INSTABILITY OF ELECTRON BEAMS AT DSR

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Abstract

Single-bunch instability of electron beams at double storage ring (DSR) in MUSES project at RIKEN was calculated. Taking into account broadband impedance estimated at the DSR ring, the calculations were performed in both the frequency domain and the time domain for small-emittance operation mode of the DSR. Results of both calculations are consistent with each other. The single-bunch threshold current for the presently designed DSR is found to be about 0.75 mA at 1 GeV and 6.6 mA at 2 GeV.

1 INTRODUCTION

In the RI beam factory project [1], we will construct accelerator complex, which is called MUSES project [2], downstream the super-conducting ring cyclotron and the fragment separators. At the DSR in the MUSES, we plan to make unique experiments such as electron-RI collision and X-ray-RI collision experiments. For these experiment [3], we have to store high-quality electron beam with large average current. Especially, for X-ray-RI collision experiment, the electron beam emittance should be order of 10^{-9} mrad to produce high-brilliant soft X ray using an undulator [4]. The design of the DSR lattice [5] for small emittance electron beam give us serious problem on the beam instability. We have to specially care about the instability to get the target current of 500 mA in the energy range of 0.3 GeV - 2.5 GeV. As the first step of study of the instability at DSR, we calculated the single bunch instability caused by broadband impedance for small emittance operation mode of the presently designed DSR. The calculations are performed in both the frequency domain and the time domain.

Table 1. List of sources of broadband impedance.

Elements	#	Z_L/n [Ω]	Z_T [MΩ/m]
Bellows	50	-0.637i	-0.087i
Tapers	2	-0.0008i	-0.0001i
Vacuum ports	90	-0.0027i	-0.0004i
Button electrodes	20	-0.076i	-0.0103i
Flanges	300	-0.003i	-0.0004i
Weldments	200	-0.0006i	-0.00008i
Valves	30	12/n -0.001i	14/n-0.001i
Space charge	-	-0.0006i	-0.00008i
Resistive wall	-	$0.6(1-i)\,1/\sqrt{n}$	$0.08(1-i)\,1/\sqrt{n}$
Cavities	2	$8000(1+i)\,1/n\sqrt{n}$	$1098(1+i)\,1/n\sqrt{n}$

2 BROADBAND IMPEDANCE

The sources of the broadband impedance and their estimated impedance are listed in Table 1. The tapers given in the table are for the undulator section. The tube radius is also assumed to be 25 mm. The biggest contribution to the broadband impedance comes from bellows.

3 CALCULATION IN FREQUENCY DOMAIN

The instability calculation in frequency domain is based on treatment of the eigenequation derived from well-known Sacherer's equation:

$$(\Omega-m\omega_s)\, R_m(r) = -\,i\,\frac{me^2\omega_0}{E\,T_0\,\beta^2}\,\frac{\varphi_0'}{r}\sum_{m'=-\infty}^{\infty}\int_0^{\infty} R_{m'}(r')r'dr'\, i^{m-m'}\sum_{p=-\infty}^{\infty}\frac{Z_L(\omega')}{\omega'}\,J_m(\omega' r)\,J_{m'}(\omega' r') \tag{1}$$

for longitudinal and

$$(\Omega-\omega_\beta-m\omega_s)\, R_m(r) = -\,i\,\frac{ve^2\omega_0^2\omega_s}{4\,\pi\,E\,\eta\,\omega_\beta}\,\varphi_0(r)\sum_{m'=-\infty}^{\infty}\int_0^{\infty} R_{m'}(r')r'dr' i^{m-m'}\sum_{p=-\infty}^{\infty} Z_T(\omega')\,J_m\!\left(\omega' r-\frac{\xi\omega_0}{\eta}r\right)J_{m'}\!\left(\omega' r'-\frac{\xi\omega_0}{\eta}r'\right) \tag{2}$$

for transverse, where $\omega'=p\omega_0+\Omega$, $R_m(r)$ is radial part of the perturbed part $\varphi_1(r)$ of particle distribution $\varphi(r)$:

$$\varphi(r) = \varphi_0(r) + \varphi_1(r) = \varphi_0(r) + \sum_{m=-\infty}^{\infty} R_m(r)\, e^{i\,m\,\phi} \tag{3}$$

in the longitudinal phase space, J_m is Bessel function, and the others are conventional notations. The solution of $\Omega-m\omega_s$ and $\Omega-\omega_\beta-m\omega_s$ is complex number; the imaginary part gives the growth rate of excited m-th mode of instability. Assuming the Gaussian distribution as $\varphi_0(r)$, we can express $R_m(r)$ by Laguerre polynomial $L_h(R)$. Finally, we can find a simple eigenequation:

$$\left(\frac{\Omega^{(m)}}{\omega_s} - m\right) a_h^{(m)} = \sum_{n=-\infty}^{\infty}\sum_{l=0}^{\infty} M_{h\,l}^{(m\,n)}\, a_l^{(n)} \tag{4}$$

for longitudinal, where $a_i^{(m)}$ is expansion coefficient in $R_m(r)$, and the matrix elements are expressed as

$$M_{h\,l}^{(m\,n)} = i\,\frac{me^2\eta\omega_0 N}{2\,\pi\,E\,T_0\beta^2\sigma^2\omega_s}\sum_{n=-\infty}^{\infty}\sum_{l=0}^{\infty}\sum_{p=-\infty}^{\infty} i^{m-n}\,\frac{Z_L(\omega')}{\omega'}\, I_{mh}I_{nl} \tag{5}$$

where σ is the bunch length and

$$I_{mh} = \int_0^{\infty} e^{-R} L_h^{(m)}(R) J_m(\omega'\sigma\sqrt{2r})\, dR \quad .$$

For transverse direction, the same kind of equation including chromatisity is easily derived. In the treatment of broadband impedance, we can find that the matrix elements are written by only real numbers.

Because user's request on the electron beam energy is mainly 1 GeV - 2 GeV, the following calculations are performed for the case of the energy of 1 GeV or 2 GeV.

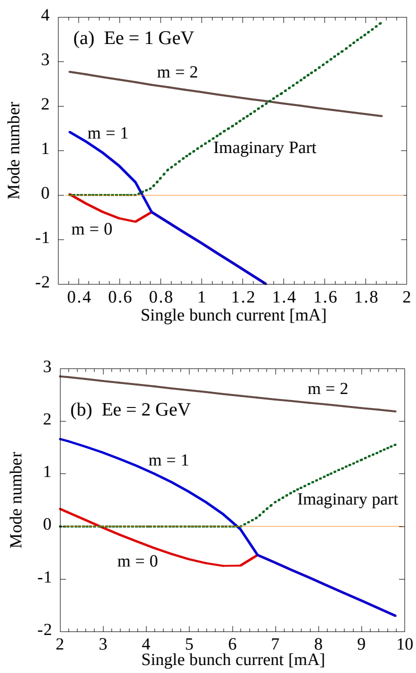

Fig. 1. Eigenvalues as a function of single-bunch current at 1 GeV and 2 GeV.

Figure 1 shows eigenvalues as a function of single-bunch current. They are obtained by solving the eq. (4) using above impedance. When we solve the equation, we took into account the bunch lengthening due to potential-well distortion. As shown in the figure, the mode coupling starts at about 0.75 mA and 6.6 mA for 1 GeV and 2 GeV, respectively. We found that the mode coupling in transverse direction also starts at 1.4 mA and 7.8 mA for 1 GeV and 2 GeV, respectively. Theses are threshold current of strong instability.

The threshold current dependson the RF voltage, because the bunch length is changed by the RF voltage. Relation between the threshold and the bunch length is shown in Fig. 2. Open circles indicate the threshold current at each RF voltage. The threshold current increase as increasing the RF voltage. Its bunch length, however, is going to be shorter. The bunch shortening limits the possible maximum current. It is supposed from the figure that the maximum is not exceeded1.5 mA for the case of 1 GeV. That is about one orderless than our target current of 500 mA (single bunch current 16 mA times 30 bunches).

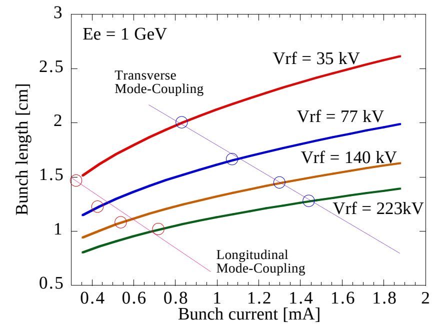

Fig. 2. Bunch lengthening due to potential-well distortion. The mode-coupling starting points are indicated by open circles together with lines for guide the eyes.

4 CALCULATION IN TIME DOMAIN

Another approach adopted here is calculation in time domain. This calculation is based on the tracking calculation using motion equations including wakepotential produced by bunched beam itself at the source of impedancelisted in Table 1. We start from well known betatron motion equations and synchrotron motion equations. They are written as

$$\frac{d\,y_i}{d\,s} = y_i', \quad \frac{d\,y_i'}{d\,s} = -\left(\frac{\omega_\beta + \xi\omega_0\delta_i}{v}\right)^2 y_i + \frac{F_{yi}(\tau,s)}{E} \ ,$$

$$\frac{d\,\tau_i}{d\,t} = -\,\eta\,\delta_i, \quad \frac{d\,\delta_i}{d\,t} = \frac{\omega_s^2}{\eta}\,\tau_i - \frac{e\,V_i(\tau,s)}{T_0E} - \frac{U_0}{T_0E}\left(1+2\delta_i\right), \qquad (6)$$

for i-th particle, where $F_{yi}(\tau,s)$ and $V_i(\tau,s)$ are the force and the potential due to wakefield, the others are conventional notations. Transverse motion in only vertical direction is took into account in the present calculation. Introducing valuables η_i and θ_i:

$$\eta_i = \frac{y_i}{\sqrt{\beta}} = \mathrm{Re}\left[a_i(\theta)\, e^{i\nu_\beta\theta}\right] \ , \qquad \theta = \frac{1}{\nu_\beta}\int^{s} \frac{1}{\beta}\, ds' \ ,$$

we can make motion equation for the amplitude $a_i(\theta)$ as [6]:

$$\frac{d\,a_i(\theta)}{d\,\theta} = i\,\Delta\nu_\beta a_i(\theta) + \frac{e}{4\pi i E}\sum_{j=1}^{Np} q_j a_j(\theta) \sum_{m=1}^{Nz} \beta_m W_{Tm}(\tau_j - \tau_i) \qquad (7)$$

for transverse and

$$\frac{d\,\delta_i}{d\,\theta} = \frac{eV_0}{T_0}\sin\left(\phi_s + \frac{h\nu}{R}\tau_i\right) - \frac{e}{T_0}\sum_{j=1}^{Np} q_j \sum_{m=1}^{Nz} W_{Lm}(\tau_j - \tau_i) - \frac{U_0}{T_0}\left(1+2\delta_i\right) \ ,$$

$$\frac{d\,\tau_i}{d\,\theta} = -\,\frac{\eta h\nu}{R}\,\delta_i \ , \qquad (8)$$

for longitudinal, where N_p is number of particle in a bunch, q_j is the charge of j-th particle, N_z is number of the source of impedance, and β_m is beta function at m-th source. Transverse radiation damping effect have to be addedto recurrenceformula obtained from eqs. (7) and (8) as:

$$\Delta a_i = -i \frac{e V_{rf}}{E} \mathrm{Im}\left[a_i^* e^{i\theta\nu_\beta}\right] e^{-i\theta\nu_\beta} , \tag{9}$$

where a_i^* is the amplitude before the cavity. The radiation excitation is also included as:

$$\Delta a_i = \sqrt{\frac{4\,\Delta T\,\varepsilon_0}{\tau_\beta}}\; v_i\, e^{i\,2\pi\, w_i} \tag{10}$$

for transverse and

$$\Delta \delta_i = \sqrt{\frac{4\,\Delta T}{\tau_\varepsilon}} \left(\frac{\sigma_\varepsilon}{E}\right) u_i \tag{11}$$

for longitudinal, where ΔT is time advance, ε_0 the equilibrium emittance, τ_β the betatron damping time, τ_ε the synchrotron damping time, σ_ε the equilibrium energy spread, v_i and u_i are random number forming gaussian distribution with rms=1, and w_i is random number forming uniform distribution. Details of the calculation is described in Ref.[6].

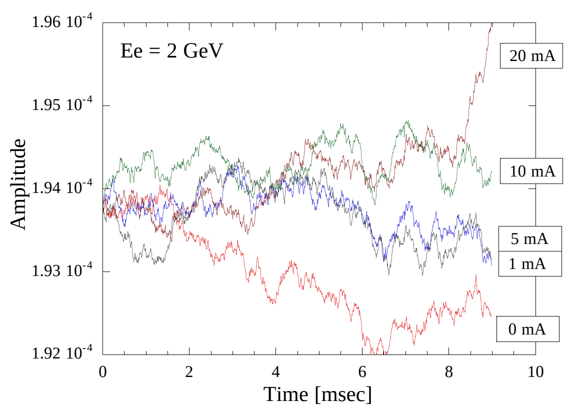

Fig. 3. Amplitude growth as a function of time.

Growth of the amplitude (a_i) at several bunch current is shown in Fig.3. The 0 mA means no wakefield; and in this case, the amplitude decreases due to radiation damping. In the case of 1 mA or 5 mA, it seems that there is no positive growth. Over 10 mA, however, we can see clearly serious growth of the amplitude.

This instability can be also seen in particle trajectory in longitudinal phase space. Figure 4 shows the trajectory of a test particle in phase space for the case of 0, 5, 10, and 20 mA of the bunch current. At lower current, the trajectory is seemed to be periodic and stable, but that is completely chaotic at more than 10 mA. These results means that the threshold current is in between 5 mA and 10 mA. That is consistent with results obtained by calculation in frequencydomain shown above.

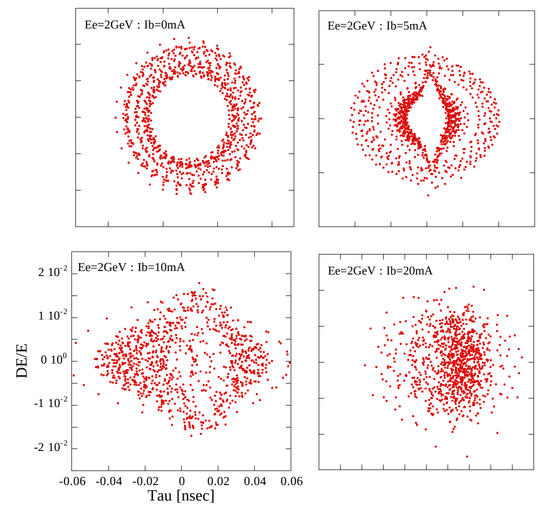

Fig. 4. Single-particle trajectory in phase space.

5 CONCLUSION

From present calculation, we found that the threshold single-bunch current is about 0.75 mA and 6.6 mA at the energy of 1 GeV and 2 GeV, respectively, for presently designed DSR. Both calculations in frequency domain and time domain give nearly the same results for the beam instability. These results make us to understand the existing state of the electron beam at the DSR and suggest how to increase the current up to the target current of 500 mA.

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