ALaDyn: A HIGH ACCURACY CODE FOR THE LASER-PLASMA INTERACTION

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Abstract

ALaDyn (Acceleration by LAser and DYNamics of charged particles) is a fully self-consistent, relativistic, parallelized PIC code to investigate the interaction of a laser pulse with a plasma and/or an externally injected beam. The code is based on compact high order finite differences schemes ensuring higher spectral accuracy compared to standard Yee schemes. We present the main features of the code (stretched grid, Boosted Lorentz frame, hierarchical particle sampling) together with a preliminary benchmarking with the PIC code VORPAL. Finally an application to the All-Optical FEL (AO-FEL) will be outlined.

INTRODUCTION

Plasma-based acceleration [1] has received much theoretical and experimental attention due to the high longitudinal electric fields that can be excited in a plasma without the limitations found in conventional accelerators. Rapid progresses are expected from the new laser generation with multiterawatt pulses of a few femtoseconds length. Even if the basic features of the laser-plasma interaction are understood, a detailed numerical 2D/3D treatment is required in order to deal with experiments and optimize the results. Several PIC codes have been already proposed in this respect [2], in all the cases the description of the fields/particles dynamics is second order accurate (fields on a Yee lattice evolved with the leap-frog scheme and particles push with the Boris algorithm) [3]. In this paper we present the relativistic, self-consistent, parallelized PIC code ALaDyn [4]. The code was developed in the framework of the PlasmonX (PLASma acceleration and MONochromatic X-rays production) experiment [5], a joint collaboration between the INFN and the Italian National Research Council (CNR). Here we shortly review some of the basic features of the code: high order schemes in space/time [6], stretched grid, hierarchical particle sampling and Boosted Lorentz Frame [7]. All these features allow us to reduce the computational needs (CPU time, memory), compared to standard PIC codes, in treating realistic problems still maintaining an acceptable accuracy. In the rest of the paper we present a preliminary benchmark of ALaDyn against the PIC code VORPAL and an application to the All-Optical Free Electron Laser (AO-FEL) [8].

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FEATURES OF ALADYN

The PIC code ALaDyn is a relativistic, fully self-consistent, electromagnetic PIC code designed to study the interaction of one or more laser pulses with a plasma and/or a charged particle beams. It is designed as a "virtual lab" where all the elements (laser, plasma and beams) can be completely defined by the user according to few simple rules. The code is organized into a library of functions written in C and parallelized with MPI (a 1D domain decomposition strategy has been implemented). The code works in 1, 2 and 3 spatial dimensions in Cartesian geometry. The key features of then code are listed below.

High Order Schemes. As in a standard PIC code electromagnetic fields are evolved by using the Ampère-Maxwell laws. An accurate representation of the wave propagation when the EM fields are discretized on a grid, requires a suitable discrete form for the *curl* operator in the Maxwell's equations. In ALaDyn all the derivatives are represented by (compact) high order finite differences schemes [6]. The increase in the computational cost, compared to standard 2^{nd} order schemes, for derivatives evaluation[1] is largely compensated by the gain in the spectral accuracy as shown in Fig. 1. We see that even with few

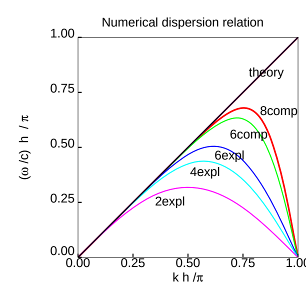


Figure 1: Numerical dispersion relation $\omega(k)$ for planar waves on a grid for several finite differences schemes at various order, h is the discretization step.

($\sim 10 \div 12$) points/wavelength the phase velocity of the wave is well reproduced. An increase in the accuracy of the spatial derivatives evaluation, which provides a better description of small scale structures, requires high order integration schemes in order to avoid instabilities. In ALaDyn time integration, for both fields and particles, is carried out by using Runge-Kutta schemes of order 4. The use of high order algorithms in space and time allows us to adopt a coarser computational grid (allowing to use a higher par-

[1] We usually use 6^{th}/8^{th} order schemes, and a tridiagonal system must be solved in case of compact operators.

ticles per cell number) and a larger time step compared to standard 2^{nd} order accurate PIC codes.

Stretched Grid. The spatial domain of interest in a typical laser-plasma simulation where we study the acceleration of an electron bunch auto or externally injected in the plasma, is the one close to the focal axis, where the bunch propagates. The simulated bunch is generally small compared to the other structures (plasma waves, laser pulse), nevertheless an accurate description of its properties (phase space, self fields) must be done. In ALaDyn we can adopt a non uniform transverse mesh which allows us to have an high resolution in the region close to the focal axis. The resolution is then gradually reduced (we use the tangent as the "stretching function") going from the centre towards the boundaries of the computational domain. For a fixed accuracy in the region of interest, this technique ensures a considerable reduction in the number of grid points and memory need compared to an uniform grid.

Hierarchical Particle Sampling. In the PIC technique the phase space distribution for each species is sampled by a (possibly) large number of numerical particles, keeping fixed the total charge and mass. The solution of the Vlasov equation is replaced by the solution of the equation of motion for the set of the numerical particles. In ALaDyn a given particle species ($e.g.$ electrons of the plasma) can be sampled by a hierarchy of particles with different charge as shown in Fig. 2. This freedom allows us to put more

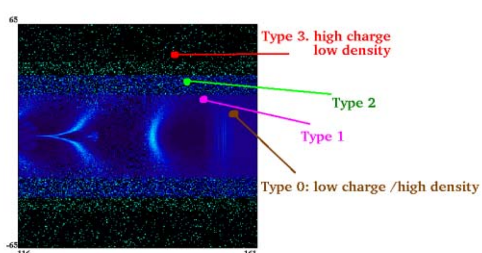


Figure 2: Hierarchical particles sampling.

particles (with small charge) inside the "dynamically interesting" zones reducing the noise level; at the same time we can decrease the particles in the outlying regions. With this approach, the total number of particles involved in the simulation can be largely reduced (without decreasing the accuracy of the results) compared to the standard sampling.

The Boosted Lorentz Frame. The simulation length in a laser-plasma interaction is determined by the ratio of the different spatial/temporal scales involved. The number of simulation steps is usually large because of the imbalance between these scales (laser wavelength $\ll$ plasma length). It has been observed [7] that the ranges of space and time scales spanned by a system in a laser-plasma simulation are not invariant under Lorentz transformation. Changing the reference frame and moving from the laboratory frame (LF) to a boosted frame (BF) the imbalance between the different spatial scales can be reduced implying the reduction of the simulation run time by a factor $\sim \gamma_{\rm BF}^2$, $\gamma_{\rm BF}$ being the relativistic factor in the transformation LF $\rightarrow$ BF. ALaDyn can perform simulations in an arbitrary reference frame setting automatically the correct initial condition. A set of utilities for particles/fields diagnostic which transform back all the quantities of interest from the BF to the LF is also available. The results obtained in the BF are generally less accurate than the ones obtained in the LF and in general the accuracy decreases as the velocity of the BF increases, nevertheless the BF is a useful tool to perform fast parameter scan.

BENCHMARK WITH VORPAL

The validation of an electromagnetic PIC code is not an easy task due to the highly nonlinear physics involved in the laser-matter interaction. A first set of validation tests of ALaDyn, where the code has been benchmarked against 1D analytical results, can be found in [4]. For the benchmark with VORPAL we consider a laser pulse ($\lambda_0 = 0.8$ μm, $I = 7.5 \cdot 10^{18}$ W/cm^2, $\tau = 17$ fs, $w_0 = 16$ μm) interacting with a cold plasma (initial density ramp: 20 μm + plateau: 30 μm @ $1.0{\cdot}10^{19}$ e/cm^3 + density transition: 10 μm, $1.0 \cdot 10^{19} \rightarrow 0.6 \cdot 10^{19}$ e/cm^3 + accelerating region: 220 μm + final down ramp: 10 μm). We consider a 2D simulation, the numerical parameters are give in Table 1. Both codes are in agreement concerning the de-

Table 1: Numerical parameters for the benchmark

	ALaDyn	**VORPAL**
domain (μm^2)	60×80	50×80
grid size	750×200	1200×320
points/λ_0	10×2	20×3.2
particles/cell	20	20
Δt (fs)	0.16	0.1
num. schemes	HO8/RK4	leap-frog-2
sim. duration @ 4 CPUs (h)	3.8	~ 14

scription of the plasma waves induced by the laser and the bunch injection at the density transition (see . 3 top line). Some discrepancy arises at late times as shown in Fig. 3 bottom line, where we plot the phase space at the end of the simulation. The difference in the details of the accelerated bunch is due to the numerical dephasing in VORPAL. Even with 20 grid points/λ_0 the leap-frog scheme in VORPAL propagates the laser pulse and, consequently, the plasma waves more slowly compared to the theoretical value. The relativistic electrons in the bunch overtake the accelerating part of plasma wave and get dephased too early. This interpretation is supported by the fact that decreasing the longitudinal resolution in VORPAL this effect is enhanced, while increasing the resolution we get a better agreement with ALaDyn. On the other hand, doubling the resolution in ALaDyn no significant change has been observed. Concerning the performances of the codes we notice that even

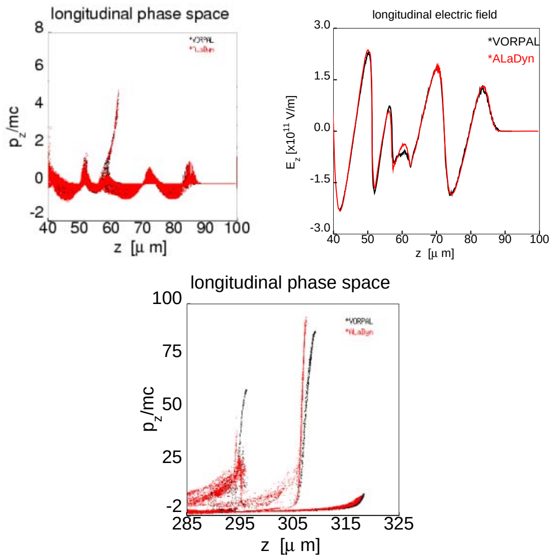
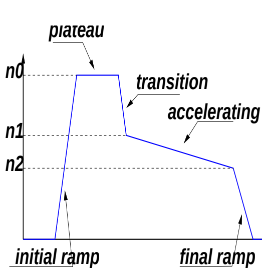


Figure 3: Top line: longitudinal phase space (left) and longitudinal electric field on the axis (right) at $t = 333$ fs. Bottom line: longitudinal phase space at the end of the simulation. The black plots refer to VORPAL, the red ones to ALaDyn.

if the overall number of particle pushes in this case study is the same for both codes, ALaDyn seems to be 3÷4 times faster than VORPAL. We notice also that the real advantage of ALaDyn is even larger since to remove the numerical dephasing in VORPAL in order to obtain meaningful results, we should consider a grid with at least 1500 points in the longitudinal direction (24 points/λ_0).

APPLICATION TO THE AO-FEL

ALaDyn has been utilized to study the generation of a low emittance, high current, mono-energetic electron bunch from laser-plasma interaction in view of achieving beam brightness of interest for FEL applications [8]. From a computational point of view the main challenges were: *i.* determine the optimal density profile (parameter scan using 2D simulations); *ii.* run a 3D simulation in the optimal case with high resolution in the bunch region. The parameter of the laser are: $\lambda_0 = 0.8\ \mu$m, $I = 8.5 \cdot 10^{18}$ W/cm^2, $\tau = 17$ fs, $w_0 = 23\ \mu$m. The optimal plasma profile obtained from the parameter scan is shown in Fig. 4. In the



initial/final	20 μm
plateau	30 μm
transition	10 μm
accel. reg.	330 μm
n_0	$1.0 \cdot 10^{19}$ e/cm^3
n_1	$0.7 \cdot 10^{19}$ e/cm^3
n_2	$0.4 \cdot 10^{19}$ e/cm^3

Figure 4: Density profile for the AO-FEL.

"high accuracy" 3D simulation we have chosen a resolution in the centre of 15 points/μm (longitudinally) and 12 points/μm (transversally) with 4 particles per cell. With a "standard" 2^{nd}-order accurate PIC code this simulation would require a mesh of (1300×1440^2) points and more than 10^9 numerical particles. Using ALaDyn (= high order schemes + stretched grid + hierarchical particle sampling) we can take a mesh with only (675×200^2) points and less than $150 \cdot 10^6$ particles. The reduction in the computational needs is evident (approximately a factor 100), keeping fixed the accuracy. In Fig. 5 we show some snapshots from the 3D simulation: the plasma waves and the accelerated bunch (left) and a close up of the bunch (right). The bunch has been analyzed for finding the slices charac-

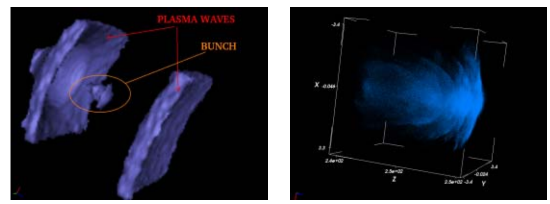


Figure 5: 3D simulation: plasma waves and accelerated bunch (left), close up of the bunch (right).

terized by the highest brightness for producing FEL radiation, the best results (emittances, energy spread, current) are summarized in Fig. 6.

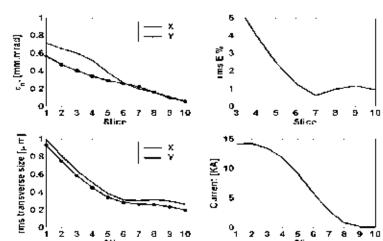


Figure 6: Slice analysis of the accelerated bunch.

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