LASER – BEAM INTERACTION AND CALCULATION OF THE SLICED BUNCH RADIATION SPECTRA FOR THE SLS FEMTO BEAM LINE

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Abstract

The FEMTO beam line at the Swiss Light Source (SLS) produces short X-ray pulses through energy modulation of the electron bunch by a fs-laser. An analytical model describes the laser-beam interaction, the propagation of the electrons leading to a temporal modulation of electron density, and the spectra of radiation emitted in a dipole magnet. A Fourier transformation technique is used to determine the coherent part of the radiation in the THz-range. For a Gaussian bunch the results are compared to tracking simulations.

INTRODUCTION

The FEMTO facility [1, 2] in the SLS storage ring produces 140 fs (FWHM) pulses of X-rays in the energy range of 3–18 keV. Inside a wiggler magnet, a laser pulse of 50 fs (FWHM in power) resonantly modulates electrons of a correspondingly thin slice of the bunch in energy. A magnetic chicane bracketing the wiggler translates the energy modulation in a horizontal separation, so synchrotron radiation from thus created “satellite bunches”, emitted in a subsequent in-vacuum undulator, can be separated from the “core” bunch radiation by a suitable set of apertures in order to extract the short X-ray pulse. The temporal modulation of the electron density in the bunch leads to emission of coherent radiation in the THz-range, which is extracted from a subsequent bending magnet and used for diagnostics of the slicing process [3,4].

ENERGY MODULATION

The energy modulation within an electron bunch propagating in a wiggler magnet at the presence of laser field is determined by the oscillating single electron interaction with the laser field. We assume that the electron motion in a planar wiggler is conditioned by the vertical magnetic field in the mid plane $B_y = B_0\cos(k_w z)$, where $k_w = 2\pi/\lambda_w$ and λ_w is the wiggler period. We are interested in the average effect of the laser-electron energy exchange per wiggler period. The energy gain (or loss) of an electron with longitudinal position s with respect to the bunch centre ($s = 0$) in a horizontally polarized laser field E_x propagating with the electron bunch along the planar wiggler axis z is given by

$$\frac{d\gamma(z,s)}{dz} = \frac{e}{m_0 c^2} E_x(z,s)\beta_x \ , \quad (1)$$

where γ is the relativistic factor, e, m_0 are the electron charge and mass, c is the velocity of light, and $\beta_x = v_x/c$ with v_x the horizontal velocity of the electron. The electron transverse and longitudinal velocities in a wiggler are given by [5]

$$\beta_x = -\frac{K}{\gamma}\sin(k_w z),\ \overline{\beta}_z = 1 - \left(1 + K^2/2\right)/(2\gamma^2)\ , \quad (2)$$

with $K = eB_0/k_w m_0 c$ the wiggler deflection parameter, B_0 the peak magnetic field, $\overline{\beta}_z$ the electron average normalized velocity along the wiggler and $z = c\overline{\beta}_z t$.

Assuming the waist point of the propagating laser field in the middle of the wiggler magnet and taking into account the laser field amplitude and phase modulation along the wiggler (Guoy phase), the electric field experienced by the electrons of the bunch is given by [6]

$$E_x(z,s) = \frac{E_0\sin\left[k_L(z-ct) + k_L s + \psi\right]}{\sqrt{1 + z^2/z_R^2}} \cdot \exp\left(-\frac{u^2}{2\sigma_L^2}\right)\ , \quad (3)$$

where k_L is the laser wave number, $z_R = k_L a_0^2/2$ the Rayleigh length, a_0 the laser waist size, $\psi = \psi_0 - \arctan(z/z_R)$, ψ_0 is the phase of the wave at the interaction origin with respect to the bunch centre, $u = z - ct + s - s_p$, s_p is the distance between the bunch center and the maximum of the laser pulse at the wiggler entrance, σ_L is the laser pulse rms length.

We assume that the laser wavelength and electron design energy satisfy the resonance condition $2\gamma_0 k_w = k_L(1 + K^2/2)$, i.e. the electron is retarding by one laser wavelength in one wiggler period so that $k_L\gamma^2(z - ct) = -k_w\gamma_0^2 ct$. For small energy spread ($\Delta\gamma/\gamma_0 << 1$) in the electron beam and $k_w << k_L$ we get $k_L(z - ct) = -k_w z$, and eq. (1) is read as

$$\frac{d\gamma}{dz} = A\frac{\sin(k_W z)\sin(k_L s - k_W z + \psi)}{\sqrt{1 + z^2/z_R^2}} \cdot \exp\left(-\frac{u^2}{2\sigma_L^2}\right) . \quad (4)$$

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For a relatively short wiggler we can neglect the term z^2/z_R^2. In this case from eq. (4) for the relative energy modulation one can obtain:

$$\delta_L(s) := \frac{\Delta\gamma}{\gamma_0}(s) = P(s)\cos(k_L s + \psi) \ , \quad (5)$$

where $P(s)$ is the energy modulation envelope given by

$$P(s) = P_0\left[erf\left(\frac{s-s_p}{\sqrt{2}\sigma_L}\right) - erf\left(\frac{s-s_p}{\sqrt{2}\sigma_L} - \frac{N_W}{N_L}\right)\right]. \quad (6)$$

Here N_W is the number of wiggler periods, $N_L = \sigma_L/\lambda_L$ is the number of optical cycles in the laser pulse, $erf(x)$ is the error function, s_p is the location of maximum laser field at the beginning of interaction, and

$$P_0 = \sqrt{\pi}\frac{eE_0 K N_L \lambda_W}{4 m_0 c^2 \gamma_0^2}. \quad (7)$$

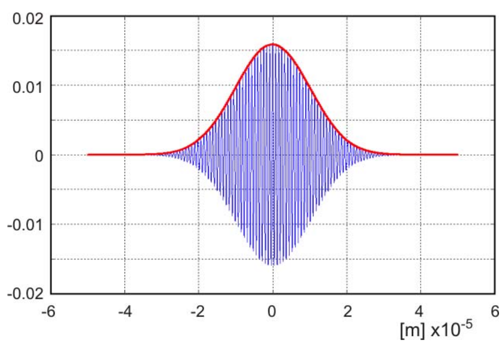

Figure 1: The electron relative energy modulation and the envelope P(s) after laser-beam interaction

From eq.(6), location and amplitude of the maximum energy modulation can be obtained:

$$s_{\max} - s_p = \sigma_L \frac{N_W}{\sqrt{2N_L}} \Rightarrow P_{\max} = 2P_0 erf\left(\frac{N_W}{2N_L}\right). \quad (8)$$

It follows, that the maximum of the energy modulation coincides with the bunch centre ($s = 0$) if the laser pulse maximum is delayed by s_p/c with respect to the electron bunch centre, where $s_p = -\sigma_L N_W / 2N_L$.

COHERENT THZ SPECTRA

The coherent part of the synchrotron radiation in a bending magnet is given by

$$\mathrm{P}(k) \cong N_b^2 P_e(k) F^2(k), \quad (9)$$

where N_b is the number of particles in the bunch, $P_e(k)$ is the radiation spectral intensity of a single electron and $F(k)$ is the bunch form factor given through the Fourier transformation of the normalized longitudinal electron distribution as $F(k) = \int \rho(s)\exp(iks)ds$.

Only electrons from a rather thin slice in the center of the bunch will be modulated in energy by interaction with the laser pulse. In order to obtain a smooth model for further multi particle tracking and analytical modeling, we may superimpose three groups of particles:

a) The electrons of the central slice, which are partially modulated in energy

b) A copy of these electrons, but with opposite charge and no modulation (i.e. only with the natural rms uncorrelated energy spread σ_E) here called “positrons”

c) The static core bunch of rms length σ_z and energy spread σ_E .

Obviously, the spectra of the three particle distributions can be superimposed: $F(k) = F_{cb}(k) + F_e(k) - F_p(k)$.

Since the core bunch has Gaussian distributions, the spectrum is simply given by $F_{cb}(k) = \exp\left(-k^2\sigma_z^2/2\right)$. For the SLS storage ring, the bunch rms length is about 1 cm, and the core beam signal is in the order of $F_{cb}(k) \approx \exp(-25)$, so the core beam does not contribute to the coherent THz spectrum, i.e. $F(k) \cong F_e(k) - F_p(k)$.

Since the interaction length $L_s \cong 2\sqrt{3}\sigma_L << \sigma_z$, the form factor of the “positron” bunch at the bending magnet is given by

$$F_p(k) = \frac{F_0(k)}{\sqrt{2\pi}\sigma_z}\int_{-L_s}^{L_s}\cos(k m_{55} s)ds \ , \quad (10)$$

where $F_0 = \exp(-k^2 m_{56}^2 \sigma_E^2/2)$, m_{55}, m_{56} are the elements of the transport matrix between the exit of the wiggler and the point of interest, i.e. the magnet.

To obtain the form factor for energy modulated electron bunch we first have to obtain the longitudinal distribution of the bunch after passing the FEMTO line [7]:

$$\Lambda(z_f) = \frac{N_{cb}}{2\pi m_{56}\sigma_z\sigma_E}\int_{-L_s}^{L_s}\exp\left(-\frac{\left(s - m_{55}s' - m_{56}\delta_L(s')\right)^2}{2\left(m_{56}\sigma_E\right)^2}\right)ds' ,(11)$$

where $\delta_L(s)$ is the energy modulation of the electrons given by eq. (5) . Assuming optimum matching between electron bunch and laser pulse according to eq. (8) makes $P(s)$ an even function. Thus the final Fourier transformation of the modulated electron bunch is

$$F_e(k) = \frac{F_0(k)}{\sqrt{2\pi}\sigma_z}\int_{-L_s}^{L_s} J_0\left[k m_{56} P(s)\right]\cos(k m_{55} s)ds \ , \quad (12)$$

where $J_0[\xi]$ is Bessel's function of zero order. The total Fourier transform of the modulated electron and the corresponding "positron" bunches is given by

$$F(k)=\frac{F_0(k)}{\sqrt{2\pi}\sigma_z}\left|\int_{-L_s}^{L_s}\{J_0[km_{56}P(s)]-1\}\cos(km_{55}s)ds\right|. \quad (13)$$

The fast oscillating term $\cos(k_L s)$, which should be included according to eqs. (5,11), has not been taken into account in order to derive eq. (13), but the difference is marginal. Fig. 2. shows the resulting spectra when the fast oscillating term is included (red line) and when it is neglected (blue line):

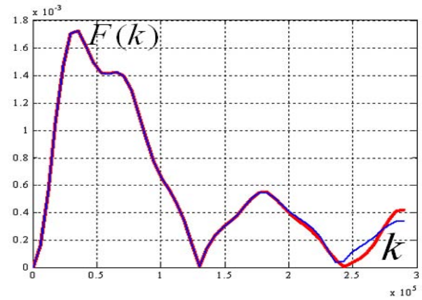

Figure 2: The radiation spectrum.

The function $P(s)$ may be approximated by a rectangular envelope of width $l_p=\sqrt{3}\sigma_P$, where σ_P is the rms length of $P(s)$ calculated as

$$\sigma_P=\sigma_L\sqrt{1+\left(N_W^2/6N_L^2\right)}. \quad (14)$$

Then for the height of the rectangular impulse we get

$$P_R=\frac{\left(N_W/\sqrt{2}N_L\right)}{\sqrt{3+\left(N_W^2/2N_L^2\right)}}P_0. \quad (15)$$

In this approximation, the form factor is given by

$$|F(k)|=\sqrt{\frac{6}{\pi}}\frac{\sigma_p}{\sigma_z}F_0(k)[1-J_0(km_{56}P_R)]\left|\frac{\sin\xi}{\xi}\right| \quad (16)$$

with $\xi=km_{55}l_p$. The peak of spectral intensity occurs at the characteristic frequency $k_{\max}$ that depends on the maximum laser induced energy spread $P_{\max}$ given by (8). Fig.3 presents the dependence of the characteristic frequency on P_0.

For small P_0 the function is well approximated by the Gaussian distribution, for large P_0 the characteristic frequency has hyperbolic dependence given by $m_{56}k_{\max}=\nu_{12}/P_0$, where ν_{12} is the second zero of the first order Bessel function.

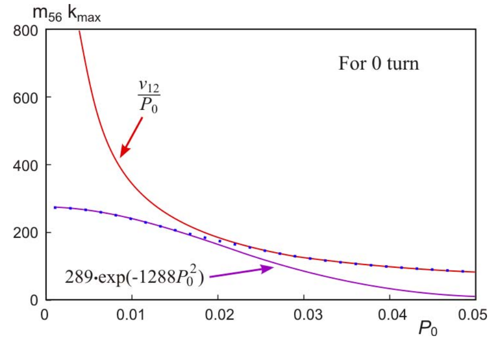

Figure 3: The characteristic frequency versus P_0.

The analytical result for the coherent spectra was compared with spectra calculated from particle tracking. We generated a group of particles with energy modulation at the exit of the wiggler, tracked them to the radiation point and obtained the radiation spectrum. The comparison is shown in Fig. 4.

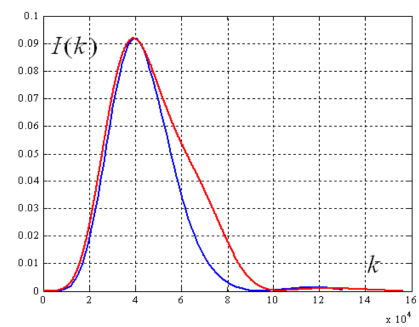

Figure 4: The coherent spectrum: analytical calculation (blue line) and particle tracking simulation (red line).

SUMMARY

Analytical expressions have been derived for the dependence of an electron's energy change on its longitudinal position in the laser field, and for the envelope of the energy modulation over the bunch.

Using these formulae we obtained the relation how the wave number at the maximum of the coherent THz-spectrum depends on the maximum energy modulation in laser-beam interaction.

REFERENCES

[1] A.Streun et al., Proc. EPAC-2006, Edinburgh, p. 3427.

[2] P. Beaud et al., Phys. Rev. Lett. 99, (2007) 174801/1-4.

[3] V.Schlott et al., Proc. EPAC-2006, Edinburgh, p.1229.

[4] K. Holldack et al., Phys. Rev. Lett. 96 (2006) 054801/1-4.

[5] H. Wiedemann , Springer, 2003, 274 p.

[6] A.A. Zholents and K. Holldack, Proc. FEL-2006, Berlin, p. 725.

[7] D.K. Kalantaryan and A. Streun, Internal report SLS-TME-TA-2007-301, 2007.