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IMPEDANCE AND COLLECTIVE EFFECTS IN JLEIC*

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Abstract

JLEIC is the high luminosity and high polarization electron-ion collider (EIC) currently under design at Jefferson Lab. Its luminosity performance relies on the beam stability under high-intensity electron and ion beam operation. The impedance budget analysis and the estimations of beam instabilities are currently underway. In this paper, we present the update status of our back-of-envelope estimations for these collective instabilities, and identify area or parameter regimes where special attentions for instability mitigations are required.

INTRODUCTION

The JLEIC baseline parameters [1] are conceived based on the unique luminosity concept of the design, featuring small bunch emittances, relatively low bunch charge, and very high bunch repetition rate [2]. These features further determine the behaviour of collective instabilities in the collider rings during bunch collision. It implies moderate single-bunch instabilities; yet it poses strong requirements on the fast feedback systems to mitigate longitudinal and transverse coupled-bunch instabilities. For a complete design study, the collective effects need to be assessed for a wide range of beam energies and ion species, and also for the entire ion bunch formation process. In this presentation, we only focus on cases for a few selected collision energies.

Ideally, the wakefield induced beam instabilities can be analytically and numerically studied once the machine impedance budget is available. However, developing impedance budget and performing instability estimations are an iterative and gradually refining process. Presently, JLEIC design is still at its early phase and the engineering design has just begun. At this stage, a preliminary estimation of impedance thresholds, for various coherent instabilities, is necessary for the engineer design to make design choices so as to minimize machine impedances and ensure beam stability. In this paper, we discuss the current status of the JLEIC impedance studies, and present our initial back-of-envelop estimations for the single and coupled bunch instabilities using the recent JLEIC baseline design parameters. The estimated impedance threshold will be compared with the expected machine impedances for the JLEIC collider rings, as inferred from the impedance budgets of some existing machines. We will also give preliminary discussions about the two-stream instabilities, i.e., the electron cloud effects in the ion ring and the ion effects in the electron ring.

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JLEIC IMPEDANCE ESTIMATIONS

In a storage ring, the electromagnetic response of the vacuum chamber to the beam current is characterized by the broadband and narrowband impedances, which could cause respectively the single-bunch and coupled-bunch collective instabilities. The narrowband impedances for the JLEIC electron and ion rings are discussed in the section on the coupled-bunch instabilities. For broadband impedances, the estimation of the impedance budget requires engineer drawings of the vacuum chamber. Yet for JLEIC, presently the machine engineering design has just begun, hence no details are available except for the element counts for most of the impedance-generating components in both rings (see Table 1). Without the actual component designs, at present we can only use the impedance budgets for some existing machines, such as PEPII, SUPERKEKB, or RHIC, as references [3-5]. One reason for using PEPII for reference is that there is consideration for the JLEIC e-ring to adopt the RF cavities, as well as the components for vacuum system and diagnostics, from PEPII HER. Another convenient feature is that the bunch length ($\sigma_z \approx 1.2$ cm) for JLEIC is comparable to that in PEPII, given that the effective impedances are bunch-length dependent. With the PEPII impedance budget and the JLEIC component counts in Table 1, and assuming these components are identical with those used in the PEPII HER, we get the estimation for the JLEIC e-ring: the inductance $L \approx 99.2$ nH, the effective longitudinal impedance $|Z_\parallel/n| \approx 0.09\ \Omega$, the loss factor $k_\parallel \approx 7.7$ V/pC, and the effective transverse impedance $|Z_\perp| \approx 60$ kΩ/m. If components in SUPERKEKB are used as reference, the JLEIC e-ring impedance estimation becomes:

$L \approx 28.6$ nH, $|Z_\parallel/n| \approx 0.02\Omega$, $k_\parallel \approx 19$ V/pC, $|Z_\perp| \approx 13$ kΩ/m,

with the note that the shorter bunch length ($\sigma_z \approx 0.5$ cm) for beams in SUPERKEKB than that in JLEIC may cause underestimation of the effective impedances.

For the JLEIC ion ring, the ion beam undergoes the bunch formation process including the injection, acceleration, bunch splitting, and finally collision. The bunch length varies through the whole process, and the short ion bunch ($\sigma_z \approx 1.2$ cm) at the collision state is made possible only by employing the envisioned high-energy electron cooling [6]. Since such short bunch length is unprecedented for the ion beams in existing ion rings, it is more appropriate [7] to use the PEPII rings rather than the existing ion rings for reference when estimating the JLEIC

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ion ring impedance budget. The ion-ring impedance at the collision scenario is thus estimated as:
$L \approx 97.6$ nH, $|Z_{\parallel}/n| \approx 0.08\ \Omega$, $k_{\parallel} \approx 8.6$ V/pC, $|Z_{\perp}| \approx 80$ kΩ/m.

Table 1: Impedance-Generating Components in JLEIC

Elements	**e-Ring**	**Ion-Ring**	**e-Cooler**
Flanges (pairs)	1215	234	104
BPMs	405	214	49
Vacuum Ports	480	92	62
Bellows	480	559	74
Vacuum valves	23	14	-
Tapers	6	6	26
Collimators	16	16	-
Forks	0	0	4
Fast kickers	0	0	2
DIP screen slots	470	-	-
Crab cavities	2	8	0
RF/SRF cavities	32	40	2
RF/SRF bellows	0	60	0
RF/SRF valves	68	24	-
Feedback kickers	2	2	-
IR chamber	1	1	-

As the JLEIC design improves and getting more complete, the counts for certain elements in Table 1, such as the collimators, feedback kickers, and ion-clearing electrodes, will be further modified. In addition, some special components unique to the JLEIC design, such as the crab cavities and IR chamber, require detailed impedance modelling and cannot use reference of impedances from the existing machines. Accurate impedance budgets of both the electron and ion rings require careful electromagnetic field calculations, which can generate the full impedance spectrum for each impedance-generating components.

SINGLE BUNCH INSTABILITIES

In this section, we discuss the beam stability at the collision scenarios for the electron beam at energies E_e=3, 5, 10 GeV and for the proton beam at E_p=100 GeV.

Longitudinal Microwave Instability (LMWI)

With the Boussard approximation, the LMWI instability threshold is given by the Keil-Schnell criterion:

$$\left|\frac{Z_{\parallel}(n)}{n}\right|^{\mathrm{th}} \approx \frac{2\pi\beta^2|\eta|E\sigma_\delta^2}{eI_{\mathrm{peak}}}.$$

For JLEIC baseline parameters in Table 2, the estimation of LMWI impedance thresholds are listed in Table 3 and compared with the expected machine impedances $|Z_{\parallel}/n|^{\mathrm{ring}}$, where “s”, “u”, and “m” denote stable, unstable and marginal respectively. It is interesting to note that unlike PEP-II LER, which is a separate ring and has different dipole configuration from that in HER, here the JLEIC e-ring uses the same dipole configuration for a wide range of beam energy, with both the dipole strength and the energy spread from synchrotron radiation scaling with the beam energy. As a result, the energy spread for beam at 3 GeV in the JLEIC e-ring is much smaller than that for the PEP-II LER beam; so the former is vulnerable to LMWI while the latter is not. This estimation indicates the necessity to apply suppression mechanisms against the microwave instability for the JLEIC e-ring at low energy. Examples of such mechanisms include use of an alternative dipole configuration, the split dipole concept in the eRHIC design [8], or damping wigglers. For the ion ring, the machine impedance is expected to be much smaller than the threshold impedance, so the beam is safe from this instability. For the electron ring, detailed simulations are to be conducted to study the bunch lengthening due to potential-well distortion below the LMWI threshold, and the turbulent bunch lengthening and energy-spread increase beyond the instability threshold.

Transverse Mode-Coupling Instability (TMCI)

The impedance threshold for the transverse mode-coupling instability (TMCI) is roughly approximated by

$$|Z_{\perp}|^{\mathrm{th}} \approx FE\,\nu_s / e\langle\beta_{\perp}\rangle I_{\mathrm{peak}},$$

with F the bunch form factor ($F \quad 2\pi$ for short bunches). The threshold results are obtained from parameters in Table 2 and listed in Table 4 for both the JLEIC electron and proton beams at selected collision energies, and are compared with the expected upper limits of the machine transverse impedances $|Z_{\perp}|^{\mathrm{ring}}$. These results show that the beams are stable with regard to TMCI. Here the machine impedances are estimated using impedance budgets of existing machines. Since there are large uncertainties in both the machine traverse impedance and the simple back-of-envelope formula, detailed studies of TMCI will be carried out when more accurate JLEIC impedance model becomes available. Such studies include solving the eigenvalue problem of the Vlasov equation [9] or macroparticle tracking that takes into account of potential-well distortion in the longitudinal phase space and many other effects [10]. Additionally, special attention needs to be paid to the Christmas- tree-like equilibrium longitudinal charge distribution for the proton bunch under strong electron cooling, which has a very dense core with long tails [11]. Space-charge effects on TMCI will also be assessed, especially for the ion bunches during their formation process [12].

Table 2: Parameters Used for Instability Estimations

	PEP-II (LER)	**JLEIC** e-Ring			**JLEIC** p-Ring
E [GeV]	3.1	3	5	10	100
I_p [A]	113	59.0	59.0	50.6	15.6
η (10^{-3})	1.31	1.09			6.22
σ_δ (10^{-4})	7.7	2.78	4.64	9.28	3.0
υ_s (10^{-2})	3.7	0.88	1.46	2.51	5.3

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$\langle \beta_\perp \rangle$ [m]	20	13	18

Table 3: Longitudinal Microwave Instability (LMWI)

	PEP-II (LER)	**JLEIC** e-Ring			**JLEIC** p-Ring
E [GeV]	3.1	3	5	10	100
$\lvert Z_\parallel/n \rvert^{ring}$ [Ω]	0.1	≤0.1 (expectation)			0.1
$\lvert Z_\parallel/n \rvert^{th}$ [Ω]	0.145	0.02 7	0.125	1.16	22.5
LMWI	s	u	m	s	s

Table 4: Transverse Mode-Coupling Instability (TMCI)

	PEP-II (LER)	**JLEIC** e-Ring			**JLEIC** p-Ring
E [GeV]	3.1	3	5	10	100
$\lvert Z_\perp \rvert^{ring}$ [MΩ/m]	≤0.1	≤0.1 (expectation)			≤0.5
$\lvert Z_\perp \rvert^{th}$ [MΩ/m]	0.28	0.22	0.60	2.4	119
TMCI	s	s			s

COUPLED BUNCH INSTABILITIES

Narrowband impedances from RF cavities can cause longitudinal or transverse coupled bunch instabilities (LCBI or TCBI). The JLEIC electron ring is expected to use the PEP-II RF cavities, with the RF HOM parameters listed in Tables 1 and 2 of Ref. [13]. For the JLEIC ion ring, an initial RF cavity design is recently developed, featuring low-cost 2-cell cavity with coaxial couplers for HOM damping. The corresponding HOM parameters for the JLEIC ion ring are listed in Table 5 and 6. In addition to HOM, we also include the resistive wall impedance and broadband impedance $\left(Z_\parallel^{BB}\right)_0 = 2\Omega$ in this study. Combining the above impedances with the JLEIC machine and beam parameters, we can estimate the growth rate for the coupled-bunch instabilities by ZAP [14] (using Sacherer-Zotter's formulas) under the assumption of even bunch filling pattern. This assumption gives an upper bound of the instability growth rate for general filling patterns. Since the growth rates are much faster than the natural damping rate, the design will rely on fast feedback systems (FBS) to mitigate the coupled-bunch instabilities. Consequently, we will assess the beam stability by comparing the instability growth time with the damping times (~ millisecond) of advanced fast feedback systems. Here a nonzero chromaticity of $\xi = 1$ and a finite betatron tune spread of 3e-04 are assumed for the TCBI calculations for both the electron and the proton beams.

Table 5: Longitudinal HOM Parameters (p-Ring)

f [MHz]	R_s [Ω]	Q
940.8	7.98e06	2.98e06
1771.9	2.25e04	5643.9
1814.0	1.00e05	5265.5
2894.8	3.33e04	9172.4
3079.4	2.23e02	2.65e04

Table 6: Transverse HOM Parameters (p-Ring)

f [MHz]	Polarization	$R_\perp$ [kΩ/m]	Q
1169.8	V	17.9	82.2
1170.1	H	18.0	90.3
1183.8	H	28.1	91.3
1183.9	V	32.3	96.5
1286.7	H	110	501.6
1290.0	V	100	474.5
1315.5	H	357	697.9
1318.7	V	503	970.5
1390.0	H	1930	36348.4
1390.2	V	27700	539455
1572.7	H	1.20	64.2
1575.2	V	2.87	94.1
1627.6	H	1.96	51.2
1629.1	V	0.43	54.1
1865.1	V	3.54	84.7
2517.1	V	7.80	9707.1
2517.1	H	2.36	8531.8

In Table 7 and 8, $\tau^\parallel_{a=1}$ and $\tau^\parallel_{a=2}$ are the growth time for the longitudinal dipole and quadruple modes respectively, and $\tau^\perp_{a=0}$ and $\tau^\perp_{a=1}$ correspond to the growth time for the transverse rigid and dipole modes. Here $\tau^\parallel_{\text{damp}}$ (or $\tau^\perp_{\text{damp}}$) for the e-ring represents the natural longitudinal (or transverse) damping time due to synchrotron radiation, while $\tau^\parallel_{\text{damp}}$ and $\tau^\perp_{\text{damp}}$ for the p-ring are the damping times for the proton beam due to the strong electron cooling [15] in the JLEIC design. Note that for the electron ring, the lowest energy beam (E_e =3 GeV) has the fastest growth time, $\tau^\parallel_{a=1}$=2.9 ms for LCBI and $\tau^\perp_{a=0}$=1.6 ms for TCBI, which are manageable by FBS as operated in mo-dern electron storage rings. For the electron beam, even though the resistive wall and broadband impedances have negligible effects on the LCBI growth rate, the resistive wall has significant effect on $\tau^\perp_{a=0}$ while the broadband impedance has significant effect on $\tau^\perp_{a=1}$. For the proton beam, because of its high energy, the fast growth times of LCBI, $\tau^\parallel_{a=0}$ =6.0 ms and $\tau^\parallel_{a=1}$ =6.0 ms, would require much stronger kicker strength for the longitudinal FBS than those found in existing proton-ring FBS. This further implies higher broadband impedance due to the demand of more kicker cavities. Recently a new RF cavity design using waveguide coupler was proposed [16], and for the p-ring it can prolong the LCBI growth time to $\tau^\parallel_{a=0}$=31 ms. However, due to resistive wall impedance, the qua-

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drupole mode growth time $\tau^{\parallel}_{a=1}$ =6.2 ms remains short. More topics for TCBI need to be addressed by computer modelling, such as (1) effects of realistic uneven bunch pattern (with injection/ejection gaps and/or ion clearing gaps), (2) the joint effects of HOMs from both the accelerating/focusing RF cavities and the crab cavities, and (3) the Landau damping effect on transverse coupled-bunch instability, from either the chromaticity or the beam-beam tune shift spread.

Table 7: LCBI in JLEIC

	e-Ring			p-Ring
E [GeV]	3	5	10	100
$\tau^{\parallel}_{a=1}$ [ms]	2.9	4.1	72.8	6.0
$\tau^{\parallel}_{a=2}$ [ms]	31	43	466	6.0
$\tau^{\parallel}_{\text{damp}}$ [ms]	187	40.5	5.1	> 30 min

Table 8: TCBI in JLEIC

	e-Ring			p-Ring
E GeV]	3	5	10	100
$\tau^{\perp}_{a=0}$ [ms]	1.6	2.7	64	23.1
$\tau^{\perp}_{a=1}$ [ms]	12.8	19.6	39.8	501
$\tau^{\perp}_{\text{damp}}$ [ms]	375	81	10	> 30 min

ELECTRON CLOUD IN THE ION RING

In an ion ring, the ionization of residual gas and the beam-loss induced surface emission provide the source for the primary electrons, while the electron cloud build-up comes mainly from the secondary electron production [17]. For different stages of ion bunch formation, the build-up of electron cloud and its impact on the ion bunch stability can behave very differently. Unlike the trailing-edge effect of electron cloud for long ion bunches found in conventional ion rings, here the high rep rate and short bunches of the ion beam in JLEIC during collision render the electron cloud build-up process similar to those in positron rings of modern lepton colliders. For the proton beam at E_p =100 GeV, the electron cloud density rapidly rises up and then saturates at around the neutralization density of

$$\rho_{sat} = \frac{N_b}{\pi b^2 L_{sep}} = 2\times 10^{12}\ \text{m}^{-3},$$

as modelled in Ref. [18] for a similar set of parameters. The threshold for the electron-cloud induced single-bunch transverse mode-coupling instability (TMCI) can be estimated using the two-particle model [19],

$$\rho_{th} = \frac{2\gamma Q_s}{\pi r_p C\langle\beta_y\rangle} = 1.7\times 10^{13}\ \text{m}^{-3}.$$

With $\rho_{sat} < \rho_{th}$, the bunch is stable from the electron-cloud induced strong head-tail instability. The electron-cloud induced coupled-bunch instability for the JLEIC ion beam can cause more concern, which is yet to be studied by detailed simulations.

ION EFFECT IN THE ELECTRON RING

The ionization scattering of the electron beam with residual gas molecules in the vacuum chamber can cause ion trapping in the electron ring. The trapped ions can cause many undesirable effects for the electron beam, such as emittance growth, tune shift, halo formation, and coherent coupled-bunch instabilities. For symmetric bunch pattern, the critical mass for the ions to be trapped in either x-motion or y-motion is given by [20]

$$A^{trap}_{x,y} = \frac{r_p N_b L_{sep}}{2\sigma_{x,y}(\sigma_x+\sigma_y)}.$$

The critical ion masses for the JLEIC electron ring are listed in Table 9, which shows that all ion molecules ($A\geq 2$) will be trapped for even bunch fill. Here constant rms bunch sizes are assumed in the estimation.

Table 9: Critical Ion Mass for Trapped Ion

E_e[GeV]	3	5	10
L_{sep} [m]	0.63	0.63	2.52
σ_x[mm]	0.15	0.26	22.2
σ_y[mm]	0.07	0.12	0.51
A^{trap}_x	0.5	0.2	0.24
A^{trap}_y	1.1	0.4	0.4

Bunch clearing gaps in electron rings are often used to clear the ions so as to prevent them from accumulating turn after turn [21]. Typically a gap in the bunch train, with a length of a few percent of the ring circumference, will help clear up the ions. However, even with the ions being cleared after each turn by a clearing gap (or gaps under multi-train operation), there is still the fast beam-ion instability (FBII) [22] that could cause coupled transverse dipole motion of the electron bunches, with the dipole amplitude increases in time and along the bunch train. Under the assumptions that (1) the force between the ion and electron beam is linear to their dipole offsets and (2) constant frequency for all ion oscillations, the FBII is characterized by the growth time τ_g by

$$y_b(t) \propto \left(t/\tau_g\right)^{-1/4} e^{\sqrt{t/\tau_g}},$$

$$\tau_g^{-1}[s^{-1}] = 5p[Torr]\frac{N_b^{3/2} n_b^2 r_e r_p^{1/2} L_{sep}^{1/2} c}{\gamma\sigma_y^{3/2}(\sigma_x+\sigma_y)^{3/2} A^{1/2}\omega_\beta}.$$

For realistic beams, Landau damping is considered as a result of ion oscillation frequency spread due to horizontal charge distribution. The dipole amplitude growth is then characterized by the e-folding time [23, 24]

$$y_b \propto e^{t/\tau_e},\quad \tau_e^{-1} \approx \tau_g^{-1}\frac{c}{4\sqrt{2\pi}L_{sep} n_b a_{bt} f_i}$$

for f_i being the coherent ion oscillation frequency, and a_{bt} the ion frequency variation. For the JLEIC electron ring, τ_g and τ_e are shown in Table 10 (for a_{bt} =0.5) for a single bunch train. Here for E_e=10 GeV, the growth

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time is comparable to its counterpart for the PEPII HER beam. However, for E_e=3-5 GeV, the growth time is one or two orders of magnitude shorter and is consequently a serious concern for the electron beam stability. Further reduction of growth rate is expected if the ion frequency spread induced by the beam-size variation due to betatron oscillation is taking into account. Possible mitigation methods include using (1) chromaticity to Landau damp the FBII, (2) clearing electrode, or (3) multiple bunch trains to reduce the growth amplitude. Comprehensive numerical modelling of FBII and the mitigation schemes in JLEIC will be performed, along with its joint effect with the beam-beam induced tune spread and the coupled-bunch beam-beam instability in the gear-change collision arrangements [25].

Table 10: Growth time of FBII for JLEIC e-Ring

E_e [GeV]	3	5	10
τ_c [μs]	0.01	0.11	13.9
τ_e [ms]	0.02	0.1	3.2

CONCLUSIONS

In this section, we present the status of our initial back-of-envelope estimations for the JLEIC beam stability for a set of selected collision energies. Our estimation shows that for the current design, the low-energy electron beam is vulnerable to the longitudinal single bunch instability, and mitigation measures need to be explored. As for the coupled bunch instabilities, both the electron and proton beams require the state-of-art longitudinal and transverse fast feedback systems---as strong as those used in PEP-II or modern storage-ring light sources. As the engineering design progresses and when more details of impedance budget are available for the JLEIC collider rings, a more in-depth modeling will be conducted for the impedance-induced single and coupled bunch instabilities. Moreover, we need to model the electron-cloud buildup and its effect on the ion beam stability, in particular the e-cloud induced coupled bunch instability, as well as the effects of chromaticity, clearing electrodes, and multi-bunch train on the mitigation of fast beam-ion instability for the electron beam. The ion effects for the ERL-based high-energy electron cooling system also require careful studies.

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