# **BEAM BLOW-UP CALCULATIONS FOR RTM AND DSM<sup>†</sup>**

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### Abstract

Estimations of the threshold current for transverse BBU by the parasitic  $TM_{110}$ -like mode are presented for a 1.5 GeV CW DSM - a possible booster behind the MAMI RTM-cascade. The RTM3 operating at MAMI was used for test calculations. Our results show, that, without any special countermeasures, the BBU threshold current for DSM is at 0.24 mA, compared with a maximum beam current of 0.1 mA projected.

### **1 INTRODUCTION**

The Mainz Microtron (MAMI) is an 855 MeV, 100 µA cascade of three continuous wave (CW) racetrack microtrons (RTM1-3, [1]). A double sided microtron (DSM, [2]) is considered as a possible fourth stage to increase the beam energy to 1.5 GeV. As for any new machine, parasitic phenomena must be considered, which could limit the beam current or deteriorate the beam quality. We present here results for regenerative beam blow up (BBU), studied with two different codes based on quite different approaches and having different capabilities. The first code, called here HBBU, is based on a consideration of steady state closed loop conditions, it was developed in the process of MAMI RTM-design [3]. The second is the time dependent code TDBBU [4], used for CEBAF design. We compared the two codes by making calculations for a simplified model of RTM3, and then we investigated the dependence of BBU threshold current on different DSM parameters.

## 2 TWO APPROACHES TO ANALYZE BBU

The HBBU-code approach is based on consideration of the steady state parasitic excitation of an RF cavity, which is included in the closed feedback loops provided by Nreturn paths of the recirculating accelerator. The main approximations used in HBBU are: the bunch structure of the beam is ignored, the linac is replaced by a single infinitely thin cavity, and vertical and horizontal planes are uncoupled both in transfer matrix and in parasitic mode polarization. For a beam oscillating transversally at the cavity position with frequency  $\mathcal{D}_1$ , the threshold current  $I_i$  is given by:

$$I_t r'_{\perp} = 2c / [ek_r L \operatorname{Im}(\overline{X})], \qquad (1)$$

where:  $r_{\perp}$  - transverse shunt impedance of parasitic mode per unit length, L - linac length,  $k_r = \overline{\omega}_r / c$  with  $\overline{\omega}_r$  - parasitic mode resonance frequency.  $\overline{X}$  is the sum of displacements at the cavity position of the beams coming from different return paths, divided by the transverse momentum amplitude given to them by the cavity. For zero displacement and transverse momentum of the initially injected beam,  $\overline{X}$  can be calculated from (we give here a simplified expression, compared to [3]):

$$\overline{X} = \sum_{n=2}^{N} \left\{ \sum_{j=2}^{n} \left[ \left( \prod_{l=j}^{n} \hat{R}_{l} \right)_{12} e^{-j\overline{\omega}_{l}\tau_{j-l}} \right] \right\},$$
(2)

where  $R_i$  - canonical transfer matrix for the *l*th orbit,  $\tau_j = T_{RF}(j-1)\{\mu + \nu j/2\}$  - transit time from 1st to *j*th cavity passage,  $T_{RF}$  - RF period of fundamental mode,  $\mu$  length of first orbit and  $\nu$  - orbit length increment per turn ( $\nu$ =1 for RTM and 2 for DSM), both in numbers of wavelengths.

An actual resonance frequency of the parasitic mode, providing a specific transverse oscillation  $\varpi_1$  of the beam, can be calculated with:

$$\overline{\omega}_r = \overline{\omega}_1 / \{1 - \operatorname{Re}(\overline{X}) / [2\operatorname{Im}(\overline{X})Q_L]\}, \qquad (3)$$

where  $Q_L$  - loaded quality factor of the cavity.

The HBBU code is quite simple and permits to get in a very short time a panoramic view of threshold current behaviour in a wide frequency range, and to investigate it's dependence on the basic RTM parameters. The price for this simplicity is the absence of a possibility for direct simulation of multi-cavity systems, in order to study effects of parasitic mode frequency-detuning and rotation of mode polarization.

We made minor modifications of the HBBU code. First, beam optics calculations were made more flexible, and an orbit matrices preparation for TDBBU was introduced. Second, an additional possibility to investigate threshold current dependence on the cavity parasitic mode resonance frequency using (3) was inserted. (In [3] there is a misprint in formulas (6), connecting  $\varpi_r$  and  $\varpi_1$ : Im and Re and the signs are to be exchanged. Moreover, in interpretation of the results these two frequencies were not strictly distinguished there.)

For comparison and for a flexible study of multi-cavity systems we used the 2-D time dependent TDBBU code,

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which simulates the time evolution of parasitic mode excitation. Basic for TDBBU is the transverse wake potential  $W(\tau)$ , defined as the transverse momentum  $\Delta p_x$  obtained by a test particle with the charge *e* in a cavity length of *L*, following at a time  $\tau$  later than an excitation particle with charge *q*, displaced at a distance *x* from the axis. For small displacements of both particles:

$$\frac{c}{e}\Delta p_x \equiv W(\tau) = \frac{r_{\perp}^2 k_r^2 L}{2Q_L} qx \sin(\varpi_{\perp}\tau) e^{-\varpi_r \tau/2Q_L}$$
(4)

The beam is pushed through the accelerator bunch by bunch during some time interval, with transverse momentum (4) applied in each cavity at each orbit. The transverse position of the bunches at output and the parasitic mode energy stored in the cavities are criteria of BBU. To estimate the threshold current for similar systems, TDBBU requires several orders of magnitude more time than HBBU.

To compare the results of HBBU and TDBBU, we repeated the BBU calculations for RTM3 of MAMI, using the well known parameters of this 90-orbit machine [1]. For the the most dangerous TM<sub>110</sub>-like parasitic mode at about 4187 MHz ([3],[5]) one has  $r_{\perp}$ '=17 MΩ/m and  $Q_{L}$ =16000. The linac with electrical length 8.87 m, consisting of 5 sections with 29 cells each, was replaced by a single cavity with equivalent shunt impedance.

In Fig. 1 we show by dotted lines the  $I_r$ -dependence on the frequency  $\varpi_r$  of parasitic mode, obtained with HBBU for the vertical plane. This dependence is formed by overlapping resonance curves. It can be seen, that for a specific frequency of parasitic mode, the single cavity can sustain several (up to the number of turns, because we have N closed loops) parasitic oscillations with different frequencies  $\varpi_1$  of the beam and different  $I_l$ .



Fig. 1: Comparison of threshold current calculations for MAMI/RTM3 with HBBU and TDBBU.

We made calculations with TDBBU for several frequencies of the parasitic mode, using the same transfer matrices as in HBBU. Results are presented in Fig. 1 by squares. For all frequencies tested we got good agreement between the two codes, only the resonance at 4187.4 MHz stayed unexcited in TDBBU.

As it was already mentioned in [3], the dependence of the kind presented in Fig. 1 is extremely sensitive to the setting of RTM parameters, including beam optics; so the value of  $I_t$  at a certain frequency can be strongly changed by small changes of these settings. The RTM3 threshold current, predicted by our calculations for all linac cells acting synchronously is at 20  $\mu$ A. At the same time, for the real machine no signs of BBU phenomena are observed for a beam current in excess of 100 µA. This fact can naturally be explained by the staggered detuning of the parasitic mode frequency of the linac cells within a frequency range of 17 MHz [6]. This detuning was accomplished by changing the angle between coupling slot pairs at opposite webs of the accelerating cells. Additional  $90^{\circ}$  rotation of the two halves of each section of RTM3 changed the plane of BBU mode polarization, further raising  $I_t$ . We did not try to reproduce this complicated experimental situation, but made TDBBU calculations with a uniform random frequency distribution of 145 cavities within 17 MHz; this gave us a gain of 15 in BBU threshold with  $I_t > 300 \mu A$ .

### **3 BBU CALCULATIONS FOR DSM**<sub>G</sub>

We considered a DSM with a vertical focusing gradient in the four  $90^0$  bending magnets (DSM<sub>G</sub>, [7]). The simplified optical scheme used in our calculations is shown in Fig. 2.

	$\mathbf{F}(\mathbf{E}) = \mathbf{F}(\mathbf{E})$					$\mathbf{F}(\mathbf{E}) = \mathbf{F}(\mathbf{E})$			
]	Linac 1	Magnets		Linac2	, 1	Magnets	Linac	1	
	L1(E)/2	L2(E)	-	L1(E)		L2(E)	L1(E)/2		

Fig. 2: Optical scheme of one DSM<sub>G</sub> orbit.

Each of the bending magnets acts as effective drift L2(E), whose length depends on energy and is different for the horizontal and vertical plane. Focusing is provided by four quadrupole doublets F(E), installed between linacs and bending magnets; the position of their principal planes is also changing with energy, therefore L1(E). The linacs operate at the first harmonic of the MAMI frequency. The most essential parameters of the DSM<sub>G</sub> are given in Table 1, they are not necessarily final for this project. Shunt impedance and quality factor of the TM<sub>110</sub>-like parasitic mode were extrapolated from that of the MAMI sections, taking into account a not exact scaling of the cavity profile (e.g. larger beam hole). The drift of synchronous phase owing to the field gradient in the bending magnets was omitted in calculations, an average value of phase was used.

Two linacs cannot be consistently incorporated to HBBU. We therefore considered two approximations: a) both linacs act coherently, oscillating at the same parasitic frequency  $\varpi_1$  with equal amplitudes, b) the linacs oscillate independently at different frequencies. The last case is more realistic: because of the essential beam energy change from one linac to the other, beta functions

and time delays over the orbit are different, providing conditions for transverse oscillations at slightly different frequencies  $\varpi_1$  even for equal frequencies  $\varpi_r$  of the parasitic mode.

Table 1: DSM<sub>G</sub> parameters used in calculations.

Injection energy	855 MeV	
Extraction energy	1500 MeV	
Number of turns	43	
Operating frequency	4899 MHz	
One linac active length	8.35 m	
Number of cells per linac	273	
Average synchronous energy gain per linac	7.5 MeV	
Effective magnet field	0.933 T	
First orbit length	65 m	
Distance between linac axes	12.6 m	
Singlet focal length at 855 MeV	2.59 m	
Frequency of most dangerous parasitic mode	~8400 MHz	
Effective shunt impedance of parasitic mode	~22 MΩ/m	
Quality factor of parasitic mode	~11000	

Panoramic view of  $I_t r_{\perp}$ ', calculated with HBBU in aproximation b) for vertical and horizontal DSM<sub>G</sub> planes over a wide frequency range is shown in Fig. 3. For two frequency regions results from TDBBU for DSM<sub>G</sub> with two single-cavity linacs are also shown; they are in good agreement with HBBU. At 8374 MHz the predicted threshold current is at 0.24 mA ( $I_t r_{\perp}$ '> 0.004 A×MΩ/m). The downgoing spikes of  $I_t$ , observed in Fig. 3, are a general property of recirculating machines, independent of their specific optics. They take place at strictly defined positions  $v\varpi_1/\varpi_{RF} = l + n/m$  (*l*,*m*,*n* integer); in this case for a *N*-orbit machine, *N/m* orbits act coherently, providing a decrease in  $I_t$ .



Fig. 3: Threshold current panoramic view for  $DSM_G$  with HBBU. Open circles and squares are TDBBU results.

We made detailed studies with TDBBU for the  $I_{l^-}$  dependence on different DSM<sub>G</sub> parameters. Detuning the frequency of the two single cavity linacs in opposite direction within 10 MHz does not produce a systematic effect on  $I_l$ . This further supports the fact, that parasitic oscillations of the linacs are nearly independent. Next we considered two linacs, consisting of seven cavities each, and studied the  $I_l$ -dependence on a equidistant detuning

with respect to several central frequencies, the same for both linacs. Gains in  $I_t$  for the vertical plane are shown in Fig. 4a. The solid curve gives the prediction by the formula in [6] for gain due to staggered cavity-detuning. The sharp resonances in  $I_t$ -dependence on cavity frequencies  $\overline{\omega}_r$  and the absence of coherence in the oscillations of initially equal tuned cavities distributed at long distances explain the difference between TDBBU results and the estimate from [6].



Fig. 4: Dependence of gain in  $I_t$  on cavity detuning range (a) and on average beta function (b).

Besides cavity-detuning, the second essential factor strongly influencing  $I_t$  is the value of the beta function ( $\beta$ ), averaged over orbits, at the center of the linacs. In Fig. 4b we show the dependence of gain in  $I_t$  on  $1/\beta$  for the vertical plane. The linacs were considered as single cavity each. The value of  $\beta$  was changed by the quadrupole settings, the right normalization end point corresponds to the project value. For comparison the straight solid line  $I_t \sim 1/\beta$  is given. The horizontal plane gain factors behave in a similar manner.

In contrast to RTM3, where the one-orbit  $\beta$  at the end of acceleration is in the range 40-60 m, DSM<sub>G</sub> is a much more strongly focused machine, with  $\beta$  below 12 m for most orbits. A small value of  $\beta$  is the main means to get a relatively high threshold current  $I_t$  without special countermeasures against parasitic modes.

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