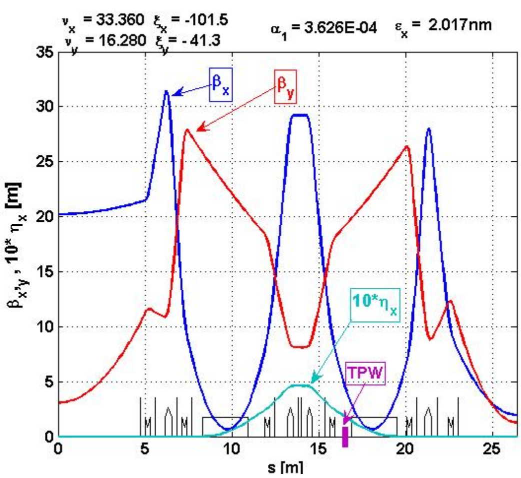

Figure 7: Optics Functions for a NSLS-II Half Cell.

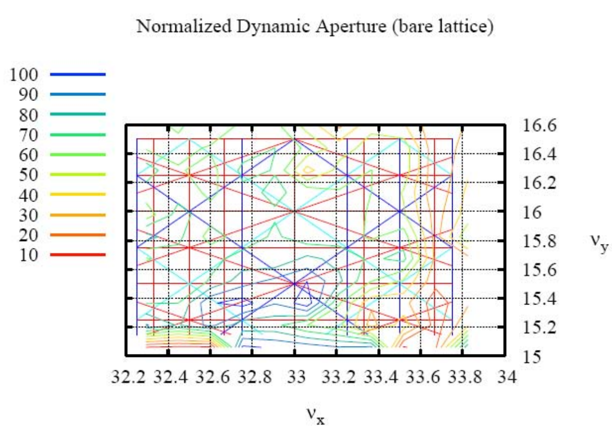

Figure 8: Tune Scan of Normalized DA ($DA/\sqrt{\beta_x \beta_y}$).

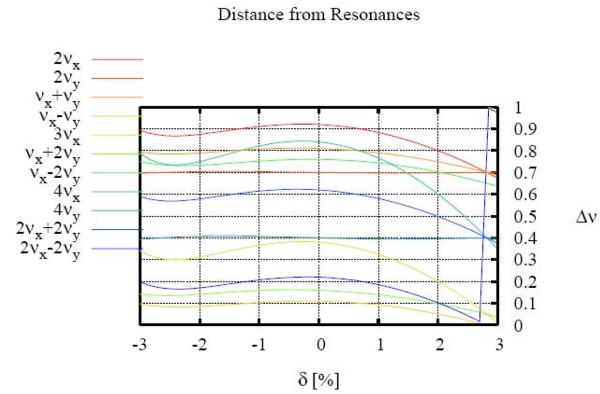

Figure 9: Avoidance of Resonance Crossing (first and second order sextupolar) due to a Momentum Deviations.

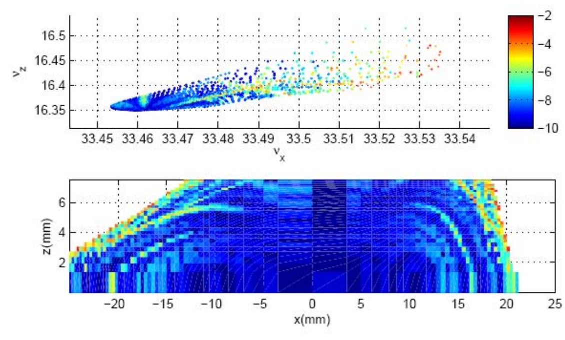

Figure 10: Frequency Map: x-y (without DWs).

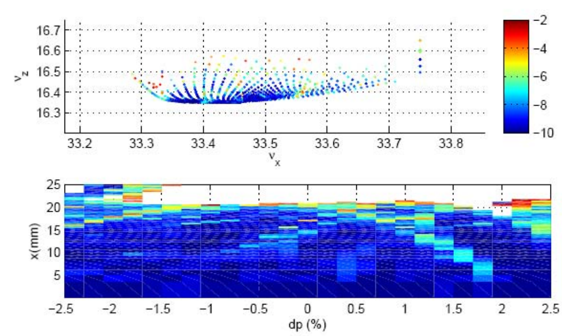

Figure 11: Frequency Map: δ-x (without DWs).

OPTICS TOLERANCES

General guidelines are obtained by evaluating the sensitivity of the DA on gradient and mis-alignment errors, see Figs. 12-13. The result is summarized by Tab. 2 which provides estimates for engineering tolerances, to what level the optics needs to be controlled when IDs are included, etc. Of course, for specific guidelines, these must be validated by detailed tracking studies.

Table 2: Optics Tolerances.

Parameter	Tolerance
$\Delta b_2/b_2$	$\sim 5\times10^{-4}$
$(\Delta\beta_{x,y}/\beta_{x,y})_{\mathrm{rms}}$	$\sim(2,3)\%$
$(\Delta\nu_{x,y})_{\mathrm{rms}}$	$\sim(3,2)\times10^{-3}$
$(\Delta x,\Delta y)_{\mathrm{rms}}$	$\sim(50,50)\ \mu\mathrm{m}$

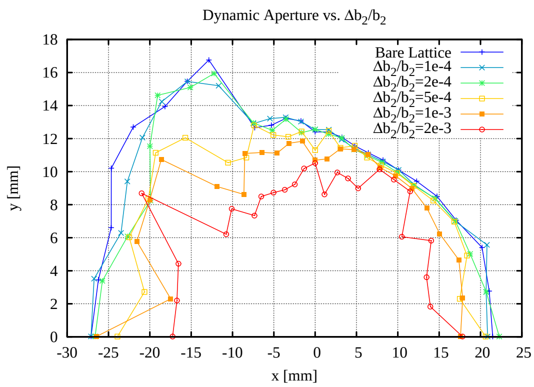

Figure 12: Sensitivity of DA on Random Gradient Errors.

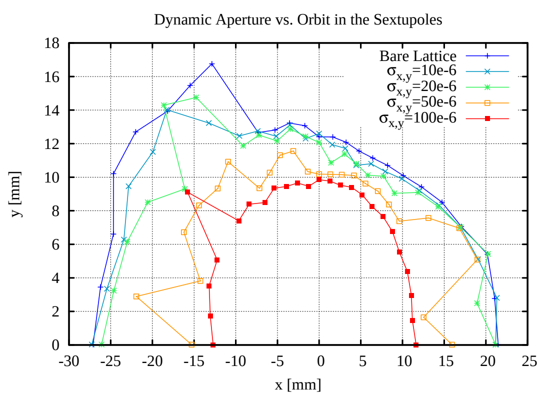

Figure 13: Sensitivity of DA on Sextupole Misalignments.

IMPACT OF INSERTION DEVICES

The averaged Hamiltonian is (planar device)

$$\langle H\rangle_\lambda = \frac{p_x^2+p_y^2}{2(1+\delta)} + \left(\frac{B_{\mathrm{w}}}{B\rho}\right)^2 \frac{\cosh(k_y y)}{4k_z^2(1+\delta)} - \delta + O(p_{x,y})^4,$$

$$k_y = k_z = \frac{2\pi}{\lambda_{\mathrm{w}}}$$

To leading order one obtains

$$\Delta\nu_y = \frac{1}{8\pi}\left(\frac{B_{\mathrm{w}}}{B\rho}\right)^2 \langle\beta_y\rangle L_w, \quad \frac{\partial\nu_y}{\partial J_y} = \frac{\pi}{4}\left(\frac{B_{\mathrm{w}}}{B\rho}\right)^2 \frac{\langle\beta_y^2\rangle L_w}{\lambda_{\mathrm{w}}^2}$$

In particular, the impact is large for medium energy rings, and IDs with short period length. Also, since

$$\beta(s) = \beta_0\left[1+\left(\frac{s}{\beta_0}\right)^2\right]$$

optimum beta for minimum impact depends on the effect:

- stay clear $\beta_0 = L/2$,
- linear optics $\beta_0 = L/2\sqrt[4]{3}$,
- nonlinear dynamics $\beta_0 = L/2\sqrt[4]{5}$.

CONCLUSIONS

- The "chromaticity wall" has been avoided by using damping wigglers. Furthermore, these turned out to be useful high-flux X-ray sources.
- The emittance can be reduced as the facility evolves.
- The nonlinear effects are taken into account for the optics design. In particular, by providing guidelines for chromaticity per cell and peak dispersion.
- The dynamic- and momentum aperture are improved by implementing a generalized higher order achromat. It is controlled by a joint optimization of the driving terms and working point.
- The ultimate low-emittance limit can be reached by this approach. It is a matter of power consumption and circumference.

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