STUDY OF LASER INJECTION OF ELECTRONS INTO PLASMA WAKEFIELDS

C. B. Schroeder, P. B. Lee, J. S. Wurtele, Dept. of Physics, Univ. of California, Berkeley, CA, USA
E. Esarey, NRL, Washington D.C., USA
W. P. Leemans, LBNL, Berkeley, CA, USA

Abstract

A proposed laser-plasma based electron source [1] using laser triggered injection of electrons is examined. The source generates ultrashort electron bunches by dephasing background plasma electrons undergoing fluid oscillations in a plasma wake. The plasma electrons are dephased by colliding two counter-propagating laser pulses. Laser intensity thresholds for trapping and optimal wake phase for injection are calculated. Numerical simulations of test particles, with prescribed plasma and laser fields, are used to verify analytic predictions and to characterize the quality of the electron bunches.

1 INTRODUCTION

The characteristic scale length of the accelerating field in a plasma-based accelerator [2] is the plasma wavelength, λ_p[m] $\simeq 3.3 \times 10^4 n_e^{-1/2}$[cm^{-3}] where n_e is the plasma density. In such short wavelength accelerators (typically $\lambda_p \sim 100\,\mu$m), production of electron beams with low momentum spread and good pulse-to-pulse energy stability requires femtosecond electron bunches to be injected with femtosecond synchronization with respect to the plasma wake. Although conventional electron sources (photocathode or thermionic RF guns) have achieved sub-picosecond electron bunches [3], the requirements for injection into plasma-based accelerators are presently beyond the performance of these conventional electron sources. Novel schemes which rely on laser triggered injection of plasma electrons into their own plasma wake have been proposed [4] to generate the required femtosecond electron bunches.

Recently a new optical injection scheme was proposed [1] which uses two relatively low intensity counter-propagating laser pulses in addition to a pump laser pulse for plasma wake excitation. This colliding pulse scheme has the ability to produce femtosecond electron bunches with low fractional energy spreads using relatively low injection laser pulse intensities compared to the pump laser pulse ($a_{inj}^2 \ll a_{pump}^2 \sim 1$). Here $a = eA/mc^2 \simeq 8.5 \times 10^{-10}\lambda[\mu\text{m}]I^{1/2}$[W/cm^2] is the normalized vector potential, I is the laser pulse intensity and λ is the laser wavelength. The colliding pulse concept also offers detailed control of the injection phase through the position of the forward injection pulse with respect to the pump pulse, the beat wave velocity via the injection pulse frequencies, and the number of trapped electrons via the injection pulse intensities.

2 PHASE SPACE ANALYSIS

The colliding pulse optical injection scheme employs three short laser pulses (shown in Fig. 1): an intense ($a_0^2 \simeq 1$) pulse (denoted by subscript 0) for plasma wake generation, a forward going injection pulse (subscript 1), and a backward going injection pulse (subscript 2). The pump pulse generates a plasma wake with phase velocity near the speed of light. The injection pulses collide some distance behind the pump pulse. When the injection pulses collide, they generate a beat wave with a phase velocity $v_b = \Delta\omega/\Delta k \simeq \Delta\omega/2k_0$, where the frequency difference of the injection pulses is $\Delta\omega = \omega_1 - \omega_2$ and the wavenumber difference is $\Delta k = k_1 - k_2$ with $k_1 \simeq |k_2| \simeq k_0$. During the time when the two injection pulses overlap, the slow beat wave injects plasma electrons into the fast plasma wake for acceleration to high energies.

The colliding pulse injection mechanism can be studied using a Hamiltonian approach. The electron motion in the plasma wake is described by the Hamiltonian

$$H(\gamma,\psi) = \gamma - \beta_\varphi\left[\gamma^2 - 1\right]^{1/2} - \phi(\psi), \qquad (1)$$

where $\gamma m_e c^2$ is the electron energy and $\beta_\varphi = (1 - \omega_p^2/\omega_0^2)^{1/2}$ is the plasma wake phase velocity (group velocity of the pump pulse). Here $\omega_p = k_p c = (4\pi e^2 n_e/m_e)^{1/2}$ is the plasma frequency with $-e$ the electron charge, m_e the electron mass, and c the speed of light. It is assumed that $\gamma_\varphi = (1-\beta_\varphi^2)^{-1/2} \gg 1$. The scalar potential of the plasma wake is assumed to have the form $\phi(\psi) = \phi_o \cos\psi$ where the wake phase is $\psi = k_p z - \omega_p t$ and the normalized wake potential amplitude is $\phi_o = e\Phi_o/m_e c^2$. The amplitude of the wake potential is determined by the pump pulse amplitude and shape. The normalized axial momentum of the electron in an orbit H of the plasma wake is

$$(\gamma\beta_z) = \beta_\varphi\gamma_\varphi^2[H+\phi(\psi)] \pm \gamma_\varphi\left(\gamma_\varphi^2[H+\phi(\psi)]^2 - 1\right)^{1/2}. \qquad (2)$$

The boundary between trapped and untrapped orbits is given by the separatrix orbit $H(\gamma = \gamma_\varphi, \psi = \pi) = \gamma_\varphi^{-1} + \phi_o$.

The colliding injection pulses lead to formation of a beat wave with phase space buckets (separatrices) of width $2\pi/\Delta k \simeq \lambda_0/2$ (much shorter than those of the wake field λ_p). The motion of the electron in the beat wave is described by the beat wave Hamiltonian,

$$H_b(\gamma,\psi_b) = \gamma - \beta_b[\gamma^2 - \gamma_\perp^2(\psi_b)]^{1/2}, \qquad (3)$$

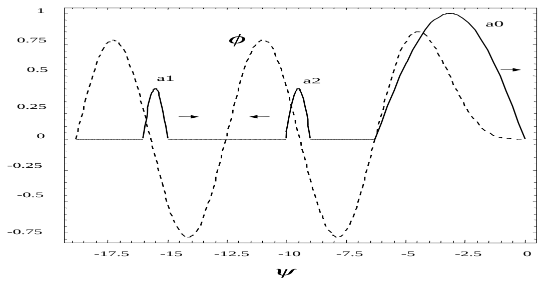

Figure 1: Normalized potential profiles of the pump laser pulse a_0, the plasma wake ϕ, forward injection laser pulse a_1, and the backward injection laser pulse a_2.

where $\beta_b = \Delta\omega / c\Delta k = (\lambda_2 - \lambda_1)/(\lambda_2 + \lambda_1)$ is the beat wave phase velocity, $\psi_b = \Delta k(z - \beta_b ct)$ is the beat wave phase, and $\gamma_\perp^2(\psi_b) = 1 + \hat{a}_1^2 + \hat{a}_2^2 + 2\hat{a}_1\hat{a}_2\cos\psi_b$ with $\hat{a}_1$ and $\hat{a}_2$ the rms amplitudes of the forward and backward injection pulses respectively. The separatrix orbit in phase space of the beat wave Hamiltonian has the value of $H_b(\gamma = \gamma_b\gamma_\perp(0), \psi_b = 0) = \gamma_\perp(0)\gamma_b^{-1}$ where $\gamma_b = (1 - \beta_b^2)^{-1/2}$. The maximum and minimum normalized axial momentum of an electron in a beat wave orbit (extrema of the separatrix) are

$$[(\gamma\beta_z)_{beat}]_{max/min} = \gamma_b\beta_b\gamma_\perp(0) \pm 2\gamma_b(\hat{a}_1\hat{a}_2)^{1/2}. \quad (4)$$

The threshold injection laser pulse intensities required for trapping of background plasma electrons into the plasma wake can be estimated by considering the effects of the plasma wake and the beat wave individually and requiring resonance overlap, namely that the maximum momentum of the beat wave separatrix exceed the minimum momentum of the plasma wake separatrix and the minimum momentum of the beat wave separatrix be less than the fluid momentum of electrons in the plasma wake. The maximum and minimum momentum of an electron in a beat wave orbit are given in Eq. (4). The momentum of an electron undergoing fluid oscillations is given by Eq. (2) with $H = 1$. The momentum of an electron in a trapped orbit of the plasma wake is given by Eq. (2) with $H \leq H(\gamma = \gamma_\varphi, \psi = \pi) = \gamma_\varphi^{-1} + \phi_o$, and for an electron in a trapped and focused orbit, the momentum is given by Eq. (2) with $H \leq H(\gamma = \gamma_\varphi, \psi = \pi/2) = \gamma_\varphi^{-1}$. A trapped and focused electron orbit is one where the wake phase remains in a range such that the transverse electric field due to the plasma wake is always providing a focusing force on the electron.

Requiring resonance overlap yields the threshold beat wave amplitude parameter for trapping plasma electrons

$$(\hat{a}_1\hat{a}_2)_{th}^{1/2} = \frac{1 - H}{4\gamma_b(\beta_\varphi - \beta_b)}, \quad (5)$$

and the optimal wake phase for injection

$$\cos\psi_{opt} = \phi_o^{-1}\left[\gamma_b\left(1 - \beta_\varphi\beta_b\right)\gamma_\perp(0) - \frac{1}{2}\left(1 + H\right)\right]. \quad (6)$$

Here $H = \gamma_\varphi^{-1} + \phi_o$ for a trapped plasma wake orbit and $H = \gamma_\varphi^{-1}$ for a trapped and focused plasma wake orbit. For example, if $\phi_o = 0.7$ and $\beta_{beat} = 0$, then $(\hat{a}_1\hat{a}_2)_{th}^{1/2} \simeq 0.25$ for injection of plasma electrons into a trapped and focused orbit.

Minimizing the injection pulse amplitudes (operating near the threshold amplitude given by Eq. (5)) will minimize the laser power $P[GW] \simeq 21.5(a_i r_i/\lambda_i)^2$ required for trapping and is therefore important for the experimental realization of this injection scheme. For illustration, if the injection pulses have a wavelength of 0.8 μm and a spot size of 15 μm, then the injection laser pulse power required for trapping is $P < 1$ TW.

3 NUMERICAL STUDIES

To further evaluate the colliding pulse scheme and to test the analytic predictions for the trapping thresholds presented in Sec. 2, the motion of test particles in the combined wake and laser fields was simulated by numerically solving the equations of motion for the electrons using an adaptive stepsize Runge-Kutta method.

We assume the laser pulses are linear polarized fundamental Gaussian beams with half-period cosine longitudinal envelopes. The polarizations of the laser pulses are chosen to be $\hat{e}_{\perp 0} = \hat{x}$ and $\hat{e}_{\perp 1} = \hat{e}_{\perp 2} = \hat{y}$ such that $\vec{a}_0 \cdot \vec{a}_2 \simeq 0$ and thus there is no beating (no wake generation) from the interaction of the pump pulse and the counter-propagating injection pulse. The plasma wakefields produced by the injection pulses can be neglected ($\phi_1 \sim \phi_2 \ll \phi_0$) since the injection pulse amplitudes required for trapping are much less than the pump pulse amplitude and the pulse lengths of the injection pulses can be chosen to provide poor coupling between the plasma response and the injection pulses.

Assuming $a_0^2 < 1$, the axial and radial components of the electric field (to lowest order in pump pulse amplitude) due to the excited plasma wake near the waist of the pump pulse are $E_z = (m_e c^2 k_p/e)\phi_0 \exp[-2r^2/r_{s0}^2]\sin\psi$ and $E_r = (m_e c^2/e)(4r/r_{s0}^2)\phi_0\exp[-2r^2/r_{s0}^2]\cos\psi$, where $\phi_o = \pi a_0^2/4$ and r_{s0} is the pump pulse spot size. The radial electric field will provide a focusing force for an electron at a plasma wake phase of $\cos\psi > 0$ and a defocusing force for $\cos\psi < 0$.

The plasma was assumed to be initially homogeneous and cold such that the test particles were loaded uniformly with no initial momentum. Unless otherwise stated, the parameters used in the numerical simulations are listed in Table 1. Simulations indicate good agreement with analytic estimates, Eqs. (5) and (6).

The fraction of loaded test electrons which become trapped as a result of the colliding injection pulses was found to peak at an injection wake phase (plasma wake

Table 1: Simulation Parameters

Plasma wavelength λ_p	40 μm
Pump laser strength a_0	0.94
Plasma wake potential ϕ_o	0.7
Pump pulse length $L_0 = \lambda_p$	40 μm
Pump pulse wavelength λ_0	0.8 μm
Laser spot size $r_{s0} = r_{s1} = r_{s2}$	15 μm
Injection laser pulse strength $a_1 = a_2$	0.3
Injection pulse length $L_1 = L_2 = \lambda_p/2$	20 μm
Injection pulse (forward) wavelength λ_1	0.83 μm
Injection pulse (backward) wavelength λ_2	0.80 μm

phase where the maxima of the injection pulses collide) of $\psi_{opt} \simeq \pm 1.0$ which agrees well with the analytic predictions Eq. (6). Significant trapping of electrons occured for an injection wake phase region of $-1.5 < \psi_{inj} < 1.5$. This indicates that the two colliding injection pulses must be synchronized with the plasma wake with an accuracy of ~ 10 fs, which is not a serious timing constraint for present laser technology.

The quality of the electron bunch can be examined as the beat wave amplitude parameter $(\hat{a}_1 \hat{a}_2)^{1/2}$ is increased beyond the threshold value for injection into a trapped and focused orbit, Eq. (5) with $H \leq \gamma_\varphi^{-1}$. Fig. 2(a) shows the fraction of loaded test electrons which become trapped and focused, as well as the bunch duration for trapped electrons versus the beat wave amplitude parameter. As an example, for a plasma density of $n_e = 7 \times 10^{17}\mathrm{cm}^{-3}$, the maximum fraction corresponds to a bunch number of $N_b \sim 0.5 \times 10^7$ electrons. Fig. 2(b) shows the asymptotic fractional energy spread $\sigma_\gamma / \langle\gamma\rangle$ and the transverse normalized rms emittance of the electron bunch versus the beat wave amplitude parameter. As expected, the rms phase spread (bunch duration) is constant for a highly relativistic bunch, the fractional energy spread is asymptotic for large interaction lengths, and the transverse normalized rms emittance is conserved for large pump laser spot size.

4 SUMMARY

The colliding pulse optical injection scheme has the ability to generate ultrashort electron bunches by colliding laser pulses to dephase background plasma electrons undergoing fluid oscillations in a plasma wake. Simulations indicate femtosecond electron bunches with low fractional energy spread ($< 10\%$) and low normalized transverse emittance (~ 1 mm mrad) can be produced. The colliding pulse scheme requires relatively low laser power compared to the pump pulse $a_1^2 \sim a_2^2 \ll a_0^2$, and allows for detailed control of injection process through the injection phase (position of the forward injection laser pulse), beat wave velocity (frequencies of the injection laser pulses), and the beat wave amplitude parameter (injection pulse intensities).

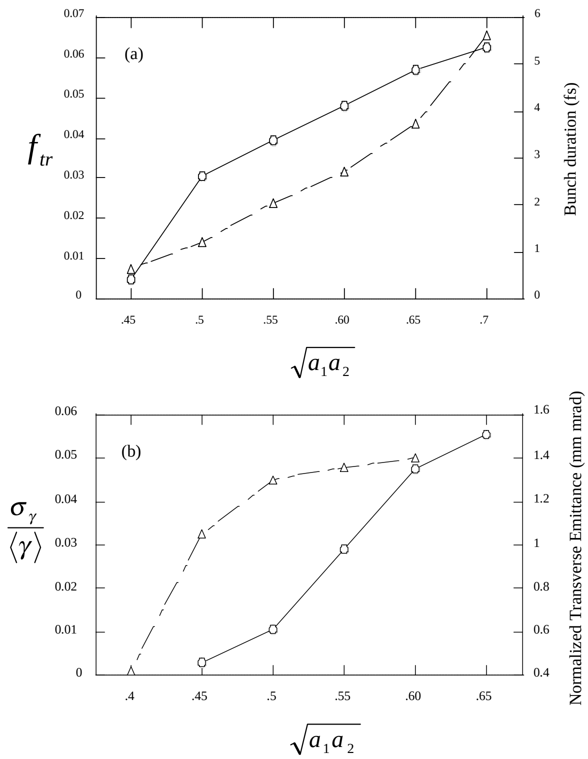

Figure 2: (a) Fraction of loaded test electrons which become trapped and focused (solid line) and bunch duration (dashed line) versus beat wave amplitude parameter. (b) Asymptotic fractional energy spread $\sigma_\gamma / \langle\gamma\rangle$ (solid line) and normalized transverse rms emittance $\varepsilon_\perp$(mm-mrad) (dashed line) of trapped electron bunch versus beat wave amplitude parameter.

5 REFERENCES

[1] E. Esarey, R. F. Hubbard, W. P. Leemans, A. Ting, and P. Sprangle, Phys. Rev. Lett. **79**, 2682 (1997).

[2] E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE Trans. Plasma Sci. **24**, 252 (1996).

[3] P. Kung, H Lihn, H. Wiedemann, D. Bocek, Phys. Rev. Lett. **73**, 967 (1994).

[4] D. Umstadter, J. K. Kim, and E. Dodd, Phys. Rev. Lett. **76**, 2073 (1996).