A New COD Correction Method for Orbit Feedback

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Abstract

We present a new COD correction method for orbit feedback, which can exactly fix the beam positions at selected BPMs and simultaneously correct COD around the whole ring. This method can realize a single feedback which has both functions of global and local feedbacks. A computer simulation of COD correction for the VSX ring confirms that the new correction method is very useful for orbit feedback.

1 INTRODUCTION

Photon beam positions or closed orbits in synchrotron radiation sources are usually stabilized by global and/or local feedbacks. The global feedback efficiently corrects COD around the whole ring with the harmonic method, the least squares method or the eigenvector method, while the local orbit feedback tightly fix the beam position at a photon source point by the local orbit bump method. However, the two feedbacks may interfere with each other and deteriorate the orbit stability when they are operated at the same time. We propose a new correction method, the eigenvector method with constraints, which can have both functions of global and local COD corrections. In the paper, the new COD correction method for orbit feedback is presented and compared with the ordinary eigenvector method.

2 THEORY

2.1 Ordinary Eigenvector Method

In this sub-section, the eigenvector method is reviewed.
The measured COD at beam position monitors (BPM), the kick angle strengths of the correctors and the response matrix are also denoted by y, θ and R respectively. Here, the numbers of BPMs and M and N, and R is an MxN matrix. The norm of $\vec{\Delta}$ defined by

$$\vec{\Delta} \equiv R\vec{\theta} + \vec{y} \qquad (1)$$

should be zero or as small as possible.

For the betatron functions and phases of i-th BPM (β_i,ϕ_i^B) and j-th corrector (β_j,ϕ_j^C) the element of the response matrix R_{ij} is given by

$$R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2\sin \pi\nu} \cos \nu(\pi - |\phi_i^B - \phi_j^C|)$$

The norm of $\vec{\Delta}$ becomes minimum when its derivatives with respect to $\theta_j (j = 1,2,\cdots\cdots,N)$ are zero. The kick angle vector $\vec{\theta}$ is then determined by

$$R^T R\vec{\theta} + R^T \vec{y} = 0 \quad . \qquad (2)$$

Since $R^T R$ is a real symmetric matrix, it can be decomposed by

$$R^T R = U\Lambda U^T , \qquad (3)$$

where the superscript "T" stands for the transposed matrix or vector. U and Λ are written as

$$U = (\overrightarrow{U_1}, \overrightarrow{U_2}, \cdots\cdots\cdots, \overrightarrow{U_n})$$
$$\Lambda = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix} , \qquad (4)$$

where U_i and λ_i are the i-th eigenvector and eigenvalue of the matrix $R^T R$, respectively.
From Eqs. (2) and (3), $\vec{\theta}$ is obtained as

$$\vec{\theta} = -R^+ \vec{y} \quad , \qquad (5)$$

where

$$R^+ = (U\Lambda U^T)^{-1} R^T = U\Lambda^{-1} U^T R^T$$

$$\Lambda^{-1} = \begin{pmatrix} \frac{1}{\lambda_1} & & & 0 \\ & \frac{1}{\lambda_2} & & \\ & & \ddots & \\ 0 & & & \frac{1}{\lambda_n} \end{pmatrix} \quad (6)$$

For small eigenvalues, their reciprocals in the matrix Λ^{-1} is usually replaced with zeros in order to avoid very large kick angles of correctors and to reduce the error effects of BPM and corrector.

2.2 New Correction Method

A new COD correction method is the eigenvector method with constraints. Here, the constraint conditions may be given by

$$\overrightarrow{C_i^T}\vec{\theta} + z_i = 0 \ \text{(i=1,……….., L)}, \quad (7)$$

where L means the number of constraints.

We minimize the norm of $\vec{\Delta}$ in (1) under the constraint conditions (7) using Lagrange's method of indeterminate multipliers. The function of S is given by

$$S = \frac{1}{2}(R\vec{\theta} + \vec{y})^2 + \sum_i^L \mu_i (\overrightarrow{C_i^T}\vec{\theta} + z_i). \quad (8)$$

Putting the derivatives of S with respect to $\vec{\theta}$ and μ_i to zeros, we obtain

$$\begin{cases} A\vec{\theta} + R^T\vec{y} + C\vec{\mu} = 0 \\ C^T\vec{\theta} + \vec{z} = 0 \end{cases}, \quad (9)$$

where

$$\vec{z} = \begin{pmatrix} z_1 \\ \cdot \\ \cdot \\ z_L \end{pmatrix}, \quad \vec{\mu} = \begin{pmatrix} \mu_1 \\ \cdot \\ \cdot \\ \mu_L \end{pmatrix},$$

$$C = (\overrightarrow{C_1}\overrightarrow{C_2}\cdots\cdots\cdots\overrightarrow{C_L}), \quad A = R^T R.$$

When the reciprocals of small eigenvalues are replaced with zeros in the same manner as subsection 2.1, the inverse matrix A^{-1} can be expressed by the n eigenvectors as follows,

$$A^{-1} = \sum_{i=1}^n \frac{\overrightarrow{U_i}\overrightarrow{U_i^T}}{\lambda_i}. \quad (10)$$

From the first and second equations in (9), we obtain

$$\vec{\mu} = P^{-1}\vec{z} - P^{-1}C^T A^{-1} R^T \vec{y} \quad (11)$$

and

$$\vec{\theta} = (-A^{-1} + A^{-1}CP^{-1}C^T A^{-1})R^T\vec{y} - (A^{-1}CP^{-1})\vec{z} \quad (12)$$

where $P = C^T A^{-1} C$.

The condition of $n \geq L$ is required for the existence of the inverse matrix P^{-1}. If z_i in Eq.(7) is taken as the beam position observed at an arbitrarily selected BPM and C as the corresponding part of the response matrix R, the beam position can be fixed at zero. Further, if BPMs on both sides of a photon source such as insertion device are selected for constraints, we can keep the beam position and angle of the source point at zero and simultaneously correct the COD around the whole ring.

3 RESULTS OF SIMULATION

The VSX project aims at constructing third-generation synchrotron light sources in the Kashiwa campus of Tokyo University. The 2GeV VSX ring is 388m in circumference. 14 insertion devices will be installed there. 128 BPMs and 112 correctors will be used for orbit feedback.

A computer simulation for the new COD correction method has been carried out for the VSX ring. The constraints adopted here are that the positions at BPMs on both sides of 14 insertion devices are zero i.e. the number of the constraints is 28. Figure 1 shows a typical COD before correction. Here, we assumed that the alignment error of quadruple magnet has a gaussian distribution with σ=5.0x10^{-5}.

Figures 2 and 3 show the rms ratio of CODs before and after correction. The new method are compared with the ordinary method in the figures. For the number of eigenvalues more than 40, the new method has almost the same rms ratio as the ordinary method.

Figure 4 and 5 shows the ratio of kick angles for the two methods. The maximum and rms of the kick angles for the new method are almost equal to those for the ordinary method.

4 CONCLUSIONS

The new COD correction method was formulated and a computer simulation was carried out. The results of this simulation show that the new correction method fully satisfies constraint conditions and has almost the same correction performance as the ordinary method. The kick angle strengths of the correctors are also comparable with those in the ordinary method. Since, the global and local feedbacks are compatible in the new method, it is most suitable for an orbit feedback system of synchrotron light sources. The effects of BPM reading and corrector setting errors are discussed elsewhere.

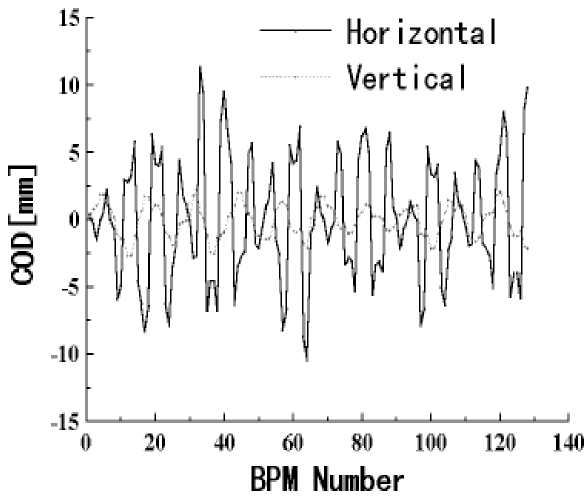

Figure 1: A typical COD without correction around the whole ring for the VSX ring.

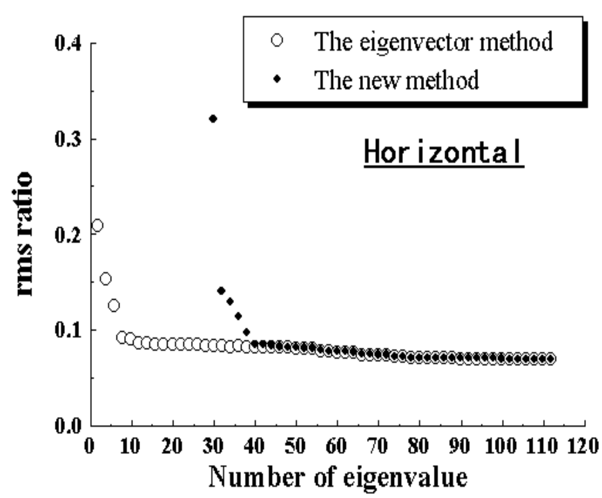

Figure 2: Horizontal rms ratio.

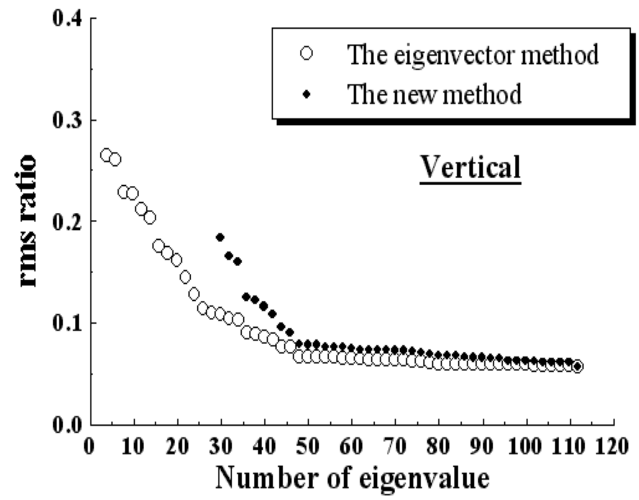

Figure 3: Vertical rms ratio.

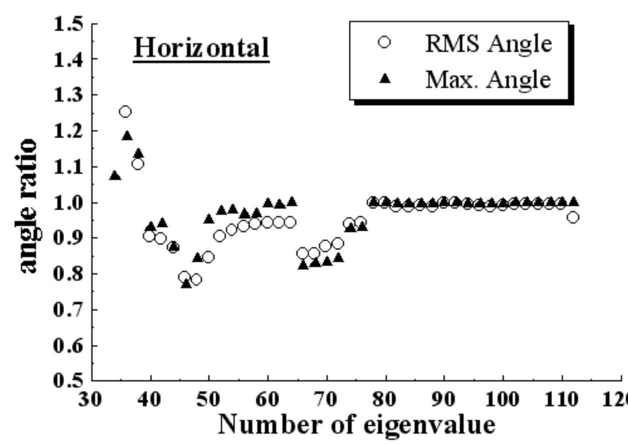

Figure 4: Ratio of kick angles of horizontal correctors.

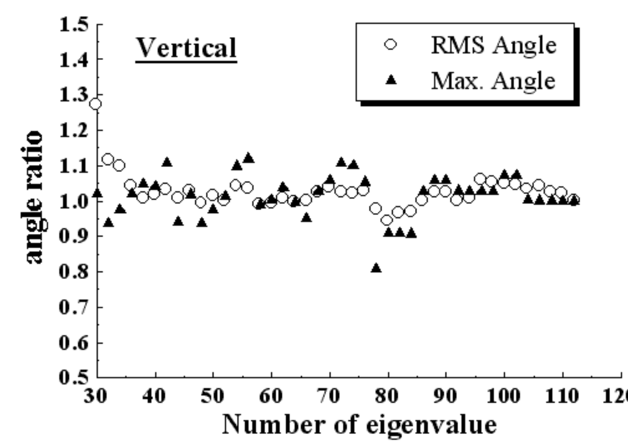

Figure 5: Ratio of kick angles of vertical correctors.