LIMITS TO SHORTENING BUNCH LENGTHS BY REDUCING MOMENTUM COMPACTION FACTORS IN ELECTRON OR POSITRON STORAGE RINGS

K. Soutome, M. Takao, H. Tanaka, SPring-8, Hyogo 679-5198, Japan
Y. Shoji, LASTI, Himeji Institute of Technology, Hyogo 671-2201, Japan

Abstract

When the momentum compaction factor is reduced to a small value in order to achieve a very short bunch length in electron or positron storage rings, fluctuations of path lengths due to the emission of photons become non-negligible. This effect determines the achievable lower limit of the equilibrium bunch length. Analytical expressions for this limit are formulated using lattice parameters of a storage ring. Some numerical examples are presented.

1 INTRODUCTION

In electron or positron storage rings, it has been studied to shorten the equilibrium bunch length by reducing the momentum compaction factor in order to realize free electron laser with an isochronous storage ring, coherent synchrotron radiation, etc.[1-3]. All of these studies are based on the following conventional formula which gives the relation between the equilibrium bunch length σ_τ and the momentum compaction factor α_0:

$$\sigma_\tau = \left(\alpha_0/\Omega_s\right)\sigma_\delta \propto \alpha_0^{1/2}, \tag{1}$$

where Ω_S is the angular frequency of synchrotron oscillation and σ_δ is the equilibrium momentum spread. According to this formula, the equilibrium bunch length can eventually become zero at the limit of $\alpha_0 = 0$.

In deriving Eq. (1), however, it was neglected that the path length of each electron in a bunch is fluctuating by emitting photons. The magnitude of this fluctuation depends on local parameters of the ring, like dispersion functions, at the positions where photons are emitted. If we want to take account of this effect, we can no longer adopt the conventional method of calculating bunch lengths by rotating equilibrium momentum spread in a longitudinal phase space with a constant value of α_0.

In the following we derive a set of formulas to calculate the equilibrium bunch length when the fluctuation of path lengths are taken into account[4].

2 EQUILIBRIUM BUNCH LENGTHS

If we denote the fluctuation of path lengths in one revolution due to the emission of photons by ΔL, the expectation value of a bunch spread caused by this fluctuation is written as

$$\sqrt{\left\langle\left\langle\left(\Delta L - \left\langle\left\langle\Delta L\right\rangle\right\rangle\right)^2\right\rangle\right\rangle}\ ,$$

where the average represented by the double parenthesis <<...>> is to be taken over both the ensemble (i.e. over electrons in a bunch) and the path (i.e. positions of photon emission). This bunch spread will grow turn by turn if damping in the longitudinal direction does not exist. In actual rings, however, there exists the radiation damping. Since the diffusion rate is $\langle\langle(\Delta L - \langle\langle\Delta L\rangle\rangle)^2\rangle\rangle/T_0$ and the damping rate is $-2A^2/\tau_\varepsilon$, where T_0, A and τ_ε are the revolution period, "amplitude" of the bunch spreading and the longitudinal damping time, respectively, the equilibrium half-bunch length due to the fluctuation of path lengths is given by[5]

$$A_{eq} = \sqrt{\left(\tau_\varepsilon / 4T_0\right)\left\langle\left\langle\left(\Delta L - \left\langle\left\langle\Delta L\right\rangle\right\rangle\right)^2\right\rangle\right\rangle}\ . \tag{2}$$

We define here the "fluctuation" part of the momentum compaction factor by

$$\alpha_s \equiv \sqrt{\frac{\left\langle\left\langle\left(\Delta L - \left\langle\left\langle\Delta L\right\rangle\right\rangle\right)^2\right\rangle\right\rangle}{L_0^2\left\langle\left(\Delta p / p_0\right)^2\right\rangle}}\ , \tag{3}$$

where L_0 is the circumference of the ring, $\Delta p/p_0$ is the deviation of momentum due to the emission of photons in one revolution, and the single parenthesis <...> means the average over the ensemble (i.e. over electrons in a bunch). By using Eq. (3), we can rewrite Eq. (2) as follows:

$$A_{eq} = L_0\,\alpha_s\,\sigma_\delta\ . \tag{4}$$

Then, combining Eq. (1) and Eq. (4), we obtain the following expression for the equilibrium bunch length when the fluctuation of path lengths exists:

$$\sigma_\tau = \sqrt{\left(\alpha_0/\Omega_s\right)^2 + \left(T_0\alpha_s\right)^2}\ \sigma_\delta. \tag{5}$$

When $\alpha_0 >> \alpha_s$, the bunch length is proportional to the square-root of α_0, as shown in Eq. (1). The effect of the fluctuation of path lengths sets in when α_0 is reduced to about zero by controlling optics of the ring[6-8]. In the case of $\alpha_0 = 0$, Eq. (5) gives the lower limit of

$$\sigma_\tau = T_0\,\alpha_s\,\sigma_\delta. \tag{6}$$

This limit is different from the stability limit caused by nonlinearity of the momentum compaction factor[6-8].

3 ANALYTICAL FORMULAS

In this section we derive analytical expressions for the "fluctuation" term α_s. To do this, we assume the following:

- Circulating electrons or positrons are ultra-relativistic and the ratio of a momentum variation to the nominal value is equivalent to that of an energy variation.

- There is no correlation among all events of photon emission in one revolution. In other words, the radiation loss per revolution is quite small compared with the reference energy of circulating particles.
- The equilibrium momentum spread σ_δ is also small compared with the reference energy of circulating particles and the variation of radiation loss for each particle can be neglected as the first approximation.
- All events of the photon emission are purely stochastic and averaging these over ensembles can be performed independently.

Under these assumptions, we start from an expression for the fluctuation of path length caused by the emission of photons. In the lowest order of perturbation it is written as

$$\Delta L = \sum_{i=1}^{N} \int_{s_i}^{L_0} ds \; \frac{\eta(s)\delta_i + x_\beta(s;\, s_i, \delta_i)}{\rho(s)} \,, \tag{7}$$

where $\rho(s)$ is the radius of curvature of the orbit at the position s , $\eta(s)$ is the horizontal dispersion function, s_i is the position where the i-th photon is emitted, N is the number of emitted photons in one revolution, $\delta_i \equiv (-u_i)/E_0$ is the ratio of the energy change due to the photon emission to the nominal energy, and x_β represents the horizontal betatron oscillation excited by the photon emission. Then, the average $\langle\langle \Delta L \rangle\rangle$ can be calculated as

$$\langle\langle \Delta L \rangle\rangle = \langle\langle \Delta L_\eta \rangle\rangle + \langle\langle \Delta L_\beta \rangle\rangle \,, \tag{8}$$

where $\langle\langle \Delta L_\eta \rangle\rangle$ and $\langle\langle \Delta L_\beta \rangle\rangle$ are contributions from equilibrium orbit shifts and betatron oscillations, respectively, and are given by

$$\langle\langle \Delta L_\eta \rangle\rangle = \frac{1}{L_0} \int_0^{L_0} ds_i \int_{s_i}^{L_0} ds \; \frac{\eta(s) \bar{N} \langle \delta \rangle}{\rho(s)} \,, \tag{9a}$$

$$\langle\langle \Delta L_\beta \rangle\rangle = \frac{1}{L_0} \int_0^{L_0} ds_i \int_{s_i}^{L_0} ds \; \frac{x_\beta(s; s_i, \bar{N} \langle \delta \rangle)}{\rho(s)} \,. \tag{9b}$$

The square of a deviation from the average is given by

$$(\Delta L - \langle\langle \Delta L \rangle\rangle)^2 = \Delta_1 + \Delta_2 + \Delta_3 \,, \tag{10}$$

where

$$\Delta_1 = \sum_{i=1}^{N}\sum_{j=1}^{N} \int_{s_i}^{L_0} ds \int_{s_j}^{L_0} ds' \; \frac{\eta(s)\eta(s')\,\delta_i\,\delta_j}{\rho(s)\rho(s')} - 2\,\langle\langle \Delta L_\eta \rangle\rangle \sum_{i=1}^{N} \int_{s_i}^{L_0} ds \; \frac{\eta(s)\delta_i}{\rho(s)} + \langle\langle \Delta L_\eta \rangle\rangle^2 , \tag{11a}$$

$$\Delta_2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \int_{s_i}^{L_0} ds \int_{s_j}^{L_0} ds' \; \frac{x_\beta(s;\, s_i, \delta_i)\; x_\beta(s';\, s_j, \delta_j)}{\rho(s)\rho(s')} - 2\,\langle\langle \Delta L_\beta \rangle\rangle \sum_{i=1}^{N} \int_{s_i}^{L_0} ds \; \frac{x_\beta(s;\, s_i, \delta_i)}{\rho(s)} + \langle\langle \Delta L_\beta \rangle\rangle^2 , \tag{11b}$$

$$\Delta_3 = 2 \sum_{i=1}^{N}\sum_{j=1}^{N} \int_{s_i}^{L_0} ds \int_{s_j}^{L_0} ds' \; \frac{\eta(s)\,\delta_i\, x_\beta(s';\, s_j, \delta_j)}{\rho(s)\rho(s')} - 2\,\langle\langle \Delta L_\beta \rangle\rangle \sum_{i=1}^{N} \int_{s_i}^{L_0} ds \; \frac{\eta(s)\,\delta_i}{\rho(s)} - 2\,\langle\langle \Delta L_\eta \rangle\rangle \sum_{i=1}^{N} \int_{s_i}^{L_0} ds \; \frac{x_\beta(s;\, s_i, \delta_i)}{\rho(s)} + 2\,\langle\langle \Delta L_\beta \rangle\rangle\,\langle\langle \Delta L_\eta \rangle\rangle \,. \tag{11c}$$

From Eqs. (3) and (10), we have

$$\alpha_s = \sqrt{\alpha_{s1}^2 + \alpha_{s2}^2 + \alpha_{s3}^2} \,, \tag{12}$$

where

$$\alpha_{si} \equiv \sqrt{\frac{\langle\langle \Delta_i \rangle\rangle}{L_0^2 \bar{N} \langle \delta^2 \rangle}} \,, \quad i = 1, 2, 3. \tag{13}$$

By dividing the double summation in Eq. (11a) into diagonal and off-diagonal contributions and averaging each part over the ensemble and the path, we obtain the following expression for α_{s1} :

$$\alpha_{s1}^2 = \frac{1}{L_0^3} \int_0^{L_0} ds_i \left[\int_{s_i}^{L_0} ds \; \frac{\eta(s)}{\rho(s)} \right]^2 . \tag{14}$$

It is worth mentioning that the process of the photon emission has the Poisson-type probability distribution and hence $\langle (N - \bar{N})^2 \rangle = \bar{N}$, which we used in deriving the above expression.

The term α_{s2} can be calculated by using the following expression for the betatron oscillation:

$$x_\beta(s;\, s_i, \delta_i) = \eta(s_i)\,\delta_i\, cos\,(\,\varphi(s) - \varphi(s_i)) = \sqrt{\beta(s) H(s_i)}\;\delta_i\, cos(\varphi(s) - \varphi_{0i}) \,, \tag{15}$$

where

$$H(s) = \gamma(s)\eta(s)^2 + 2\alpha(s)\eta(s)\eta'(s) + \beta(s)\eta'(s)^2 , \tag{16}$$

$$\eta'(s) = \frac{d\eta(s)}{ds} \,, \tag{17}$$

$$\varphi_{0i} = \varphi(s_i) - Arccos\left[\eta(s_i) / \sqrt{\beta(s_i) H(s_i)} \right] \,, \tag{18}$$

and $(\alpha(s), \beta(s), \gamma(s))$ and $\varphi(s)$ are the Twiss parameters and the betatron phase, respectively[5]. By substituting Eq. (15) into Eq. (11b) and taking the average, we have

$$\alpha_{s2}^2 = \frac{1}{L_0^3} \int_0^{L_0} ds_i\, H(s_i) \times \left[\int_{s_i}^{L_0} ds \; \frac{\sqrt{\beta(s)}}{\rho(s)}\, cos(\varphi(s) - \varphi_{0i}) \right]^2 . \tag{19}$$

The term α_{s3} is calculated in a similar manner and we obtain

$$\alpha_{s3}^2 = \frac{2}{L_0^3} \int_0^{L_0} ds_i \sqrt{H(s_i)} \int_{s_i}^{L_0} ds \int_{s_i}^{L_0} ds' \; \frac{\eta(s)\,\sqrt{\beta(s')}}{\rho(s)\rho(s')} \times cos(\varphi(s') - \varphi_{0i}) \,. \tag{20}$$

Note that the phase φ runs from 0 to 2π and the integral in Eq. (20) gives almost zero. We then have

$$\alpha_{s3} \approx 0 \,. \qquad (21)$$

By substituting Eqs. (14), (19) and (21) into Eq. (12) and using Eq. (5), we can calculate the equilibrium bunch length when the fluctuation of path lengths exists.

4 NUMERICAL EXAMPLES AND DISCUSSION

In this section we give some numerical results. As an example, we take a high-energy electron storage ring of SPring-8. This ring is of double-bend achromat (DBA) type with a beam energy of 8GeV, a typical value of the natural emittance of 7nmrad and a circumference of 1436m. For this ring with a typical optics, we have $\alpha_0 = 1.46\times10^{-4}$, $\alpha_{s1} = 8.44\times10^{-4}$ and $\alpha_{s2} = 4.76\times10^{-6}$. Then, from Eq. (5), we can calculate contributions from α_0, α_{s1} and α_{s2} to the equilibrium half bunch length in the nominal operation. The resulting values are 3.6mm, 0.13mm, and 0.007mm, respectively. We then see that the effect of the fluctuation of path lengths is about 4% in this case. However, the effect of fluctuation terms becomes significant when we lower the conventional momentum compaction factor α_0.

To see the dependence of the bunch length on α_0, we performed a model calculation by breaking the achromat condition and leaking the dispersion function η in the outside of the arc section. For simplicity, we assumed that η keeps a symmetric form in the arc between two bending magnets (see Fig. 1). In this model the dispersion function in a bending magnet can be written in the form of

$$\eta(s) = \eta_{in} + \rho_0 - \rho_0 cos((s - s_{in})/\rho_0) \;, \qquad (22)$$

where η_{in} is to be varied to lower the value of α_0. The results are shown in Fig. 2, where the equilibrium half bunch length is plotted as a function of α_0. The dashed curve is a contribution from α_0, the dotted curve is that from α_s and the solid curve is the sum of these contributions. Contributions from α_{s2} were neglected in these calculations. The limit value at α_0=0 is 0.3μm.

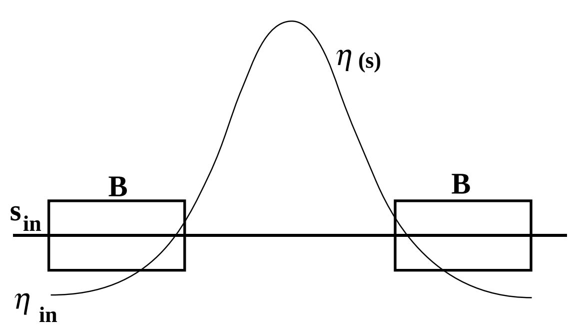

Figure 1: A model dispersion function to calculate the equilibrium bunch length.

Note that though this limit value is small for the SPring-8 storage ring, it can be significant for a small ring with a low momentum compaction factor. This is expected because α_{s1} is proportional to the third power of the bending angle (see Eq. (14)).

We finally note that essentially the same results as obtained in the previous section were derived in Ref.[9] with a different method. The derivation shown in this work is complementary to that and will be useful for further and detailed considerations.

REFERENCES

[1] H. Hama, S. Takano and G. Isoyama, Nucl. Instr. and Meth. A392 (1993) 29.

[2] A. Nadji, P. Brunelle, M. P. Level, M. Sommer and H. Zyngier, Proceedings of the 4th European Particle Accelerator Conference (1994) 128.

[3] D. Robin, R. Alvis, A. Jackson, R. Holtzapple and B. Podobedov, Stanford Linear Accelerator Center, SLAC-PUB-95-7015 (1995).

[4] H. Tanaka, unpublished note (1996). (All results presented here are based on this note.)

[5] See, e.g., M. Sands, SLAC-121 (1970), Stanford Linear Accelerator Center.

[6] A. Ando and K. Takayama, IEEE Trans. Nucl. Sci., Vol. NS-30, No.4 (1983) 2604.

[7] C. Pellegrini and D. Robin, Nucl. Instr. and Meth. A301 (1991) 27.

[8] D. Robin, E. Forest, C. Pellegrini and A. Amiry, Phys. Rev. E48 (1993) 2149.

[9] Y. Shoji, H. Tanaka, M. Takao and K. Soutome, Phys. Rev. E54 (1996) R4556. Y. Shoji, A. Ando, H. Tanaka and M. Takao, Nucl. Instr. and Meth. A390 (1997) 417.

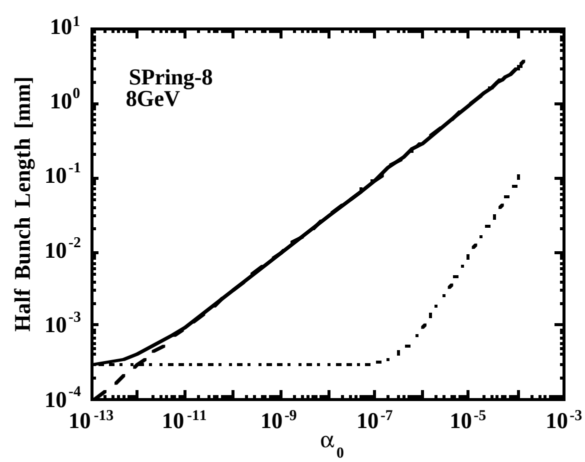

Figure 2: The equilibrium half bunch length in the SPring-8 storage ring as a function of the conventional momentum compaction factor α_0.