# EFFECT OF ENERGY-PHASE CORRELATION ON THE COHERENT EMISSION FROM AN RF MODULATED ELECTRON BEAM

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### Abstract

In the design of electron accelerators for Free Electron Lasers, the manipulation of the particle distribution in the phase space can play a crucial role to realize high efficiency generators of coherent radiation in the mmwave and far-infrared regions and at even shorter wavelengths. An RF modulated electron beam passing through a magnetic undulator emits coherent radiation at harmonics of the RF with a phase which depends on the electron drift velocity. At long wavelengths a proper energy ramping of the electrons during the bunch can be exploited to lock in phase the field radiated by the individual electrons, resulting in а significant enhancement of the coherent emission. The feasibility of a device suitable for a systematic investigation of energyphase correlation effects is presented in this paper together with a comparison between the theoretical model and computer simulations. We also describe the preliminary design of an accelerating structure composed of a beta-graded self-focusing RF linac operating at 3 GHz followed by a "Phase-Matching Section" controlled in phase and amplitude with respect to the main linac section. The accelerator will drive a compact high efficiency mm-wave generator.

# 1 COHERENT EMISSION AND ENERGY-PHASE CORRELATION

### 1.1 Coherent radiated field

The spectral distribution of the radiated power emitted by a RF modulated electron beam passing in a waveguide inserted in an undulator with period  $\lambda_u$  has been derived through the analytical approach described in Ref. [1] and [2]. The radiated power relative to the harmonic *l* of the fundamental frequency  $v_{RF}$  is calculated from the time averaged flux of the Poynting vector as:

$$P_{l} = \frac{\beta_{gl}}{2Z_{0}} \left| A_{l,0,n} \right|^{2} \tag{1}$$

where  $A_{1,0,n}$  are the amplitude coefficients of the e.m. field expansion. The total power of the coherent spontaneous emission (CSE) can be obtained by summing (1) over the number of harmonics.

If the electron bunches are treated as a collection of  $N_e$  particles of charge q, each with its energy  $\gamma_i$  and phase  $\Psi_i$ ,

the coefficients can be simply calculated as:

$$A_{l,0,n} = -\frac{Z_0}{\beta_g} \frac{q}{T_{RF}} \frac{KL}{\sqrt{ab}} F \sum_{j=1}^{N_c} \frac{1}{\beta_j \gamma_j} \frac{\sin(\theta_j/2)}{(\theta_j/2)} e^{i\left(\frac{\theta_j}{2} + i\psi_j\right)}$$
(2)

where 
$$\theta_j = \left(\frac{\omega_l}{c\beta_{zj}} - k_u - k_{0,n}\right)L$$
 (3)

with L=undulator length, K=undulator parameter, Zo =377  $\Omega_{k_u}$ =  $2\pi/\lambda_u$  wave undulator number,  $k_{o,n}$ = TE<sub>on</sub> waveguide mode number, axb = waveguide cross section,  $\beta_g$ =normalized group velocity of the waveguide mode.

The term *F* indicates a form factor describing the overlapping between the electron beam transverse distribution and the waveguide mode. The phase term  $\theta_j/2$  describes the dependence of the phase shift from resonance on the electron drift velocity, and  $\beta_{z0}$  is the drift velocity of the reference particle injected in the undulator at t = 0.

### 1.2 Phase-Energy correlation effect

It is evident from (2) that constructive interference is achieved when the electrons are distributed in the phase space  $(\psi, \gamma)$  as close as possible to the "phase-matching" curve given by:

$$\left(\frac{\theta_l(\gamma)}{2}\right) + l\psi = \Phi_l \tag{4}$$

where  $\Phi_l$  is a constant phase at the harmonic *l*. Substituting (3) in (4) and putting  $\Phi_l=0$  the phase matching relation can be reformulated as:

$$\psi = -\pi \frac{L}{cT_{RF}} \left( \frac{1}{\beta_z(\gamma)} - \frac{1}{\beta_{z0}} \right)$$
(5)

This condition corresponds to a velocity spread such that the electrons in the head of the bunch are slower than those in the tail and it can be easily shown that this velocity modulation leads to a minimum bunch length at the undulator center. Of course the energy-phase correlation method is most effective at low energy.

The effect of phase-matching has been evaluated for a set of experimental parameters providing a broad band emission in the frequency range between 60 and 180 GHz. We generated randomly an ensemble of 3000 particles filling a rectangular area of 7.2 degrees x 30 KeV and then we rotated this rectangle according to the condition (5) (Figure 1a). The spectral distribution of power computed by the relation (1) assigning a charge of 0.1 pC

to each particle for the two cases is shown in Fig. 1b. Coherent spontaneous emission occurs in both cases due to the short bunch duration. However, in spite of the identical value of longitudinal emittance and bunch length  $(360^{\circ} \text{ at } 150 \text{ GHz})$ , the correlated case shows an enhancement of the high frequency components in the spectrum, with a total radiated power one order of magnitude greater than the uncorrelated case.



Figure 1: (a) correlated and uncorrelated energy-phase distributions (b) relative CSE power vs the harmonic number *l* of the fundamental frequency  $v_{RF}$  = 3 GHz.

The dependence of the total radiated power from the slope in the longitudinal phase-space is illustrated in Figure 2 where it is plotted for two different values of the bunch central energy as a function of  $m=\Delta\gamma/(\gamma\Delta\Psi)$ .



Figure 2: Total CSE power vs the slope in the longitudinal phase-space for two bunch centre energies.

# **2 EXPERIMENT DESCRIPTION**

### 2.1 General Layout

In order to perform a systematic investigation of the energy-phase correlation effect, we have designed an experimental system that can offer the maximum versatility. It consists (Figure 3) of a linear accelerator composed by an electron gun, a short transport line and two accelerating sections followed by a permanent magnet planar wiggler located at a distance of 30 cm. In the space between the accelerator and the wiggler three matching quadrupoles are placed. The wiggler is 40 cm long, consists of 16 periods and operates with a magnetic field of 6000 Gauss corresponding to  $K_w=1$ . Inside the wiggler a rectangular waveguide is placed having cross section dimensions a x b = 10.668 x 4.318 mm<sup>2</sup>.



Figure 3: Experiment layout

#### 2.2 Accelerator System

The electron beam (1 A - 13 kV) is produced by a pulsed triode gun equipped with a 7.7 mm diameter osmium treated dispenser cathode. A suitable optical magnetic lens system matches the input admittance of the accelerator which is composed by two modules:

a) a  $\beta$ -graded on axis S band (2998 MHz) linear accelerator (LINAC), with three full cells and two halves end cells, operating in the  $\pi/2$  mode. This section accelerates an electron macro bunch current of 0.40 A, to an energy of 1.8 MeV; the coupling coefficient to the waveguide  $\beta$  is 3.2.

b) a phase matching section (PMS) placed 4 cm downstream of the LINAC output. The drift space and the phase shift of the PMS are set to have the reference electron passing through the centre of the PMS with a phase close to zero. The coupling coefficient to the waveguide  $\beta$  is 1.1. The PMS is composed by three on axis coupled cavities (one and two halves) operating in the  $\pi/2$  mode tuned at the same frequency of the LINAC. Two motorised plungers inserted at the end cavities set the frequency at the exact design value. In addition a cooling system provides a control of the LINAC and PMS temperature fixed at 30°C ± 0.05 °C.

The distribution of the bunched electrons in the phase space at the PMS output can be changed by varying the phase and the amplitude of the RF field driving the PMS with respect to the LINAC RF.

The total RF power required is about 2 MW. In order to control phase and amplitude independently each other, a suitable RF system has been designed. It makes use of a high stability low power cavity controlled oscillator, an amplifying chain and a 10 MW klystron equipped with a 3- dB power splitter: one arm feeds and controls the RF accelerating field amplitude to the linac, the other is equipped with a high power variable attenuator and a variable high power phase shifter (0°- 360°) enabling the

power and phase control of the PMS module.

# **3 NUMERICAL SIMULATIONS**

## 3.1 PARMELA Simulations

Numerical simulations of the beam dynamics were done by using the PARMELA code. The calculation uses 3000 particles leaving the gun with a charge per particle of 0.11 pC corresponding to a beam current of 1 A with an energy of 13 KeV and a unnormalized emittance of 17  $\pi$ mm mrad and takes into account the space charge effects. Extensive computer simulations were done varying the crucial parameters of the system: the distance between the linac and the PMS, the relative electric field amplitude in the two structures and the relative phase. In Figure 4 and 5 the computed phase space distributions in four points of the beam line are shown. In this case the electric field in the PMS is 60% of the electric field in the linac ( $E_{iinac} = 25$ MV/m) and the relative phase is adjusted in order to have the proper correlation at the wiggler input (m  $\approx 0.5$ ) including the effect of the drift between the PMS and the wiggler. Keeping constant the RF field in the linac and increasing the amplitude in the PMS up to a maximum value of 28 MV/m (that accordingly to our experience is reachable without sparking), it will be possible to reproduce experimentally the curve of Figure 1 until m=1.



Figure 4: Computed phase-space distribution (a) at linac output (b) at PMS output



Figure 5: Computed phase-space distribution (a) at wiggler input (b) at the center of the wiggler

# 3.2 Spontaneous Coherent Emission

PARMELA does not take into account the spontaneous radiation emission in the undulator. So we evaluated the sensitivity of the system to the phasing between the two sections inserting the PARMELA beam data at the undulator input in the relation (1) getting the curves shown in Figure 6. For a fixed amplitude in the linac and in the PMS, we moved the phase in a range of  $12^{\circ}$  respect to the value that maximises the band power and computed the emitted radiation power in the  $50^{\circ}$  harmonic and in the total band.



Figure 6: Computed power on the 50th harmonic (150 GHz) and total band power vs the phase shift between LINAC and PMS. The phase shift is defined respect to the value that maximises Pband.  $E_{PMS}$ =0.6  $E_{linac}$ .

Moving the phase moves the reference energy and it can be observed that a variation of 7 degrees produces a band power variation of about 10%. Our system will allow us to trace a curve of this type.

In these calculations the transverse beam distribution was neglected. In order to estimate the effect of the beam transverse emittance (a value RMS of 5  $\pi$  mm mrad is foreseen) we included the particles transverse coordinates as computed by PARMELA in the calculation of the coefficients  $A_{L0,n}$  considering the consequent spread in the drift velocities, the variation of the magnetic field in the undulator and a factor form F given by the actual beam distribution. In this way a drop of about 35% in the CSE band power has been estimated.

# **4 CONCLUSIONS**

Analytical and numerical calculation show that the manipulation of the particle distribution in the phase space can provide a tool for the realization of high efficiency generators of coherent radiation in the mm-wave region. A test device suitable for a systematic investigation of this effect has been completely designed and is currently under construction.

### REFERENCES

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