# BEAM THREADING IN THE LHC 

H. Grote, CERN, Geneva, Switzerland


#### Abstract

Particles circulating in the LHC will experience a considerable amount of nonlinear magnetic forces, in particular at injection. Therefore it is important to show that a first circulating beam can be achieved at all, albeit in successive steps of measurement and correction. This has been done in simulation using a realistic model of the LHC which includes all estimated magnet alignment and field errors. The automatic procedure is based on pickups and corrector magnets. The results of this study have influenced the choice of the pickup system for the LHC. In particular, they show that a sufficiently smooth initial trajectory can be achieved even with a certain percentage of "dead" pickups.


## 1 INTRODUCTION

A fully defined LHC contains alignment and field errors for all magnetic elements. These errors are simulated by choosing random errors from Gaussian distributions.

Both in simulation, and in the real machine it will happen that the beam does not circulate without "steering" with the help of corrector magnets. In order to simulate the real machine as closely as possible, a beam threading algorithm was developed and implemented in MAD[1] that makes use of pickup readings and corrector magnet settings only.

The MAD closed orbit finder always starts with a first turn in which a particle is tracked in four- or sixdimensional phase space (the latter in the presence of cavities and radiation); the tracking algorithm considers all linear and non-linear effects defined in the machine at that moment, including displacement errors. After one full turn, the closure condition provides an estimate for the starting point, based on the linearized one-turn matrix. The whole procedure is repeated until the difference between two successive closure point vectors is small enough. Therefore it is crucial to be able to track once around the full machine. However, when trying this in a fully defined LHC machine with alignment and field errors, MAD does normally not manage to complete a first turn, since the nonlinear field errors kick the particle out at relatively low trajectory amplitudes (see Figure 1 and Figure 2).

## 2 THREADER IMPLEMENTATION

### 2.1 The LHC model

The LHC model used is version 4.3, since it is the last version in which pickups and corrector magnets are implemented. There are a total of 498 double-readout pickups (providing both horizontal and vertical readings), 261 hori-
zontal, and 261 vertical corrector magnets, typically behind each arc quadrupole, correcting the projection in which the quadrupole focuses. The alignment errors for dipoles and quadrupoles, and the field errors for arc dipoles are given in Table 1 and Table 2. The field errors are visualized in Figure 1. One notices the "brick wall" around amplitudes of 10 mm if one admits maximum relative errors of $10^{-5}$ to $10^{-4}$. Since an error of $10^{-3}$ leads already to a maximum trajectory excursion of 3 mm over the length of one single arc cell (ca. 90 m ), it becomes clear from the plot that the orbit amplitudes should stay well below 10 mm at all times.

| component | best current <br> estimate (rms) |
| :---: | :---: |
| main dipole roll | 1 mrad |
| quad transverse alignment | 0.5 mm |
| quadrupole roll | 1 mrad |

Table 1: Dipole and quadrupole alignment errors.

| order | injection | end injection | collision |
| :---: | :---: | :---: | :---: |
| 1 | -9.3 | -6.51 | 0. |
| 2 | 0.246 | -0.246 | 1.994 |
| 3 | -4.3 | -3.06 | 0.456 |
| 4 | 0.11 | -0.11 | 0.13 |
| 5 | 0.093 | 0.031 | 0.122 |
| 6 | 0.0018 | 0.0018 | 0.001 |
| 7 | 0.0177 | -0.007 | 0.019 |
| 8 | 0.00004 | 0.00004 | 0.00003 |
| 9 | 0.0012 | -0.001 | 0.0063 |
| 10 | 0.00001 | -0.00001 | 0. |
| 11 | 0.0089 | 0.0089 | 0.0091 |
| 12 | 0. | 0. | 0. |
| 13 | 0.00045 | -0.00045 | -0.00045 |
| 14 | 0. | 0. | 0. |
| 15 | 0.00004 | -0.00004 | -0.00004 |

Table 2: $\Delta B / B$ (in $10^{-4}$ ) (normal component only) of main dipoles at 10 mm radial. This table was in use with version 4.3.

### 2.2 Algorithm

The threader routine inside MAD deals with trajectory excursions in a way that is close to the operation of the real machine: it only uses the pickup read-out values to correct the trajectory by setting the corrector magnet strength. The procedure treats both planes independently since this has
turned out to be sufficient. However, the algorithm is so simple that it can easily be extended to a 4-D treatment.

The procedure works as follows: MAD tracks along (including all non-linear effects in the magnets) until the trajectory excursion in the horizontal or vertical transverse coordinate at a pickup $P_{1}$ becomes too large. One then searches backwards until one finds a pickup $P_{2}$; the phase advance from $P_{2}$ to $P_{1}$ must be different from a multiple of $\pi$, otherwise the backward search continues. Once two suitable pickups are found, one now searches further backwards for two corrector magnets $C_{1}$ and $C_{2}$ for the current projection (horizontal or vertical). For these correctors as well, the phase advance from $C_{2}$ to $C_{1}$ must be different from a multiple of $\pi$, otherwise the backward search continues until two suitable corrector magnets are found.

We now want to power the correctors in such a way that the amplitudes at $P_{1}$ and $P_{2}$ become both zero. This is equivalent to making the amplitude and the angle at one pickup equal to zero (because of the phase advance condition), but of course we do not know the angle and therefore have to use two pickups.

orbit amplitude [m]
Figure 1: Relative field errors as a function of the radial distance from the zero orbit. Since the lower orders will be corrected, only the orders 4 to 15 are shown ( 12 and 14 are missing, see Table1). The steeper a line, the higher the order. The bent curve at the left gives the total of all orders shown.

Let us assume now that we deal with the horizontal projection. The pickups give the read-out values $x_{1}$ and $x_{2}$. The $2 \times 2$ transport matrix from the start of the machine to pickup $P_{1}$ is called $M_{P 1}$ etc., the transport matrix from $P_{2}$ to $P_{1}$ is called $M_{P 2-P 1}$ etc.; the kick angles necessary for the trajectory correction are $c_{1}$ and $c_{2}$, respectively.

We now want to compensate the pickup-readouts $x_{1}$ and $x_{2}$, i.e. we want the first components of the following two vectors to become zero:
$M_{C 1-P 1}\left[M_{C 2-C 1}\binom{0}{c_{2}}+\binom{0}{c_{1}}\right]+\binom{x_{1}}{0}$
$M_{C 1-P 2}\left[M_{C 2-C 1}\binom{0}{c_{2}}+\binom{0}{c_{1}}\right]+\binom{x_{2}}{0}$
This leads after some trivial matrix manipulations to the linear equation

$$
\left(\begin{array}{cc}
a_{\alpha} s_{\beta}+a_{\beta} s_{\delta} & a_{\alpha} r_{\beta}+a_{\beta} r_{\delta} \\
b_{\alpha} s_{\beta}+b_{\beta} s_{\delta} & b_{\alpha} r_{\beta}+b_{\beta} r_{\delta}
\end{array}\right)\binom{c_{2}}{c_{1}}=-\binom{x_{1}}{x_{2}}
$$

(with $\alpha \equiv 11, \beta \equiv 12, \gamma \equiv 21$, and $\delta \equiv 22$ ) where the $a_{i j}$ and $b_{i j}$ are the components of the matrices $M_{P 1}$ and $M_{P 2}$, and the $r_{i j}$ and $s_{i j}$ the components of the matrices $M_{C 1}^{-1}$ and $M_{C 2}^{-1}$.

Once the correctors have been set accordingly, the tracking resumes just before $C_{2}$, with the very important change that the check on the trajectory amplitude is not performed until one reaches $P_{1}$ again; this in order to avoid that a small (but necessary) excess of the amplitude caused by the kicks just applied, leads to another correction which partially cancels the effect of the previous one, and this could lead to an infinite correction loop. However, at $P_{1}$ one now has to check again that the amplitude is within limits, because the correction was based on the linearized transport matrix and therefore need not be sufficient. Indeed, it happens in the case of large trajectory excursions that the algorithm performs two or three successive corrections at the same place before the requested precision is reached, and it can continue.

The trajectory values just before $C_{2}$ may be obtained from the pickup-readouts in a way similar to the calculation of the kicker angles. They are the solution of the linear equation (assuming zero length for $C_{2}$ )

$$
\left(\begin{array}{cc}
a_{\alpha} r_{\alpha}+a_{\beta} r_{\gamma} & a_{\alpha} r_{\beta}+a_{\beta} r_{\delta} \\
b_{\alpha} r_{\alpha}+b_{\beta} r_{\gamma} & b_{\alpha} r_{\beta}+b_{\beta} r_{\delta}
\end{array}\right)\binom{x}{p_{x}}=\binom{x_{1}}{x_{2}}
$$

This equation can as well be used to correct the initial tracking values, if the trajectory excess occurred so close to the start that no two preceding correctors could be found. The algorithm requires that the beam passes at least two pickups.

The transport matrices from the start of the machine to all correctors and pickups have to be kept, or at least all those in a reasonable range behind the current tracking position; if this causes problems with memory, a sliding window can be used.

Once the end of the machine is reached, the orbit is closed with the help of the last two correctors. This time one uses the (known) initial values $x, p_{x}, y, p_{y}$ which leads to similar equations as above.

### 2.3 Imperfections

The magnet alignment and field imperfections have already been summarized. There are 261 horizontal and 261 vertical correctors in the LHC model used that are assumed to be flawless. With the 498 pickups measuring both horizontal and vertical beam positions, the situation is different: they will be misaligned, have electronic errors; some will not work at all and give either no signal, or random signals: it is assumed that all pickups in this last category are known and switched off. For pilot bunch intensities, the combined effect of uncorrected pickup misalignments, and of electronic errors will result in a total position uncertainty of ca. 0.7 mm r.m.s. [2]; this pickup error is simulated in the study presented here.


Figure 2: Unaided MAD tracking (dashed, big amplitudes) and tracking with threader (solid, small amplitudes), horizontal projection. The unaided particle hits the beam screen and is lost. The threaded particle amplitude stays below 11 mm over the full circumference.

## 3 RESULTS

The LHC model described here is simulated with alignment and field errors, the average $a_{2}$ (skew quadrupole) and $b_{3}$ (sextupole) components of the dipole errors are corrected with the help of spool pieces. The pickup readouts are simulated including their random error. In view of the pickup resolution, a (measured) trajectory excursion of up to 2 mm is tolerated before a correction takes place. It was found for several random number seeds that in all cases:

- without threader MAD cannot complete a first turn
- with threader, the first turn is always found and can subsequently be corrected further with the help of the MICADO algorithm
- the threader with subsequent MICADO still works when $30 \%$ of the pickups are switched off randomly.

A typical first turn without and with threading is shown in Figure 2. One can see that MAD struggles along for quite some while in this case, but with trajectory amplitudes of 10 to 20 mm and above, before a fatal kick throws it off track. This means that even if the trajectory did a full turn in MAD (respectively the pilot bunch in the machine), the MICADO algorithm would almost certainly not work. The threader trajectory stays below 5 mm most of the time, with occasional peaks up to 10 mm . This is in good agreement with the qualitative expectations, and is sufficient for a subsequent orbit refinement.

## 4 SUMMARY

A threader algorithm for the LHC has been implemented in MAD. From a simulation as realistic as possible, including magnet alignment and field errors, pickup readout errors, and "dead" pickups it appears that a first turn in the machine can be achieved safely, and can be further corrected with the help of the MICADO algorithm. This justifies the assumption that the pilot bunches in the LHC can be safely steered around the machine with a similar algorithm.

## 5 REFERENCES

[1] H. Grote and F.C. Iselin, "The MAD Program", CERN/SL/90-13 (AP) (Revision 3), 1992.
[2] Hermann Schmickler(CERN), private communication, May 1998.

