# DESIGN OF SLOW EXTRACTION SYSTEM AT BOOSTER SYNCHROTRON FOR MUSES 

T.Ohkawa, RIKEN, Wako, Saitama 351-01, Japan<br>T.Katayama, Center for Nuclear Study, Graduate School of Science, University of Tokyo Tanashi, Tokyo 188, Japan

## Abstract

The Booster Synchrotron Ring (BSR) is a part of Multi-USe Experimental Storage rings (MUSES). BSR functions exclusively for the acceleration of ion and electron beams. The maximum accelerating energy is, for example, to be 3 GeV for proton; 1.45 GeV /nucleon for light ions of $q / A=1 / 2 ; 800 \mathrm{MeV} /$ nucleon for heavy ions of $\mathrm{q} / \mathrm{A}=1 / 3$. Electron beam is accelerated to 2.5 GeV from the injection energy 300 MeV . The accelerated ion and electron beams will be fast extracted and injected into the Double Storage Rings (DSR) by one turn injection. As another operation mode, ion beams will be slowly extracted for the experiments. In this paper, injection and extraction procedures of the BSR, especially slow extraction, are presented.

## 1 BSR LATTICE DESCRIPTION

As shown in Fig.1, the BSR consists of two arc sections and two long straight sections. Each arc section is mirror symmetrical system, and there are two bending cells. The dispersion in the straight sections outside the arc section is zero. The lattice is specified for eight families of quadrupoles; QF1 and QD1, QF4 and QD4 in the arcs, QF2 and QD2, QF3 and QD3 in the long straight sections.

The BSR lattice is designed to be able to operate with two different extraction mode for ion beams; fast extraction and slow extraction.


Fig. 1 A Layout of BSR
As seen in Fig.2, an electrostatic septum(ES), four septum magnets(SM1, SM2, SM3, SM4) and three bump magnets(BM1) are used for the electron beam multi-turn injection. These magnets are used for ion beam slow extraction which is carried out by using third order resonance ( $\mathrm{v}_{\text {res }}=20 / 3$ ).

## 2 SLOW EXTRACTION

On slow extraction process, at the first, sextupole magnets as a resonance exciter are excited. Secondly horizontal tune is moved from the operating value 6.691
- Ion beam


Fig. 2 An arrangement of straight section A
to a value near the third order resonance $(20 / 3)$ by changing excitation currents of the quadrupole magnets. Beams which have deviated to the distance more than 25 mm inside from the central orbit at the entrance of the ES are deflected inward as large as 3 mrad by the static high voltage of the ES.

The betatron oscillation can be expressed by the following relation

$$
\begin{aligned}
\binom{\mathrm{x}}{\mathrm{y}} & =\left(\begin{array}{cc}
\cos \mu+\alpha \sin \mu & \sqrt{1+\alpha^{2}} \sin \mu \\
-\sqrt{1+\alpha^{2}} \sin \mu & \cos \mu-\alpha \sin \mu
\end{array}\right)\binom{x_{0}}{y_{0}} \\
& =\left(\begin{array}{cc}
1+\alpha \varepsilon & \sqrt{1+\alpha^{2}} \varepsilon \\
-\sqrt{1+\alpha^{2}} \varepsilon & 1-\alpha \varepsilon
\end{array}\right)\binom{x_{0}}{y_{0}}
\end{aligned}
$$

here we put
$y=\frac{\beta}{\sqrt{1+\alpha^{2}}} x^{\prime}$.
Let $\Psi i$ be the deflection due to the i-th sextupole magnet SEi
$\Psi_{i}=-g_{i} x_{i}^{2}, g_{i}=\frac{B_{i}^{\prime \prime}}{2 B \rho}$,
where xi is the displacement of the orbit from the equilibrium orbit at SEi and gi is the strength of the Sei. Here we assume that the transfer matrix $\Gamma$ i from electric septum to sextupole magnet is
$\Gamma_{i}=\left(\begin{array}{ll}a_{i} & b_{i} \\ c_{i} & d_{i}\end{array}\right)$.
In the third resonance, particles pass each sextupole magnets three times a period. Therefore, taking summations of contributions from each magnet over one period, we have

$$
\begin{aligned}
H & =-\frac{\varepsilon}{2}\left(X^{2}+Y^{2}\right)-\frac{\beta}{3 \sqrt{1+\alpha^{2}}} \Sigma g_{i} x_{i}^{3} \\
& =-\frac{\varepsilon}{2}\left(X^{2}+Y^{2}\right)-\frac{\beta}{3 \sqrt{1+\alpha^{2}}} \Sigma\left(\left(X^{3}-3 X Y^{2}\right) g_{i} A_{i}+\left(Y^{3}-3 X^{2} Y\right) g_{i} B_{i}\right\}
\end{aligned}
$$

where
$X=\frac{1}{\sqrt{2}}\left[\sqrt{1+\alpha^{2}}-\alpha\right]^{\frac{1}{2}}(x-y)$
$\mathrm{Y}=\frac{1}{\sqrt{2}}\left[\sqrt{1+\alpha^{2}}+\alpha\right]^{\frac{1}{2}}(\mathrm{x}+\mathrm{y})$
$A_{i}=\frac{3}{8 \sqrt{2}}\left(\sqrt{1+\alpha^{2}}+\alpha\right)^{\frac{3}{2}}\left\{\left(a_{i}-b_{i}\right)^{3}-3\left(\sqrt{1+\alpha^{2}}-\alpha\right)^{2}\left(a_{i}+b_{i}\right)^{2}\left(a_{i}-b_{i}\right)\right\}$
$\mathrm{B}_{\mathrm{i}}=\frac{3}{8 \sqrt{2}}\left(\sqrt{1+\alpha^{2}}-\alpha\right)^{\frac{3}{2}}\left\{\left(\mathrm{a}_{\mathrm{i}}+\mathrm{b}_{\mathrm{i}}\right)^{3}-3\left(\sqrt{1+\alpha^{2}}+\alpha\right)^{2}\left(\mathrm{a}_{\mathrm{i}}-\mathrm{b}_{\mathrm{i}}\right)^{2}\left(\mathrm{a}_{\mathrm{i}}+\mathrm{b}_{\mathrm{i}}\right)\right\}$
$\Sigma$ means the summation over one revolution. Putting the angle of rotation of the coordinates as $\Psi$, given by
$\tan 3 \Psi=-\mathrm{B} / \mathrm{A}$, the coordinates are transformed as
$\binom{\mathrm{X}^{\prime}}{\mathrm{Y}^{\prime}}=\left(\begin{array}{cc}\cos \Psi & \sin \Psi \\ -\sin \Psi & \cos \Psi\end{array}\right)\binom{\mathrm{X}}{\mathrm{Y}}$.
Then, we can rewrite the Hamiltonian as
$H=-\frac{\varepsilon}{2}\left(X^{\prime 2}+Y^{\prime 2}\right)-\frac{\beta g \sqrt{A^{2}+\mathrm{B}^{2}}}{3 \sqrt{1+\alpha^{2}}}\left(X^{\prime 3}-3 X^{\prime} Y^{\prime 2}\right)$,
from which we can obtain following three unstable fixed points:

$$
\begin{aligned}
& \mathrm{A}:\left(\mathrm{X}^{\prime}{ }_{1}, \mathrm{Y}^{\prime}{ }_{1}\right)=\left(-\frac{\varepsilon \sqrt{1+\alpha^{2}}}{\beta \mathrm{~g} \sqrt{\mathrm{~A}^{2}+\mathrm{B}^{2}}, 0}\right) \\
& \mathrm{B}:\left(\mathrm{X}^{\prime}{ }_{2}, \mathrm{Y}^{\prime}{ }_{2}\right)=\left(-\frac{1}{2} \mathrm{X}^{\prime}, \frac{\sqrt{3}}{2} \mathrm{X}^{\prime}{ }_{1}\right) \\
& \mathrm{C}:\left(\mathrm{X}^{\prime}{ }_{3}, \mathrm{Y}^{\prime}{ }_{3}\right)=\left(-\frac{1}{2} \mathrm{X}^{\prime}, \frac{\sqrt{3}}{2} \mathrm{X}^{\prime} 1\right)
\end{aligned}
$$

The separatrix can be given by
$X^{\prime}=-\frac{X^{\prime} 1}{2}, Y^{\prime}=\frac{1}{\sqrt{3}}\left(X^{\prime}-X^{\prime}{ }_{1}\right), Y^{\prime}=-\frac{1}{\sqrt{3}}\left(X^{\prime}-X^{\prime}{ }_{1}\right)$
Transforming back to the ( $\mathrm{x}, \mathrm{x}^{\prime}$ ) plane, we obtain the following unstable fixed point:
$A:\left(x_{1}, x^{\prime}{ }_{1}\right)=\left(-G \cos (\psi-\chi), \frac{\sqrt{1+\alpha^{2}}}{\beta} \operatorname{Gcos}(\psi+\chi)\right)$
$B:\left(x_{2}, x^{\prime} 2\right)=\left(-G \cos \left(\psi-\chi+\frac{2}{3} \pi\right), \frac{\sqrt{1+\alpha^{2}}}{\beta} G \cos \left(\psi+\chi+\frac{2}{3} \pi\right)\right)$
$C:\left(x_{3}, x^{\prime} 3\right)=\left(-\operatorname{Gcos}\left(\psi-\chi-\frac{2}{3} \pi\right), \frac{\sqrt{1+\alpha^{2}}}{\beta} \operatorname{Gcos}\left(\psi+\chi-\frac{2}{3} \pi\right)\right)$
where
$\tan \chi=\sqrt{1+\alpha^{2}}-\alpha$
and
$G=\frac{\varepsilon\left(1+\alpha^{2}\right)^{\frac{3}{4}}}{\beta g \sqrt{A^{2}+B^{2}}}$.
The area of the triangular separatrix in the ( $\mathrm{x}, \mathrm{x}^{\prime}$ ) plane is given by
$S=\frac{3 \sqrt{3}}{4}|\sin 2 \chi| \frac{\sqrt{1+\alpha^{2}}}{\beta} G^{2}$
The turn separation $\Delta \mathrm{x}$ during three turns can be derived from the Hamiltonian as follows:
$\Delta x=\frac{\left[\sqrt{1+\alpha^{2}}+\alpha\right] \frac{1}{2}\left(\Delta X^{\prime} \cos \psi-\Delta Y^{\prime} \sin \psi\right)+\left[\sqrt{1+\alpha^{2}}-\alpha\right]_{2}^{1}\left(\Delta X^{\prime} \sin \psi+\Delta Y^{\prime} \cos \psi\right)}{\sqrt{2}}$
where
$\Delta \mathrm{X}^{\prime} \equiv \frac{\mathrm{d} \mathrm{X}^{\prime}}{\mathrm{d} v}=-\frac{\partial \mathrm{H}}{\partial \mathrm{Y}^{\prime}}=\varepsilon \mathrm{Y}^{\prime}-\frac{2 \beta \mathrm{~g} \sqrt{\mathrm{~A}^{2}+\mathrm{B}^{2}}}{\sqrt{1+\alpha^{2}}} \mathrm{X}^{\prime} \mathrm{Y}^{\prime}$
$\Delta Y^{\prime} \equiv \frac{d Y^{\prime}}{d v}=\frac{\partial H}{\partial X^{\prime}}=-\varepsilon X^{\prime}-\frac{\beta \mathrm{g} \sqrt{\mathrm{A}^{2}+\mathrm{B}^{2}}}{\sqrt{1+\alpha^{2}}}\left(\mathrm{X}^{2}-\mathrm{Y}^{\prime 2}\right)$.
If we use two sextupole magnets as a resonance exciter, the strengths of the two sextupole magnets are given by
$\mathrm{g}_{1}=\frac{\left(\mathrm{A}_{2} \tan 3 \psi+\mathrm{B}_{2}\right) \cos 3 \psi \sqrt{\mathrm{~A}^{2}+\mathrm{B}^{2}}}{\mathrm{~A}_{2} \mathrm{~B}_{1}-\mathrm{A}_{1} \mathrm{~B}_{2}} \mathrm{~g}$
$g_{2}=-\frac{A_{1} \tan 3 \psi+B_{1}}{A_{2} \tan 3 \psi+B_{2}} g_{1}$.

But in above expressions, there is a little difference between the separatrices obtained by the theory and tracking of the orbit with a computor, because we take one of the non-linear terms which has the largest contribution to the resonance orbit.

For example, if we assume $S$ to be $10 \pi \mathrm{~mm} . \mathrm{mrad}$ and $\Delta x$ to be 5 mm , we have

$$
\begin{aligned}
& \Psi=32^{\circ} \\
& \mathrm{g} 1=1.461 \mathrm{~m}^{-3} \\
& \mathrm{~g} 2=-0.333 \mathrm{~m}^{-3} .
\end{aligned}
$$

Result of tracking is shown in Fig.3, where the outgoing trajectory at the entrance of ES is defferent from that obtained by the theory.


Fig. 3 The separatrices and outgoing trajectories at the entrance of ES $\left(\Psi=32^{\circ}\right)$

We calculate tracking of the orbit with a computor changing $\psi$ as a parameter and select the best strenghs of the sextupole magnets.

$$
\begin{aligned}
& \Psi=29^{\circ} \\
& \mathrm{g} 1=1.166 \mathrm{~m}^{-3} \\
& \mathrm{~g} 2=0.025 \mathrm{~m}^{-3} .
\end{aligned}
$$

Result of tracking is shown in Fig.4, where the outgoing trajectory at the entrance of ES nearly coincide with that obtained by the theory.


Fig. 4 The separatrices and outgoing trajectories at the entrance of ES $\left(\Psi=29^{\circ}\right)$

Next we calculate the tracking for the beam with $\pi \varepsilon \mathrm{x}=10 \pi \mathrm{~mm} . \mathrm{mrad}$ and within the limit of $\Delta \mathrm{p} / \mathrm{p}= \pm 10^{-3}$. The tracking results show that the extraction efficiency exceeds $90 \%$.

## 3 SUMMARY

We have found that ion beam slow extraction is carried out by using third order resonance ( $v_{\text {res }}=20 / 3$ ). This procedure is required to be further investigated in detail for the installation of all the necessary hardwares.

## REFERENCES

[1] Y.Kobayashi, "Theory of the resonant beam ejection from synchrotrons", Nuclear Instruments and Methods 83, p.77~87, 1970.
[2] T. Ohkawa et al., Proc. 1997 Particle Accelerator Conf., Vancouver, to be published.

