SOME REMARKS ABOUT CHARACTERIZATION OF MAGNETIC BLOCKS WITH HELMHOLTZ COILS

G. Tosin and R.A. Pimenta
LNLS - Laboratório Nacional de Luz Síncrotron, CP 6192, 13084-971, Campinas,SP, Brazil

Abstract

The usage of Helmholtz coils for magnetic block characterization is a widespread technique since it is little sensitive to errors in the block positioning and has good level of precision. In this paper, some calculations concerned to the block positioning are done for the cases where the Helmholtz condition is not satisfied. Also, the comparison between a model based on point dipolar magnetic moment and magnetized blocks with real dimensions is analyzed, as well as the corrections associated to the effect of self-demagnetization of the blocks.

INTRODUCTION

The characterization system based on Helmholtz coils is a largely used technique to determine the magnetic moment of magnetized blocks [1-2]. However, considerations about the advantages of this coil configuration and the approach employed can not be found neither in a compact way nor in an easy search in the scientific literature. The main topics studied in this paper are the following: the approach of point dipole, its validity when compared to the real block, considering geometric dimensions and magnetic permeability, and the advantages of two parallel coils (not being necessary exactly the Helmholtz geometry).

FLUX GENERATED BY A POINT DIPOLE

Figure 1 shows the definitions used for the analysis of the problem. A point dipole with magnetic moment **m** rotates over the z axis, in the center of two parallel coils, placed at a distance r_{dc} from them. The mean coil radius is r_c. Here, the Helmholtz condition, $2r_{dc}=r_c$, is not being applied yet.

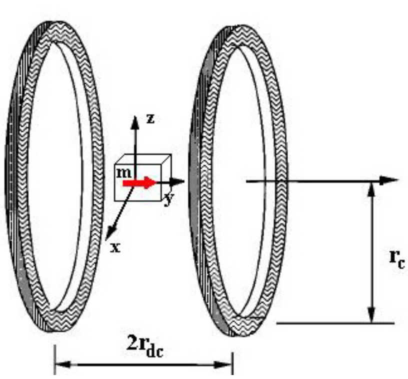

Figure 1: Definition of the reference system associated to two parallel coils.

Equation (1) describes the magnetic field **B** generated by a point magnetic dipole [3]:

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left[\frac{-\boldsymbol{m}}{r^3} + \frac{3(\boldsymbol{m}.\boldsymbol{r})\boldsymbol{r}}{r^5}\right] \quad (1)$$

where $\boldsymbol{m} = m_x\boldsymbol{i}+m_y\boldsymbol{j}+m_z\boldsymbol{k}$ e $\boldsymbol{r} = r_x\boldsymbol{i}+r_y\boldsymbol{j}+r_z\boldsymbol{k}$.

The interest is in the B_y magnetic component, since only it contributes to the magnetic flux through the coils

$$B_y = \frac{\mu_0}{4\pi}\left[\frac{-m_y}{(r_x^2+r_y^2+r_z^2)^{3/2}} + \frac{3(m_x r_x + m_y r_y + m_z r_z) r_y}{(r_x^2+r_y^2+r_z^2)^{5/2}}\right] \quad (2)$$

The integration over two coils with N turns each, for one point dipole located exactly on the origin of the reference system, conduces to the magnetic flux Φ_T that can be expressed in terms of a geometric factor F_g:

$$\Phi_T = \frac{\mu_0 N m_y r_c^2}{(r_c^2 + r_{dc}^2)^{3/2}} = \mu_0 m_y F_g \quad (3)$$

In order to verify if this coil configuration is little sensitive to small errors in the dipole position, it is considered a magnetic dipole displaced from central position by the vector **Δ**

$$\boldsymbol{r} = \boldsymbol{r}_0 - \boldsymbol{\Delta} = (r_{0x} - \Delta x, r_{0y} - \Delta y, r_{0z} - \Delta z) \quad (4)$$

where $\boldsymbol{r}$ is the vector from the dipole to one point on the coil surface and $\boldsymbol{r_0}$ is the vector connecting the origin of the reference system to the same point on the coil surface.

Despite of the dipole be free to Δy displacements, only the magnetization m_y gives a contribution for the total flux:

$$\Phi_T = \frac{\mu_0 N m_y r_c^2}{2}\left\{\frac{1}{\left[r_c^2+(\Delta y - r_{dc})^2\right]^{3/2}} + \frac{1}{\left[r_c^2+(\Delta y + r_{dc})^2\right]^{3/2}}\right\} \quad (5)$$

Being Δy small enough, Equation (5) can be expanded in Taylor series up to 2nd order:

$$\Phi_T = \mu_0 N m_y r_c^2 \left\{\frac{1}{\left[r_c^2+r_{dc}^2\right]^{3/2}} - \frac{3\Delta y^2 (r_c^2 - 4 r_{dc}^2)}{2\left[r_c^2+r_{dc}^2\right]^{7/2}}\right\} \quad (6)$$

Displacements Δx and Δz are symmetric with respect to the coils, and their influence on the magnetic flux Φ_T is evaluated expanding Equation (3), with the correspondent transformation shown in equation (4), in a Taylor series for small Δx and Δz, before the integration on the coils surfaces.

On the plane xz (y=0), the contribution of m_x and m_z to Φ_T is null due to symmetry reasons and, again, only m_y generates a net magnetic flux in the coils.

Any displacement ΔR ($\Delta R^2 = \Delta x^2 + \Delta z^2$) on the plane xz (y=0), independently of the values of Δx and Δz, presents the same geometry relative to the coils.

The integration of Equation (2), expanded up to 2nd order in ΔR gives

$$\Phi_T = \mu_0\, N\, m_y\, r_c^2 \left\{ \frac{1}{\left[r_c^2 + r_{dc}^{\,2}\right]^{3/2}} + \frac{3\,\Delta R^2\,(r_c^2 - 4\,r_{dc}^{\,2})}{4\left[r_c^2 + r_{dc}^{\,2}\right]^{7/2}} \right\} \qquad (7)$$

It is observed from equations (6) and (7) that the magnetic flux becomes independent of the position errors Δx, Δy and Δz when $2r_{dc}=r_c$, which is the Helmholtz condition. However, looking at Figure 2, we can notice that such condition is not so rigid, obtaining even good results when $2r_{dc}$ is a little different of r_c. Such graphics show the ratio between the flux detected due to a magnetic dipole displaced the amount Δy *or* ΔR from the coils central point (origin of coordinate system - Figure 1) and the ideal flux produced by a dipole placed at origin (Equation (3)). The three dimensional graphics present such flux difference as a function of the ratio r_{dc}/r_c and the position error. Even when the displacement is as huge as $0.1r_c$ and r_{dc}/r_c is far from Helmholtz condition, the discrepancy between the fluxes is smaller than 1%.

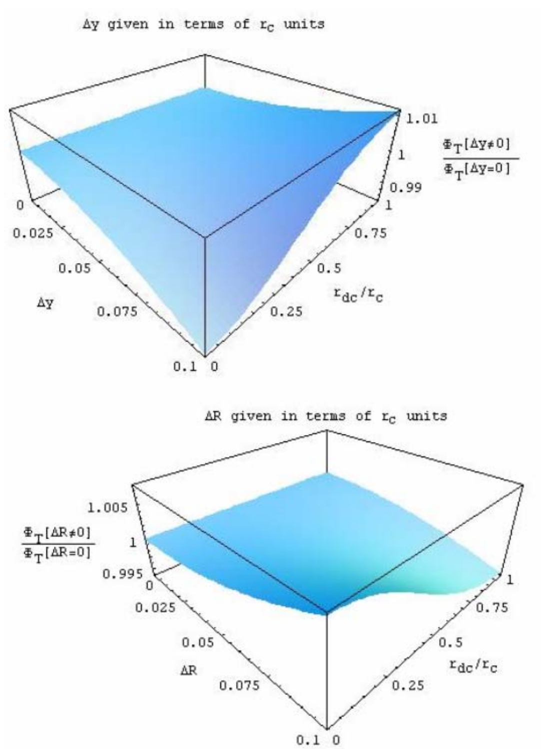

Figure 2: Relative difference between the magnetic flux generated by a dipole displaced Δy or ΔR from the point centered between the two coils and the flux produced with the dipole at this point. All geometric dimensions are given in terms of coil radius length r_c.

FINDING THE MAGNETIZATIONS

This section explains how to determine the dipolar magnetic moment **m** from the induced voltage in the coils.

When the magnetic moment is rotating in the middle of the two parallel coils, as shown in Figure 1, the Induction Faraday´s Law predicts the appearing of an electromotive force *V*, according to $V = -d\Phi/dt$, which is the magnetic flux derivative with respect to the time.

In our case, a digital integrator PDI5025 [4] is used to detect induced voltage. In this way, the sequence of integrator readings corresponds directly to the increment of magnetic flux $d\Phi$.

Only the first harmonic (dipolar coefficient) is expected when a Fast Fourier Transformer (FFT) is applied on the signal produced by one point magnetic dipole. The proof of this statement is easily obtained by looking at equation (6). Just m_y contributes for the magnetic flux, and m_y changes in a sinusoidal way when the magnetic dipole is rotating over axis z. For this reason, the magnetic flux increment can be written as

$$d\Phi(n\,d\theta) = A_1 \cos(n\,d\theta) + B_1 \sin(n\,d\theta) \qquad (8)$$

where $d\theta$ is the increment in the angular position and n is integer corresponding to the encoder angular position.

Equation (8) can be made a function of $n.d\theta$, in the following way:

$$\Phi(n\,d\theta) = \mu_0\, F_g\, [\, m_y \cos(n\,d\theta) + m_x \sin(n\,d\theta)\,] \qquad (9)$$

and, consequently, $d\Phi$ is

$$d\Phi(n\,d\theta) = \mu_0\, F_g \left(\left\{ m_y\, [1-\cos(d\theta)] + m_x \sin(d\theta) \right\} \cos(n\,d\theta) + \left\{ m_x\, [1-\cos(d\theta)] - m_y \sin(d\theta) \right\} \sin(n\,d\theta) \right) \qquad (10)$$

Finally, from the equality between equations (8) and (10), the components m_x and m_y of the magnetic moment are obtained:

$$m_x = \frac{1}{\mu_0\, F_g} \, \frac{A_1 \sin(d\theta) + B_1\, [1-\cos(d\theta)]}{2\,[1-\cos(d\theta)]} \qquad (11)$$

$$m_y = \frac{1}{\mu_0\, F_g} \, \frac{-B_1 \sin(d\theta) + A_1\, [1-\cos(d\theta)]}{2\,[1-\cos(d\theta)]} \qquad (12)$$

The three components of the magnetization can be determined by changing the block orientation.

COMPARISON BETWEEN A POINT DIPOLAR MAGNETIC MOMENT AND A REAL MAGNETIZED BLOCK

An important question is how good the point dipole approach is when the real dimensions and magnetic permeability of the block are taken into account. In order to answer this question, a block homogeneously magnetized with remanent field (B_r) of 1.2 teslas and with relative permeabilities of 1.06 for easy direction and 1.17 for hard direction is used for the analysis. The flux generated by the block is calculated using Radia [5] magnetic field simulator. The block rotates around z axis and the coils are always in Helmholtz configuration (Figure 1).

Three configurations of blocks relative to the coils are considered: a perfect cube of sides {L,L,L} and two rectangular blocks of sides {L,L/4,L} and {L,L,L/4}.

The ratio between the magnetic flux amplitude of a point dipole and the flux of a block homogenously magnetized without permeability changes like an exponential profile. Depending on the block shape, this ratio can either increases or decreases. It begins in 1 for the three cases and reaches, the higher ratio of 0,996 when L= $0.7r_c$, for the cubic geometry. To make this comparison, the dipole has its magnetic moment **m** equal to the product V**M**, where V is the block volume and **M** is its magnetization.

Also when the shape of the magnet is considered, the magnetic flux captured by the coils does not present a sinusoidal profile anymore. Thus, decomposing the flux in a Fourier series, other coefficients besides those related to dipolar terms, will be present (sextupoles, decapoles).

Next graphs are showing the ratio between FFT dipolar coefficients for point model (C_d) and for a real block (C_r). These curve profiles do not depend on the coil radius.

In general, the way to input the magnets in the simulating code to calculate the magnetic field of the insertion devices, for instance, is to place homogenously magnetized blocks with the magnetization vector calculated from measurements in which the block is treated as a point dipole.

Each graph in Figure 3 has two curves: one related to the total effect of both permeability and shape, got from the point dipole (C_d) and real block (C_r), and the other concerning only to permeability, which proceeds from the comparison between the real block and the homogenously magnetized block without permeability (C_s). Looking at the graphs, it can be concluded that for these proposed block geometries, the permeability effect is much more important than the shape influence, reaching some percents (~ 4% for L, L/4, L). For calculations with permeability, the blocks were segmented in cells with lengths of L/16.

In order to reconstitute the correct magnetization of the homogenously magnetized block, it is necessary to multiply the dipolar coefficient obtained for the real block (or measured block) by the correspondent value of the shape/permeability curve. To do these corrections, the block geometry, with respect to the coils, must be observed.

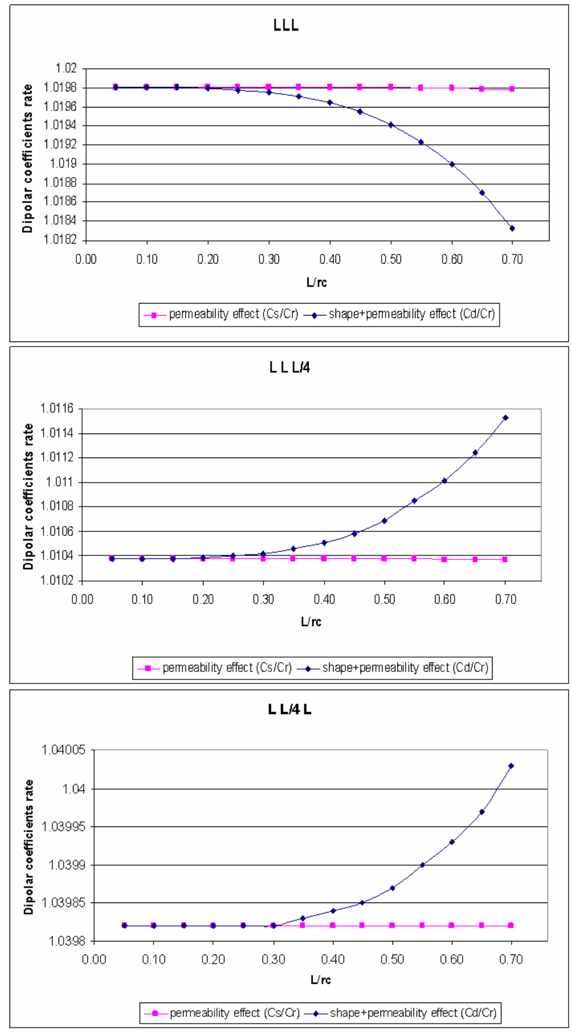

Figure 3: Graphs showing the ratio betwen the dipolar coefficients (A1) obtained for a point dipole and for a real block. The curves considering only the permeability effect are also shown.

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