ISOCHRONOUS PION DECAY CHANNEL FOR ENHANCED MUON CAPTURE*

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Abstract

Intense muon beams have many potential applications, including neutrino factories and muon colliders. However, muons are produced in tertiary beams into a diffuse phase space. To make useful beams, the muons must be rapidly cooled before they decay. A promising new concept for the collection and cooling of muon beams is being investigated, namely, the use of a nearly Isochronous Helical Transport Channel (IHTC) to facilitate capture of muons into RF bunches. Such a distribution could be cooled quickly and coalesced into a single bunch to optimize the luminosity of a muon collider. We describe the IHTC and provide simulations demonstrating isochronicity, even in the absence of RF and absorber.

INTRODUCTION

A nearly Isochronous Helical Transport Channel (IHTC) is being investigated as an alternative to a significant portion of the baseline front end for a neutrino factory or muon collider. The study 2A [1] front end consists of a target solenoid (20 T), a tapered capture solenoid (20 T to -2T, 12 m long), drift section (99 m), RF buncher (50 m), an energy-phase rotator (54 m), and cooling region (80 m). The IHTC might be developed to replace all but the last cooling stage, and this cooling section may be replaced by a Helical Cooling Channel (HCC) [2]. The IHTC offers a more natural match into the potentially more efficient HCC.

The IHTC concept takes advantage of the larger effective RF buckets with respect to particles on isochronous orbits, because, in this operating region, the electric fields have a longer time-integrated effect on these particles. Critical components of an IHTC system, as presently conceived, include the following: (1) a helical magnetic field that creates helical particle trajectories near a reference orbit of a selected muon momentum, (2) RF cavities that capture particles in stable bunches, and (3) an absorber that reduces the energy of particles that would otherwise be too energetic to be captured. In this paper, we focus only on the aspect of helical orbits that exhibit approximate isochonicity.

ISOCHRONOUS CONDITION

A helical channel [2] combines solenoidal, dipole, and quadrupole fields, causing charged particles to move along helical trajectories according to the relationship:

$$p(a) = \frac{\sqrt{1+\kappa^2}}{k}\left[B - \frac{1+\kappa^2}{\kappa} b_\phi\right] \qquad (1)$$

where:

- B is the solenoidal magnetic field
- b_φ is the dipole component
- $\kappa = p_\perp / p_z$ is the pitch angle; $\kappa = k a$
- $k = 2\pi/\lambda$; λ is the helical wavelength
- p is design momentum; a is reference radius

This novel concept entails designing an IHTC with the beam gamma near the transition gamma (γ_T) for the purpose of operating near the isochronous condition. An expression for γ_t for an IHTC is identified after a derivation of the slip factor η in [2] and setting it to zero:

$$\eta = \frac{\sqrt{1+\kappa^2}}{\gamma_T \beta^3}\left(\frac{\kappa^2}{1+\kappa^2}\hat{D} - \frac{1}{\gamma_T^2}\right) = 0 \qquad (2)$$

where the dispersion factor $\hat{D}$ relates to apparatus quantities and design momentum via:

$$\hat{D}^{-1} = \frac{a}{p}\frac{dp}{da} = \frac{\kappa^2 + (1-\kappa^2)[(B\sqrt{1+\kappa^2}/pk) - 1]}{1+\kappa^2} - \frac{(1+\kappa^2)^{3/2}}{pk^2}\frac{\partial b_\phi}{\partial \rho} \qquad (3)$$

with

- $\frac{\partial b_\phi}{\partial \rho}$ being the quadrupole component

Figure 1, shows the trajectories of μ^+s generated with a gaussian momentum spread of 200 ± 50 MeV/c that are initially directed along the path of the reference muon with momentum 200 MeV/c.

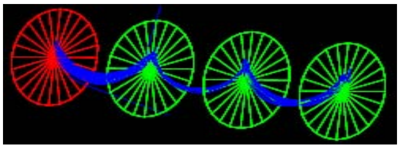

Figure 1: Display of μ^+s propagating down an IHTC. The designed IHTC period is 2 meters, which is the spacing between the green virtual detectors; the red circle indicates start of IHTC.

The reduction of the spread of flight times is shown in Figure 2, in which plot (a) shows the normal time broadening due to the momentum width for muons propagating 14 meters down a straight channel versus the narrower time spread in (b) for muons traversing 10 meters down the IHTC. Muons with p = 250 MeV/c and p = 150 MeV/c are spread by about 7 nsecs in the straight drift, while muons of same momentum band in the IHTC are spread by ~1.5 nsecs.

An important kinematic aspect is that muons produced from pion decays necessarily have similar velocities as their parent pions. Hence, designing an RF capture system for pions at a particular velocity also captures the decay muons as well. That means the capture process can

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begin even before the pions have decayed. Figure 3 shows the momentum/time dependence for pions and muons at downstream longitudinal propagation distances of 10m and 20m, where the initial beam consisted of only pions with a gaussian momentum spread of 200+/-50 MeV/c.

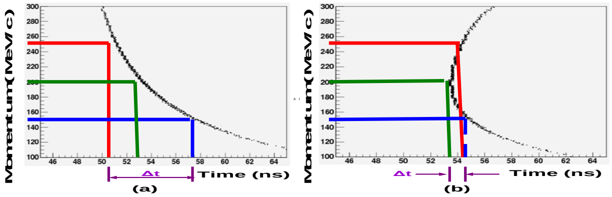


Figure 2: Momentum (MeV/c) vs. time (ns) of μ^+s generated with gaussian momentum spread of 200 ± 50 MeV/c. (a) Muons at 14 meters in straight drift channel. (b) Muons at 10 meters in an IHTC operating at γ_T for muons with p=200 MeV/c.[†]

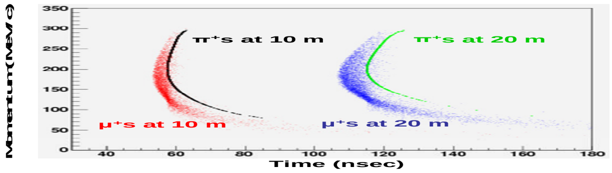
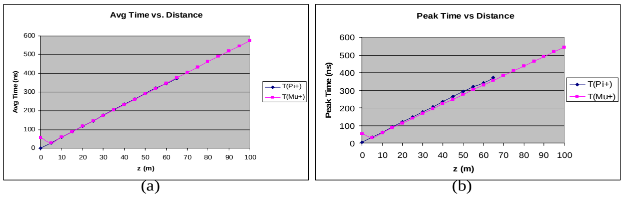


Figure 3: Momentum versus time for pions and decay product muons at 10 and 20 meters in an IHTC. Pions are injected at z=0 and t=0 with gaussian momentum spread 200+/-50 MeV/c along reference direction.

The time distributions at any particular z are asymmetric. The average and peak values for each time distribution at 5 m increments along z are shown in Figure 4. Both average and peak time values along z are nearly identical for pions and muons, verifying the timing that allows capture of muons before the parent pions have decayed.



(a) (b)

Figure 4: Time versus longitudinal propagation distance for pions and muons. Pions are injected at z=0 with gaussian momentum spread of 200+/-50 MeV/c. (a) Average time vs. longitudinal distance. (b) Peak time vs. longitudinal distance. Horizontal axes are 0 to 100 m; vertical axes are 0 to 600 nsec.

TRANSVERSE OSCILLATIONS AND SCALABILITY

The first order analytic solution for transverse oscillations about the reference helical orbit requires [2]:

$$0 < G < R^2 \tag{4}$$

where:

$$G = \left(\frac{2q + \kappa^2}{1 + \kappa^2} - \hat{D}^{-1} \right) \hat{D}^{-1} \tag{5}$$

and

$$R = \frac{1}{2} \left(1 + \frac{q^2}{1 + \kappa^2} \right) \tag{6}$$

with $q = (B_{solenoid} / p_z)/k - 1$ (7)

and $\hat{D}^{-1} = \frac{a}{p}\frac{dp}{da} = \frac{\kappa^2 + (1-\kappa^2)q}{1+\kappa^2} - \frac{(1+\kappa^2)^{3/2}}{pk^2}\frac{\partial b_\phi}{\partial \rho}$ (8)

The relationship between q and $B_{solenoid}$:

$$B_{solenoid} = \frac{pk(1+q)}{\sqrt{1+\kappa^2}} \tag{9}$$

allows changes in $B_{solenoid}$ to directly influence the range of *p* that provides solutions for transverse oscillations about the reference orbit. Simulations were performed for several values of $B_{solenoid}$[‡] with the following constraints:

- $P_{\mu+,reference} = p = 200$ MeV/c, κ = 1
- $R_{reference} = a = 0.320$ m, λ = 2 m

Figure 5 shows the momentum vs. time at 10 meters into an IHTC for muons produced on the reference orbit generated from a flat 100 MeV/c < *p* < 600 MeV/c distribution for configurations with $B_{solenoid}$ of 5 T, 10 T, 20 T, and 30 T. Distributions at 20 m are similar, but with the spread doubled along horizontal time axis, as expected. One sees that the $B_{solenoid}$ cannot be arbitrarily increased to enhance the momentum range of muons captured into transverse oscillating orbits about the reference particle. For the chosen constraints, $B_{solenoid}$ = 10 T appears to be optimal.

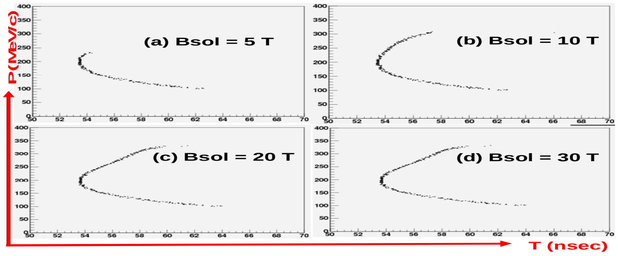


Figure 5: Dependence on $B_{solenoid}$ for fixed helix period of 2 meters. Muons are from a flat 100 MeV/c < *p* < 600 MeV/c distribution at 10 meters into an IHTC. Horizontal axes are time from 50 to 75 nsec; vertical axes are momentum from 0 to 600 MeV/c.

Although the optimal operating $B_{solenoid}$ is 10 T for our chosen set of constraints, it is desirable to lower the value of $B_{solenoid}$ and maintain capture efficiency. We utilize the relationship in equation (9) to lower the required $B_{solenoid}$ by increasing the overall dimensions. Recalling that q is related to solutions for transverse oscillations (G and R in equations 4-6) for a given pitch κ and dispersion, $B_{solenoid}$ is inversely proportional to the helical period (k=2π/λ). The desire is to have $B_{solenoid}$ = 2.2T, which matches the end of a capture tapered solenoid similar to study 2A [1].

[†] Because of the helix pitch angle, the actual particle trajectory is longer than z by reciprocal of cosine of the angle (factor of 1.414 here).

[‡] Once $B_{solenoid}$ is chosen, the dipole component is determined from $B_{solenoid}$ to maintain helical trajectories via equation (1).

Scaling approximately from the previous configuration for $B_{solenoid}$=10T with λ = 2m gives $B_{solenoid}$ = 2.2 T with λ = 10m. Figure 6 illustrates the scaling.

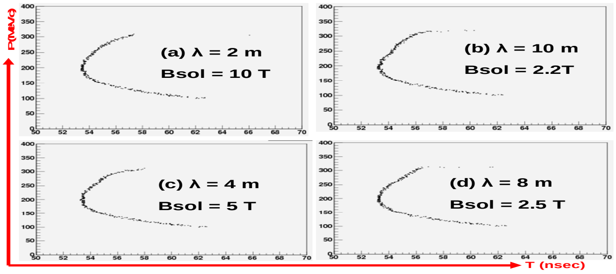


Figure 6: Scaling between $B_{solenoid}$ and helix period. Muons are from a flat 100 MeV/c < p < 600 MeV/c distribution at 10 meters into an IHTC. Horizontal axes are time from 50 to 75 nsec; vertical axes are momentum from 0 to 600 MeV/c.

APPLICATION TO NEUTRINO FACTORY/MUON COLLIDER TARGET

To demonstrate the effectiveness of the IHTC in a potential neutrino factory or muon collider, we subject our apparatus with $B_{solenoid}$= -2.2T and λ=10m to pions and muons produced by a target system similar to MERIT [3]. Eight GeV protons are aimed onto a Hg target with MERIT geometry, but with $B_{solenoid}$ in the target region of 20T to match the capture solenoid in study 2A that tapers from 20T to ~2T. Figure 7 shows trajectories of pions and muons that emerge from a capture solenoid similar to study 2A [1] and travel 20m inside the IHTC.

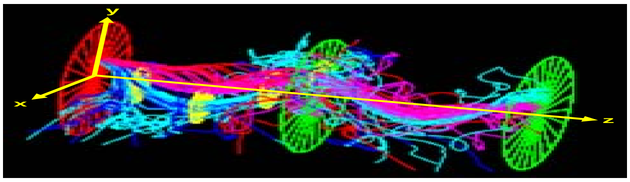


Figure 7: Trajectories of pions and muons emerging from end of tapered capture solenoid (red disk) and traversing 20m inside an IHTC (green disks are 10m apart). Particles were produced by 8 GeV protons on Hg target and propagated through 10m study 2A [1] type capture solenoid. π^+s are dark blue, μ^+s are light blue, π^-s are red, and μ^-s are purple. Yellow discs are virtual detectors (r=1.6m) centered along reference trajectory at z = 2.5m, 5.0m, 7.5m, and 10m.

Figure 8 illustrates the evolution of momentum/time for pions and muons in the IHTC and a short free drift. Figure 8(a) is the free drift equivalent with respect to time of that in the IHTC in Figure 8(b). At any z position, the momentum/time relationship is determined from the free drift and isochronous fields; at a downstream position dominated by the IHTC in Figure 8(d), the design momentum emerges as the earliest time arrival. Hence, a desired momentum/time relationship that maintains a correlation over a narrow time window (distributions as Fig. 8(a) over extended z) to achieve a level of isochronicity should be attainable by a judicious choice of amounts of free drift and fields in the IHTC.

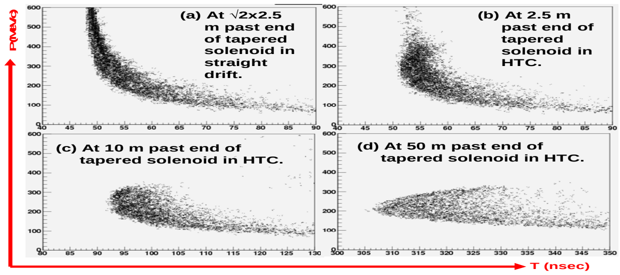


Figure 8: P(MeV/c) vs. T(nsec) for pions and muons at distance equivalent to quarter IHTC period in straight drift in (a), at quarter period in IHTC in (b), full period in (c), and 5 periods in (d) where designed momentum of 200 MeV/c emerges at peak of earliest arrival. Horizontal axes are time from 50 to 75 nsec; vertical axes are momentum from 0 to 600 MeV/c.

CONCLUSIONS AND FUTURE PLANS

We have shown the theory behind the IHTC design and demonstrated that isochronicity is maintained throughout the decay process of pions to muons. We have also explained the scalability between $B_{solenoid}$ and period length (λ) of the IHTC that allows freedom in its design. Application to a distribution of particles expected from a neutrino factory or muon collider illustrated the power and flexibility of the IHTC to shape momentum/time distributions, even without benefits of RF or absorber.

Besides adding RF and absorber to engage the full power of the IHTC, we will also consider the effects of:

1. Applying judicious amounts of free drift and the IHTC fields to maintain p/t correlation over a narrow time window (quasi isochronicity) to enhance RF capture via longer time integrated E field influence.
2. Incorporating small amount of RF phased to accelerate earliest arrivals, since exclusive application of IHTC fields results in reference particle being earliest arrival at lowest momentum.
3. Improved cooling by operating at higher momenta.

Finally, the IHTC could, in principle, start from the target to optimize its effect. The target and capture B fields could be superimposed into the IHTC with RF, although the absorber would need to be held off until enough pions have decayed.

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[3] http://www.cap.bnl.gov/mumu/conf/collider-071203/talks/HKirk1-71203.ppt