# SLOW EXTRACTION METHOD FOR THE COOLER INJECTOR SYNCHROTRON

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#### Abstract

The Cooler Injector Synchrotron (CIS) is a compact synchrotron, presently being commissioned at Indiana University Cyclotron Facility. It is designed to accelerate high intensity polarized proton (deuteron) beam from 7 MeV (5 MeV) to 200 MeV (105 MeV). A slow extraction method is studied to extract a smooth, uniform proton beam by using the half integer resonance. With the uniform beam by the slow extraction, CIS can be used for medical applications.

#### **1 INTRODUCTION**

The Cooler Injector Synchrotron (CIS) at Indiana University Cyclotron Facility (IUCF) was jointly found by National Science Foundation (NSF) and Indiana University in 1994 [1]. The CIS will replace the cyclotron as an injector of polarized ions into the IUCF Cooler ring. CIS can fill the Cooler to about  $10^{11}$  protons in a few second for research. Injection, accumulation and acceleration of  $1.2 \times 10^{10}$  protons to the energy of 235 MeV has been achieved successfully in the past few months [2]. Beam from CIS will be injected into Cooler after finishing of the extraction beam line in August of 1998.

CIS is a very compact machine with low cost of construction and operation. Currently in CIS, single-turn extraction is performed by a fast kicker and a Lambertson magnet [3]. In order to extend the application of the CIS to medical and material sciences, slow extraction is needed.

# 2 PROPERTIES OF COOLER INJECTOR SYNCHROTRON

Figure 1 shows the configuration of CIS which has four superperiods [4]. Each superperiod is composed of a drift space, a dipole magnet with  $90^0$  bending angle and  $12^0$  edge angle at both ends. Four trim quadrupoles are used in order to have the flexibility of adjusting betatron tunes. Figure 2 shows the tune diagram of the CIS ring. At the working point, lattice parameters are shown in Table 1 and lattice functions are shown in Figure 3. Including the chiane dipoles, the horizontal betatron tune becomes 1.48.

# **3 SLOW EXTRACTION SCHEME**

Half integer extraction method is chosen because CIS horizontal tune 1.48 of CIS is close to 1.5. This method is also



Figure 1: The general layout of the Indiana University Cyclotron Facility Cooler Injector Synchrotron at IUCF

Table 1: IUCF CIS lattice parameters		
Parameters	Symbol	Values
Circumference(m)		17.364
Horizontal tune	$\nu_x$	1.4633
Vertical tune	$\nu_z$	0.7788
Dipoles:		
Length(m)	L	2.0
Edge angle	$ heta_e$	12°
Bending radius(m)	ho	1.273
Transition Energy	$\gamma_T$	1.271
Beta x: Maximum(m)	$eta_x$	4.373
Minimum(m)		0.996
Beta y: Maximum(m)	$\beta_y$	3.786
Minimum(m)	0	3.380
Dispersion	$D_x$	
Maximum(m)		1.759
Minimum(m)		1.617
Chromaticity(x)	$C_x$	-0.529
Chromaticity(z)	$C_z$	-0.156



Figure 2: Tune shift by four symmetrical trim quads. Where  $\bigcirc$ stand for CIS working point, riangle for the calculation curve, ullet for the experimental measurement, and  $\Box$  for the tune with chicane dipoles but zero trim quad current.



Figure 3: The CIS lattice functions  $\beta_x$ ,  $\beta_z$  and  $D_x$  for L = $2.0 \text{ m}, \theta_e = 12^0$ 

used at Fermilab [5]. Quadrupoles are used to drive the half integer resonances and octupoles are used to extract the beam.

The vector potential of octupole and quadrupole are:

$$A_{\text{oct}} = \frac{B_3}{24} (x^4 - 6x^2z^2 + z^4),$$
  

$$A_{\text{quad}} = \frac{B_1}{2} (x^2 - z^2),$$
(1)

where  $B_1 = \frac{\partial B_z}{\partial x}$ ,  $B_3 = \frac{\partial^3 B_z}{\partial x^3}$ . The total Hamiltonian is

$$H = \frac{1}{2} [x' + K_x(s)x^2] + \frac{1}{2} [z' + K_z(s)z^2] + \frac{1}{24} \frac{B_3}{B\rho} (x^4 - 6x^2z^2 + z^4) + \frac{1}{2} \frac{B_1}{B\rho} (x^2 - z^2).$$
(2)

In CIS ring, beam orbit is excited in the horizontal direction for extraction. Using Floquet transformation, the Hamiltonian in for x motion is

$$H = \nu_x J_x + \frac{1}{2} \alpha_{xx} J_x^2 + g J_x \cos(2\psi_x - l\theta) + \cdots$$
 (3)

where  $J_x$  and  $\psi_x$  are action-angle coordinates,  $\alpha_{xx}$  is the strength of octupoles, g is the half integer resonance amplitude, and l/2 = 1.5.  $\alpha_{xx}$  and g can be derived as

$$\alpha_{xx} = \frac{1}{16\pi} \frac{1}{|B\rho|} \beta_x^2 \oint B_3 ds \tag{4}$$

$$g = \frac{1}{4\pi} \frac{1}{|B\rho|} \oint \beta_x B_1 \cos 2\chi_x ds \qquad (5)$$

on the condition that

$$\oint \beta_x B_1 \sin(2\chi_x) ds = 0 \tag{6}$$

to ensure the beam trajectory at the septum is in the upright direction.

Using the generating function

$$F_2 = \left(\psi_x - \frac{l}{2}\right) J,\tag{7}$$

the Hamiltonian in Eq. (3) becomes

$$H = \Delta \nu J + \frac{1}{2} \alpha_{xx} J^2 + g J \cos 2\psi \tag{8}$$

where  $\Delta \nu = \nu_x - \frac{l}{2}$ .

To find the beam trajectory in (x, p) space, rewrite the above Hamiltonian as

$$H = \frac{1}{2}\Delta\nu(X^{2} + P^{2}) + \frac{1}{8}\alpha_{xx}(X^{2} + P^{2})^{2} + \frac{g}{2}(X^{2} - P^{2}), \qquad (9)$$

where  $X = \sqrt{2J}\cos\psi$ ,  $P = \sqrt{2J}\sin\psi$ . The actual particle position is given by  $x = \sqrt{\beta_x} X$ . The unstable fix point is  $[X_{\rm U}, P_{\rm U}] = [0, \frac{2}{\alpha_{xx}} (\Delta \nu - g)]$ . The trajectory of the separatrix is

$$\frac{1}{2}(\Delta\nu + g)X^{2} + \frac{1}{2}(\Delta\nu - g)(P^{2} - P_{U}^{2}) + \frac{1}{8}\alpha_{xx}[(X^{2} + P^{2})^{2} - P_{U}^{4}] = 0$$
(10)

or

$$[(X-a)^2 + P^2 - r_0^2][(X+a)^2 + P^2 - r_0^2] = 0 \quad (11)$$

where

$$a^2 = -\frac{2}{\alpha_{xx}}g \qquad r_0^2 = -\frac{2}{\alpha_{xx}}\Delta\nu \tag{12}$$

This means the trajectory in phase space at the septum is circle as shown in Figure 4.



Figure 4: Extraction beam trajectory in phase space

The area of the stable region is

$$S = 4\frac{1}{\beta_s} \left(\frac{1}{2}\theta r_0^2 - \frac{1}{2}r_0^2\sin\theta\cos\theta\right)$$
$$= \frac{2\pi r_0^2}{\beta_s}\Gamma(k), \qquad (13)$$

where

$$\Gamma(k) = \frac{1}{\pi} [\sin^{-1} \sqrt{1 - k} - \sqrt{k - k^2}]$$
(14)

and  $k = g/\Delta \nu$ .

The step increase per turn of the extracted beam position

$$\frac{dX}{dn} = 2\pi \frac{\partial X}{\partial \psi} = 2\pi \frac{\partial H}{\partial P}$$
$$= 2\pi \left[ (\Delta \nu - g)P + \frac{\alpha_{xx}}{2}P(X^2 + P^2) \right]$$
$$= 2\pi P \Delta \nu \left[ 1 - k - \frac{1}{r_0^2}(X^2 + P^2) \right]$$
(15)

#### **4** PRACTICAL CONSIDERATION

For the CIS, the vacuum chamber is about 50 mm. We have set the the wire septum at 15 mm, an available aperture of about 200  $\pi$ mm-mrad.

Figure 5 shows the change of step size per turn versus the horizontal position. In order to get the maximum extraction efficiency, the step size at the septum should maximum. From the calculation, the radius of the beam circle is  $r_0 = 12$  mm. The emittance of CIS beam at extraction energy is 10  $\pi$ mm-mrad and the betatron function in the straight section is unfortunately only 1 m. For practically application, we can install the wire septum as near the dipole as possible.

# **5** CONCLUSIONS

A half integer resonance slow extraction scheme is possible for the CIS in order to extend its application to the medical science. By putting quadrupoles in the proper positions, the extraction efficiency can be maximized.



Figure 5: Step size of the extracted beam versus horizontal position. Wire septum is at x = 15 mm.  $r_0 = 12$  mm, a = 8.5 mm.

### **6 REFERENCES**

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