MODELING FLAT FIELDS, CREATED BY SMOOTH POLES

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Abstract

In work the opportunity of construction on sources of flat fields with the help of structures of poles formed by smooth curves is considered. The conformal map of area $0 \le \mathrm{Im}(z) \le H$ on a strip with any bounds without intermediate map to a half-plane is received. The communication of factors of decomposition of a field in a Taylor series with parameters of conformal map realising recuired distribution of a field is determined. The influence of various sites of a structure of a pole on distributions of a field is analysed. On the basis of the received analytical expression connecting the form of a pole with a field, the construction of a structure of a pole is analysed. The received results were used in the computer program for synthesis of the pole on required distribution of a field.

1. INTRODUCTION

Usual method of reaching of demanded parameters of modern installations [1,2] has become application of magnets with complicated multipolar structure. The necessity of modelling of such complicated fields (stipulated by demanded allowable deviations ~ 10^{-4}) generates desire to have expression connecting a field and geometric performances of a magnet, namely with the form of a pole. Even the saturation of iron can be compensated by preliminary geometric distorions of a pole shape [3].

As is known, the field is possible to express both through the complex potential $z(\omega)$, and through the inverse function to the complex potential $\omega(z)$ ($\omega(z)$ - conformal map of a band $0 \le \mathrm{Im}(z) \le H$ on a pole [4]).

$$B = -i\overline{(dz/d\omega)} = -i/\overline{(d\omega/dz)} \qquad (1)$$

Following exposition is devoted to deriving of a map of a line band $0 \le \mathrm{Im}(z) \le H$ on a band of a pole area $\omega = x+iy$.

2. THE “BAND TO BAND” CONFORMAL MAPPING

Let’s consider a conformal map of a line band $0 \le \mathrm{Im}(z) \le H$ of an area $z=t+ih$ on a band of a pole area $\omega = x+iy$ (Fig.1). Let the angle of declination $v=v(t)$ of a tangent to L in apoint ω that is appropriated to point t is known in each point t of boundaries of a band $0 \le \mathrm{Im}(z) \le H$. Let also $dz=dt$ and $d\omega = |d\omega| \exp[iv(t)]$ are elements of a boundary of a band and contour L, appropriate one another in considered conformal mapping, then

$$\frac{d\omega}{dz} = \exp[iv(t)]\frac{|d\omega|}{dt} \qquad (2)$$

Let's remark that:

$$-i\ln\frac{d\omega}{dz} = v(t) - i\ln\frac{|d\omega|}{dt} = g(z), \qquad (3)$$

where g (z) - function, which real part on boundaries of a band accepts significances $v(t)$. Obviously, that a deriving map has a form:

$$\omega(z) = C\int_{z_0}^{z}\exp[ig(z)]dz + C_0\,; \qquad (4)$$

where C, C_0 are constants of an integration.

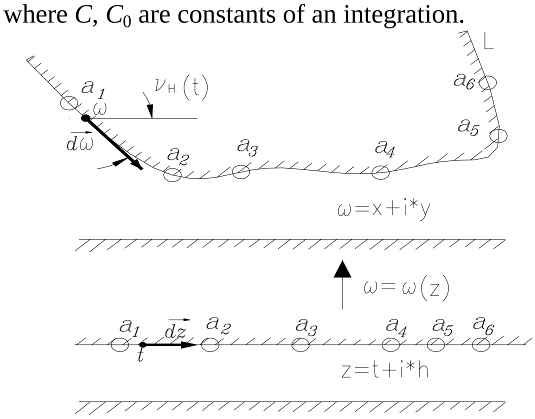

Fig.1 Map of a band $0 \le \mathrm{Im}(z) \le H$ on a band with any boundaries.

Let's establish the correspondence $\omega(0)=0$; that is $C_0=0$. For $\omega(z)$, the ratio of real and imaginary parts is important. Therefore let's assume $C=1$. The function g(z), by virtue of an above-stated property (3), is restored by an integral of the Schwarz for a band. For map of a circle to any symply connected region the formula of a kind (4) is known as the formula Chizoti [4].

2.1. Integral of the Schwarz for a band

The integral of the Schwarz for a circle $|\zeta| \le 1$ has a form

$$G(\zeta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} V(\exp(i\tau))\frac{\exp(i\tau)+\zeta}{\exp(i\tau)-\zeta}\,d\tau\,; \qquad (5)$$

where τ it a an angular coordinate of a plane ζ containing a circle $|\zeta| \le 1$. Let's consider a conformal mapping of a band $0 \le \mathrm{Im}(z) \le H$ of a plane $z=t+ih$ on a circle$|\zeta| \le 1$ of a plane $\zeta = r \cdot exp(i\tau)$,

$$\zeta(z) = \mathrm{th}(\pi(2z - iH)/4H), \qquad (6)$$

transferring the lower and upper boundaries of a band in the lower and upper semicircles accordingly, also we shall designate $G[\zeta(z)] = g(z)$, $V[\exp(i\tau)] = v(t)$. By allocating two intervals of an integration, $\tau\in[-\pi,0] \Rightarrow t\in[-\infty,\infty]$; $\tau\in[0,\ \pi] \Rightarrow t\in[\infty,-\infty]$ and, by making a change of variables, we shall receive

$$g(z)=\frac{i}{2H}\left\{-\int_{-\infty}^{\infty} v_0(t)\left[\operatorname{cth}\frac{\pi(t-z)}{2H}-\operatorname{th}\frac{\pi t}{H}\right]dt+\int_{-\infty}^{\infty} v_H(t)\left[\operatorname{th}\frac{\pi(t-z)}{2H}-\operatorname{th}\frac{\pi t}{H}\right]dt\right\}. \qquad (7)$$

The first integral responces for degrees of a symmetry, the second - for the form of a pole. The preservation of an addend th($\pi t/H$) under the integral (in [4] it is brought in constants of an integration) allows, by substituting (7) in (4) to receive analytical expression for map "band to band". The formulas (4,7) allow to describe practically all " Halbach geometries" [5].

3. INFLUENCE OF VARIOUS AREAS OF A STRUCTURE THE FIELD

The formulas (4,7) result in expression for a conformal mapping "band to band".

$$\omega(z)=\int_{z_0}^{z}\exp[G(z)]dz\,; \qquad (8)$$

where

$$G(z)=\frac{1}{2H}\int_{-\infty}^{\infty} v_0(t)\left[\operatorname{cth}\frac{\pi(t-z)}{2H}-\operatorname{th}\frac{\pi t}{H}\right]dt-\frac{1}{2H}\int_{-\infty}^{\infty} v_H(t)\left[t\,\mathrm{h}\frac{\pi(t-z)}{2H}-\operatorname{th}\frac{\pi t}{H}\right]dt \qquad (9)$$

We shall define the function describing the behaviour of an angle of declination of a structure of the pole $v_H(t)$, as

$$v_H(t)=\begin{cases} const=U_{-\infty}, & t\in[-\infty,a_1];\\ q_j(t), & t\in[a_j,a_{j+1}],\quad j=1\cdots M-1;\\ const=U_{-\infty}, & t\in[a_1,\infty]; \end{cases} \qquad (10)$$

M, is number of points on the upper coast of a band, between which $v_H(t)$ is continuous. In case of a multipolar symmetry, $v_0(t)$ is determined as:

$$v_0(t)=\begin{cases} const=U_{mult}, & t\in[-\infty,0]\\ const=0, & t\in[0,\infty] \end{cases} \qquad (11)$$

For a dipole symmetry U_{mult}=0; for a quadrupole symmetry $U_{mult}=-\pi/2$; for sextupole $U_{mult}=-\pi 2/3$; etc.

We shell rewrite (8) under of accepted definition of the angle of declination of a pole structure (10) as:

$$\omega(z)=\int_{z_0}^{z}\exp\left[G_{mult}(z)+G_{le}(z)+G_{re}(z)+\sum_{j=1}^{M-1}G_j(z)\right]dz\,;(12)$$

where

$$\exp[G_{mult}(z)]=\left[1-\exp(-\pi z/H)/\sqrt{2}\right]^{U_{mult}/\pi}\,; \qquad (13)$$

This factor originating from the first integral of the formula (9) describes influence of the degree of the symmetry on the field. So for the dipole symmetry $\exp[G_{mult}(z)]=1$.

$$G_{le}(z)=\frac{U_{-\infty}z}{2H}-\frac{U_{-\infty}}{\pi}\left(\ln\sqrt{2}+\frac{\ln[\operatorname{ch}(\pi(a_1-z)/2H)]}{\sqrt{\ln[\operatorname{ch}(\pi a_1/H)]}}\right) \qquad (14)$$

This member responces for the left-hand edge of a pole.

$$G_{re}(z)=\frac{U_{\infty}z}{2H}+\frac{U_{\infty}}{\pi}\left(\ln\sqrt{2}+\frac{\ln[\operatorname{ch}(\pi(a_2-z)/2H)]}{\sqrt{\ln[\operatorname{ch}(\pi a_2/H)]}}\right) \qquad (15)$$

This member responces for the right-hand edge of a pole. $G_{le}(z)$, $G_{\hat{e}e}(z)$ is an outcomes of an integration of the second integral of expression (9) in limits $[-\infty,a_1]$, $[a_2,\infty]$ accordingly.

$$G_j(z)=-\frac{1}{2H}\int_{a_j}^{a_{j+1}} q_j(t)\left[\operatorname{th}\frac{\pi(t-z)}{2H}-\operatorname{th}\frac{\pi t}{H}\right]dt \qquad (16)$$

This member responces for a area of a pole between points a_j, a_{j+1}.

Formally, each of the members $G_{le}(z)$, $G_{\hat{e}e}(z)$, $G_j(z)$ can be considered as function generating a conformal mappings of a rectilinear band on a band which at upper boundary is tangent to the lower one everywhere, except the area described by the appropriate member. Let's a pole of a dipole magnet is determine by a specific behaviour of an angle of declination of a structure of a pole between 6 points in Fig.2.à. To each of areas between points $-\infty$, a_1, a_2, a_3, a_4, a_5, a_6, ∞ we shall get in accordance with the conformal mapping.

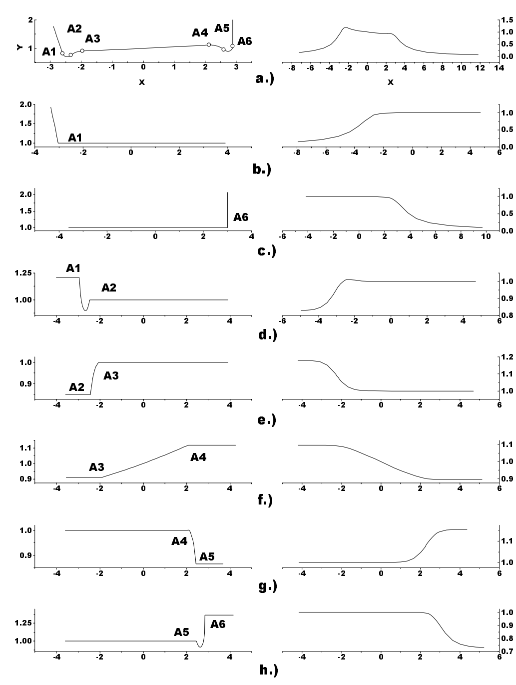

Fig.2 Influence of areas of the pole structure to the field. At the left there are area of a pole structure. On the right fields, appropriate to these areas of a pole structure.

For example, we shall consider the contribution for a left-hand slope of a pole ($-\infty$,a_1). Let angle of declination forming of a pole structure $v_H(t)$ (10) is determined in such a manner that: $(q_j(t)=0,(j=1,M),U_\infty=0)$. Then the expression (12) acquires a form

$$\omega(z)=\int_{z_0}^{z}\exp[G_{le}(z)]dz\,; \tag{17}$$

It is a conformal mapping of a rectilinear band on a band, which the lower boundary is a direct line , upper: an angle between direct lines is tangent to axes 0X, the other one intersect the first line under the angle U_∞ in a point A_1 Fig.2.b. Similarly it is possible to construct maps, therefore are fields for each site of a pole structure. Fig.2 illustrates the contribution separate site of a pole structure between points $\grave{a}_j$ in a field for a dipole magnet.

The influence of a different areas of a structure on a field consists of the following: *the field, in coordinates of a plane containing a rectilinear band, is equal to the product of fields from elementary conformal mappings, which are determined by the initial parameters of the areas.*

4. THE MULTIPOLAR ANALYSIS OF A FIELD

Using expression (1,9) we shall derive an expression for the field in coordinates of the band $0\le \mathrm{Im}(z)\le H$

For the dipole symmetry $v_0(t)=0$

$$B(z)=\exp[-F(z)] \tag{18}$$

where

$$F(z)=-\frac{1}{2H}\int_{-\infty}^{\infty}v_H(t)\left[t\,\mathrm{h}\frac{\pi(t-z)}{2H}-\mathrm{th}\frac{\pi t}{H}\right]dt \tag{19}$$

as

$$d\omega(z)=\exp[F(z)]dz$$

that

$$\frac{d^iB(z)}{d\omega(z)^i}=\exp[-F(z)]\frac{d}{dz}\left[\frac{d^{i-1}B(z)}{d\omega(z)^{i-1}}\right]. \tag{20}$$

By applying this procedure so many times as many derivatives on the field we know, we shall get a system , which ñan be solved concerning a derivatives of function $F(z)$. And, as we are free to establish the correspondence $\omega(0)=0$,:

$$\begin{cases} B(0)=\exp[-F(0)] \\ B'(0)=-F'(0)B(0)^2; \\ B''(0)=\left(2F'(z)^2-F''(z)\right)B(0)^3; \\ B^{(3)}(0)=\left(-6F'(0)^3+7F'(0)F''(0)-F^{(3)}(0)\right)B(0)^4 \\ \ldots \end{cases} \tag{21}$$

For the quadrupole symmetry

$$v_0(t)=\begin{cases} -0.5\pi, & t\in[-\infty,0]; \\ 0, & t\in[0,\infty]. \end{cases} \tag{22}$$

$$dw(z)=\sqrt{\sqrt{2}/[1-\exp(-\pi z/H)]}\,\exp[F(z)]dz \tag{23}$$

$$B(z)=\sqrt{[1-\exp(-\pi z/H)]/\sqrt{2}}\,\exp[-F(z)] \tag{24}$$

$$\frac{d^iB(z)}{d\omega(z)^i}=\frac{\sqrt{[1-\exp(-\pi z/H)]/\sqrt{2}}}{\exp[F(z)]}\frac{d}{dz}\left[\frac{d^{i-1}B(z)}{d\omega(z)^{i-1}}\right] \tag{25}$$

$$\begin{cases} B'(0)=\dfrac{\pi}{H2\sqrt{2}\exp[2F(0)]}; \\ B^{(3)}(0)=\dfrac{-\pi^2(\pi+4HF'(0))}{H^3 8\exp[4F(0)]}; \\ B^{(5)}(0)= \\ \dfrac{\pi^3\left(2\pi^2+13H\pi F'(0)+26H^2F'(0)^2-9H^2F''(0)^2\right)}{H^5 8\sqrt{2}\exp[6F(0)]}; \\ \ldots \end{cases} \tag{26}$$

All the even derivatives are equal to zero at the point $\omega=0$. The influence of the multipolar member (13) consists of that. The analysis of expressions (26) shows, that if $F(z)+\pi/4$ is the odd function (the right member of a pole is symmetric left-hand), the only derivatives allowed for a quadrupole symmetry, with numbers n = 1 + 4*i are not equal to zero.

Using the determined with the help of (26) the value of the derivatives of F(z), one can restored the function F(z) itself, and the inverse to the complex potential function as well. Thus, knowing a field it is possible to restore the pole structure which will realize this field.

5. CONCLUSION

The above mentioned expressions allow to calculate the field and the forms of poles of multipoles if the function of an angle of declination forming of a pole structure $v_H(t)$ - in coordinates of a rectilinear band $0\le \mathrm{Im}(z)\le H$ is known. The inverse solution is posible to get as well: by a field is discovered $v_H(t)$, so also form of a pole.The form of function $v_H(t)$ can be those, that of a pole will take the form necessary of technological reasons. These circumstances have allowed to develop the program of synthesis of a pole structure. Input data of the program are: the field specified explicity or as aseries, and some design data of a pole. The Fig.2 is ploted using this program.

6. REFERENCES

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