# SLOW EXTRACTION METHOD FOR THE COOLER INJECTOR SYNCHROTRON 

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#### Abstract

The Cooler Injector Synchrotron (CIS) is a compact synchrotron, presently being commissioned at Indiana University Cyclotron Facility. It is designed to accelerate high intensity polarized proton (deuteron) beam from 7 MeV (5 $\mathrm{MeV})$ to $200 \mathrm{MeV}(105 \mathrm{MeV})$. A slow extraction method is studied to extract a smooth, uniform proton beam by using the half integer resonance. With the uniform beam by the slow extraction, CIS can be used for medical applications.


## 1 INTRODUCTION

The Cooler Injector Synchrotron (CIS) at Indiana University Cyclotron Facility (IUCF) was jointly found by National Science Foundation (NSF) and Indiana University in 1994 [1]. The CIS will replace the cyclotron as an injector of polarized ions into the IUCF Cooler ring. CIS can fill the Cooler to about $10^{11}$ protons in a few second for research. Injection, accumulation and acceleration of $1.2 \times 10^{10}$ protons to the energy of 235 MeV has been achieved successfully in the past few months [2]. Beam from CIS will be injected into Cooler after finishing of the extraction beam line in August of 1998.

CIS is a very compact machine with low cost of construction and operation. Currently in CIS, single-turn extraction is performed by a fast kicker and a Lambertson magnet [3]. In order to extend the application of the CIS to medical and material sciences, slow extraction is needed.

## 2 PROPERTIES OF COOLER INJECTOR SYNCHROTRON

Figure 1 shows the configuration of CIS which has four superperiods [4]. Each superperiod is composed of a drift space, a dipole magnet with $90^{\circ}$ bending angle and $12^{0}$ edge angle at both ends. Four trim quadrupoles are used in order to have the flexibility of adjusting betatron tunes. Figure 2 shows the tune diagram of the CIS ring. At the working point, lattice parameters are shown in Table 1 and lattice functions are shown in Figure 3. Including the chiane dipoles, the horizontal betatron tune becomes 1.48.

## 3 SLOW EXTRACTION SCHEME

Half integer extraction method is chosen because CIS horizontal tune 1.48 of CIS is close to 1.5 . This method is also


Figure 1: The general layout of the Indiana University Cyclotron Facility Cooler Injector Synchrotron at IUCF

Table 1: IUCF CIS lattice parameters

| Parameters | Symbol | Values |
| :--- | :---: | :--- |
| Circumference(m) |  | 17.364 |
| Horizontal tune | $\nu_{x}$ | 1.4633 |
| Vertical tune | $\nu_{z}$ | 0.7788 |
| Dipoles: |  |  |
| Length(m) | L | 2.0 |
| Edge angle | $\theta_{e}$ | $12^{\circ}$ |
| $\quad$ Bending radius(m) | $\rho$ | 1.273 |
| Transition Energy | $\gamma_{T}$ | 1.271 |
| Beta x: Maximum(m) | $\beta_{x}$ | 4.373 |
| $\quad$ Minimum(m) |  | 0.996 |
| Beta y: Maximum(m) | $\beta_{y}$ | 3.786 |
| $\quad$ Minimum(m) |  | 3.380 |
| Dispersion | $D_{x}$ |  |
| $\quad$ Maximum(m) |  | 1.759 |
| $\quad$ Minimum(m) |  | 1.617 |
| Chromaticity(x) | $C_{x}$ | -0.529 |
| Chromaticity(z) | $C_{z}$ | -0.156 |



Figure 2: Tune shift by four symmetrical trim quads. Where $\bigcirc$ stand for CIS working point, $\triangle$ for the calculation curve, $\bullet$ for the experimental measurement, and $\square$ for the tune with chicane dipoles but zero trim quad current.


Figure 3: The CIS lattice functions $\beta_{x}, \beta_{z}$ and $D_{x}$ for $L=$ $2.0 \mathrm{~m}, \theta_{e}=12^{0}$
used at Fermilab [5]. Quadrupoles are used to drive the half integer resonances and octupoles are used to extract the beam.

The vector potential of octupole and quadrupole are:

$$
\begin{align*}
A_{\mathrm{oct}} & =\frac{B_{3}}{24}\left(x^{4}-6 x^{2} z^{2}+z^{4}\right) \\
A_{\text {quad }} & =\frac{B_{1}}{2}\left(x^{2}-z^{2}\right) \tag{1}
\end{align*}
$$

where $B_{1}=\frac{\partial B_{z}}{\partial x}, B_{3}=\frac{\partial^{3} B_{z}}{\partial x^{3}}$.
The total Hamiltonian is

$$
\begin{align*}
H=\quad & \frac{1}{2}\left[x^{\prime}+K_{x}(s) x^{2}\right]+\frac{1}{2}\left[z^{\prime}+K_{z}(s) z^{2}\right] \\
& +\frac{1}{24} \frac{B_{3}}{B \rho}\left(x^{4}-6 x^{2} z^{2}+z^{4}\right) \\
& +\frac{1}{2} \frac{B_{1}}{B \rho}\left(x^{2}-z^{2}\right) \tag{2}
\end{align*}
$$

In CIS ring, beam orbit is excited in the horizontal direction for extraction. Using Floquet transformation, the Hamiltonian in for $x$ motion is

$$
\begin{equation*}
H=\nu_{x} J_{x}+\frac{1}{2} \alpha_{x x} J_{x}^{2}+g J_{x} \cos \left(2 \psi_{x}-l \theta\right)+\cdots \tag{3}
\end{equation*}
$$

where $J_{x}$ and $\psi_{x}$ are action-angle coordinates, $\alpha_{x x}$ is the strength of octupoles, $g$ is the half integer resonance amplitude, and $l / 2=1.5$. $\alpha_{x x}$ and $g$ can be derived as

$$
\begin{align*}
\alpha_{x x} & =\frac{1}{16 \pi} \frac{1}{|B \rho|} \beta_{x}^{2} \oint B_{3} d s  \tag{4}\\
g & =\frac{1}{4 \pi} \frac{1}{|B \rho|} \oint \beta_{x} B_{1} \cos 2 \chi_{x} d s \tag{5}
\end{align*}
$$

on the condition that

$$
\begin{equation*}
\oint \beta_{x} B_{1} \sin \left(2 \chi_{x}\right) d s=0 \tag{6}
\end{equation*}
$$

to ensure the beam trajectory at the septum is in the upright direction.

Using the generating function

$$
\begin{equation*}
F_{2}=\left(\psi_{x}-\frac{l}{2}\right) J, \tag{7}
\end{equation*}
$$

the Hamiltonian in Eq. (3) becomes

$$
\begin{equation*}
H=\Delta \nu J+\frac{1}{2} \alpha_{x x} J^{2}+g J \cos 2 \psi \tag{8}
\end{equation*}
$$

where $\Delta \nu=\nu_{x}-\frac{l}{2}$.
To find the beam trajectory in $(x, p)$ space, rewrite the above Hamiltonian as

$$
\begin{align*}
H= & \frac{1}{2} \Delta \nu\left(X^{2}+P^{2}\right)+\frac{1}{8} \alpha_{x x}\left(X^{2}+P^{2}\right)^{2} \\
& +\frac{g}{2}\left(X^{2}-P^{2}\right) \tag{9}
\end{align*}
$$

where $X=\sqrt{2 J} \cos \psi, P=\sqrt{2 J} \sin \psi$. The actual particle position is given by $x=\sqrt{\beta_{x}} X$. The unstable fix point is $\left[X_{\mathrm{U}}, P_{\mathrm{U}}\right]=\left[0, \frac{2}{\alpha_{x x}}(\Delta \nu-g)\right]$. The trajectory of the separatrix is

$$
\begin{array}{r}
\frac{1}{2}(\Delta \nu+g) X^{2}+\frac{1}{2}(\Delta \nu-g)\left(P^{2}-P_{\mathrm{U}}^{2}\right) \\
+\frac{1}{8} \alpha_{x x}\left[\left(X^{2}+P^{2}\right)^{2}-P_{\mathrm{U}}^{4}\right]=0 \tag{10}
\end{array}
$$

or

$$
\begin{equation*}
\left[(X-a)^{2}+P^{2}-r_{0}^{2}\right]\left[(X+a)^{2}+P^{2}-r_{0}^{2}\right]=0 \tag{11}
\end{equation*}
$$

where

$$
\begin{equation*}
a^{2}=-\frac{2}{\alpha_{x x}} g \quad r_{0}^{2}=-\frac{2}{\alpha_{x x}} \Delta \nu \tag{12}
\end{equation*}
$$

This means the trajectory in phase space at the septum is circle as shown in Figure 4.


Figure 4: Extraction beam trajectory in phase space

The area of the stable region is

$$
\begin{align*}
S & =4 \frac{1}{\beta_{s}}\left(\frac{1}{2} \theta r_{0}^{2}-\frac{1}{2} r_{0}^{2} \sin \theta \cos \theta\right) \\
& =\frac{2 \pi r_{0}^{2}}{\beta_{s}} \Gamma(k) \tag{13}
\end{align*}
$$

where

$$
\begin{equation*}
\Gamma(k)=\frac{1}{\pi}\left[\sin ^{-1} \sqrt{1-k}-\sqrt{k-k^{2}}\right] \tag{14}
\end{equation*}
$$

and $k=g / \Delta \nu$.
The step increase per turn of the extracted beam position

$$
\begin{align*}
\frac{d X}{d n} & =2 \pi \frac{\partial X}{\partial \psi}=2 \pi \frac{\partial H}{\partial P} \\
& =2 \pi\left[(\Delta \nu-g) P+\frac{\alpha_{x x}}{2} P\left(X^{2}+P^{2}\right)\right] \\
& =2 \pi P \Delta \nu\left[1-k-\frac{1}{r_{0}^{2}}\left(X^{2}+P^{2}\right)\right] \tag{15}
\end{align*}
$$

## 4 PRACTICAL CONSIDERATION

For the CIS, the vacuum chamber is about 50 mm . We have set the the wire septum at 15 mm , an available aperture of about $200 \pi \mathrm{~mm}$-mrad.

Figure 5 shows the change of step size per turn versus the horizontal position. In order to get the maximum extraction efficiency, the step size at the septum should maximum. From the calculation, the radius of the beam circle is $r_{0}=12 \mathrm{~mm}$. The emittance of CIS beam at extraction energy is $10 \pi \mathrm{~mm}-\mathrm{mrad}$ and the betatron function in the straight section is unfortunately only 1 m . For practically application, we can install the wire septum as near the dipole as possible.

## 5 CONCLUSIONS

A half integer resonance slow extraction scheme is possible for the CIS in order to extend its application to the medical science. By putting quadrupoles in the proper positions, the extraction efficiency can be maximized.


Figure 5: Step size of the extracted beam versus horizontal position. Wire septum is at $x=15 \mathrm{~mm} . r_{0}=12 \mathrm{~mm}$, $a=8.5 \mathrm{~mm}$.

## 6 REFERENCES

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