

ABBERATION-FREE MUON TRANSPORT LINE FOR EXTREME IONIZATION COOLING: A STUDY OF EPICYCLIC HELICAL CHANNEL*

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Abstract

In order to achieve cooling of muons in addition to 6D helical cooling channel (HCC) [1], we develop a technique based on parametric resonance. The use of parametric resonance requires alternating dispersion, minimized at locations of thin absorbers, but maximized in between in order to compensate for chromatic aberrations [2]. Standard HCC is characterized by a rotating dispersion that is constant in magnitude. In this work we consider a transport line that includes a solenoid with superimposed transverse dipole fields that vary with *two* characteristic periods. Such a configuration allows combining requirements of alternating dispersion of beam periodic orbit with best conditions for maintenance of stable beam transport in continuous solenoid-type field.

PARAMETRIC RESONANCE IONIZATION COOLING

Muon beam ionization cooling is a key element in designing next-generation high-luminosity muon colliders. To reach adequately high luminosity without excessively large muon intensities, it was proposed to combine ionization cooling with techniques using parametric resonance [2]. In the linear approximation, a half-integer resonance is induced such that normal elliptical motion of $x-x'$ phase space becomes hyperbolic, with particles moving to smaller x and larger x' at the channel focal points. Thin absorbers placed at the focal points of the channel then cool the angular divergence of the beam by the usual ionization cooling mechanism where each absorber is followed by RF cavities. Further compensation for chromatic aberrations in this channel requires the regions with large dispersion. On the other hand, the absorbers for ionization cooling have to be located in the region of small dispersion. In order to satisfy both these requirements within a single transport line, we suggest here a design of a cooling channel characterized by alternating dispersion and stability provided by a (modified) magnetic field of a solenoid.

SIMULATIONS OF EPICYCLIC HELICAL COOLING CHANNEL

We demonstrate results of simulations for the muon trajectory for the proposed transport line. The magnetic

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field in z -direction B_z is chosen uniform and constant, as provided by a straight long solenoid. The charged particle motion in this field is characterized by a cyclotron wave number k_c . Then we superimpose alternating transverse dipole fields B_{T1} and B_{T2} on the solenoid field B_z . Each transverse field is periodic as a function of z -position, with a period defined by wave numbers k_1 and k_2 :

$$B_T = B_{T1} + B_{T2} = |B_1| e^{ik_1z} + |B_2| e^{ik_2z}.$$

Such a structure of the magnetic field brings a new feature to the helical cooling channel, namely, a variable dispersion function that is modulated with a frequency proportional to (k_1-k_2) . It can be shown (in a certain approximation) that if the following relation is satisfied,

$$\frac{|B_1|}{(k_1 - k_c)^2} = \frac{|B_2|}{(k_2 - k_c)^2}, \quad k_1 \neq k_2,$$

the dispersion has periodic nodes with a frequency proportional to the difference (k_1-k_2) .

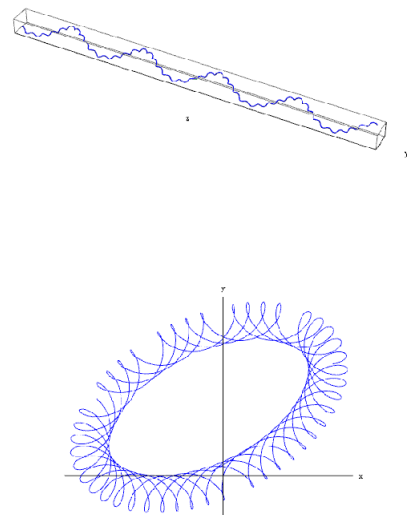


Figure 1: Particle 3D-trajectory in lab (upper plot) and its projection on the transverse plane (lower plot). Adiabatic case: $k_1=-k_2 \ll k_c$.

We solved numerically with *Mathematica* the equations of motion for a charged particle in such a double-periodic magnetic field and identified corresponding closed trajectories.

If the transverse field has slow variation over the distance characterized by cyclotron wavelength, we obtain an adiabatic solution shown in Figure 1. The choice of transverse magnetic field wave numbers $k_1 = -k_2$ results in a trajectory bound by ellipses. Different ratios and relative signs k_1 and k_2 give different constraints on the particle trajectories and in some cases resemble planet orbits in Ptolemy's system of epicycles. (Hence the word 'epicyclic' for the name of the transport line). Examples of such constraints are shown in Figure 2, where the same-sign (opposite-sign) commensurate wave numbers k_1 and k_2 produce the curves known as epitrochoids (hypotrochoids).

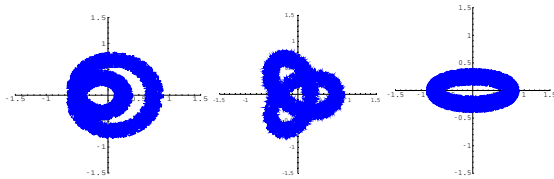


Figure 2: Transverse-plane constraints on particle trajectories in the epicyclic transport line in the adiabatic limit: $k_1=2k_2$ (left plot); $k_1=-2k_2$ (middle plot); $k_1=-k_2$ (right plot).

To maximize efficiency of the epicyclic helical cooling channel, we choose wave numbers k_1 and k_2 compatible with the cyclotron wave number k_c , and therefore the adiabatic case of Figure 1 no longer applies. Still, it appears possible to find a closed orbit in this case, an example of which is shown in Figure 3. Corresponding transverse phase-space plots are shown in Figure 4.

Above examples are based on simplified magnetic fields used to demonstrate the need for double-periodic transverse field configuration that results in alternating dispersion. For the next stage of simulations, it is essential that we use realistic magnetic fields. Ways to obtain such fields are briefly discussed in the following Section.

DESIGNING EPICYCLIC HELICAL COOLING CHANNEL

There are several technical possibilities for implementation of the epicyclic HCC. The most straightforward one would be a direct superposition of transverse helical fields, each having a selected spatial period. Another possibility would be along the lines suggested by V. Kashikhin and collaborators for single-periodic HCC, c.f. [3] and references therein. HCC for muon beams is under development by Muons, Inc., in Batavia, IL, USA [4]. A series of close and parallel conducting rings centred on the parametric curves as shown in Figure 2 may produce the desired double-

periodic spatial structure of the epicyclic HCC. In this case the simplest solution is elliptic, Figure 2 (right).

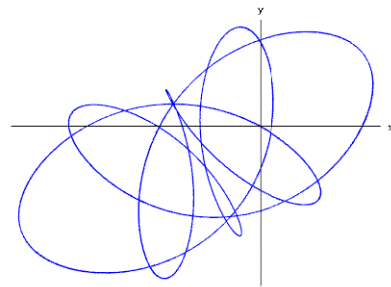
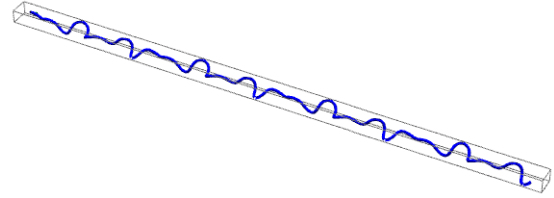


Figure 3: Same as in Figure 1, the periodic orbit for a non-adiabatic case: $k_1 = -k_2 = k_c/2$.

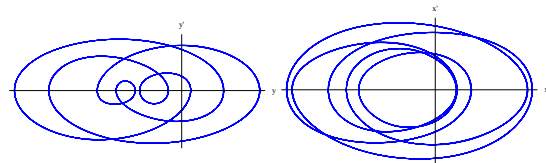


Figure 4: Transverse phase-space diagrams for the orbit of Figure 3.

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