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POLARIZATION ISSUES AT CEPC

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Abstract

We study a possibility of obtaining transversely polarized electron/positron beams at the CEPC collider. At the beam energy of 45 GeV, this requires the use of the special wigglers to speed up the radiative self-polarization process. A numerical estimation of the depolarizing effect of the collider field errors is made, taking into account the modulation of the spin precession frequency by synchrotron oscillations. In addition, we consider an alternative possibility of obtaining polarization by accelerating the polarized particles in the booster and then injecting them into the main ring. This option saves time spent on the polarization process, and can also be crucial for obtaining longitudinal polarization.

INTRODUCTION

This work is devoted to obtaining transverse and longitudinal polarizations within the framework of the CEPC Collider project and is based on materials presented by the author in [1, 2].

Particle transverse polarization at least of 10% is needed to apply the resonant depolarization technique [3] in the experiment on precise Z-pole mass measurement. Because of too long time of radiative polarization in the basic version of the magnetic structure of CEPC, it becomes necessary to use strong non-uniform wiggler magnets to speed up the polarization process. Such wigglers can cause a significant increase in the spread of the spin precession frequency. In turn, this leads to an intensification of the depolarizing effect of quantum fluctuations in the presence of the guiding field imperfections. The calculations of this effect should take into account the synchrotron modulation of the spin tune. We consider the main obstacles to obtaining the radiative self-polarization at CEPC with 45 and 80 GeV. It is necessary to determine the critical level of errors in the CEPC magnet alignment by calculating their response in the spin motion.

In addition, we are trying to imagine an alternative way of obtaining polarization at CEPC. The rate is made for acceleration of polarized electrons in the booster using the Partial Siberian Snake technique [4] for crossing spin resonances. Injection into the collider can provide for two modes - with transverse and longitudinal polarizations. The kinematic scheme of longitudinal polarization can include the restoration of vertical polarization in the arcs and two spin rotators at the ends of the section with IP.

SPEEDING UP POLARIZATION PROCESS

The well-known Sokolov-Ternov mechanism of radiation self-polarization of particles in an ideal storage ring is characterized by the time τ_p of polarization build-up to the extent $P_0 = 0.92$ [5]:

$$\frac{1}{\tau_p} = \frac{5\sqrt{3}}{8}\frac{r_e\Lambda_e c\gamma^5}{R^3}\left\langle K^3\right\rangle, \tag{1}$$

where r_e, Λ_e, γ are the electron radius , Compton wave length and relativistic factor respectively; K is the orbit curvature in units of the inverse machine radius R; $< .. >$ is averaging over the storage ring azimuth (ϑ). The design time of the radiative polarization in the 100 km CEPC is huge: 260 hrs at 45 GeV! At 80 GeV, this time falls as $(45/80)^5$ to 16 hrs. To speed up the polarization process, it is possible to apply the long-known method [5] based on the use of N_w special wiggler magnets (the so-called shifters) with such a distribution of the vertical field along the orbit that $\int B_w ds = 0$ and $\int B_w^3 ds \neq 0$. Let every shifter consist of three bending magnets. The field of edge magnets (B_-) is much smaller in magnitude than the field of the central one (B_+) and opposite in sign to it . The field of the latter is directed like the bending filed in the arcs. Since $|B_+|^3 >> |B_-|^3$, the equilibrium degree of polarization in the ideal case is close to the maximum (P_0). The shifters decrease the polarization time in accordance with the equation (L_- and L_+ are the corresponding magnet lengths):

$$\tau_p^w = \tau_p\left[1 + N_w\frac{B_+^3L_+ + 2|B_-|^3L_-}{2\pi R < B_0 > B_0^2}\right]^{-1}. \tag{2}$$

The fraction of radiation energy loss enhancement is

$$u = N_w\frac{B_+^2L_+ + 2B_-^2L_-}{2\pi R < B_0 > B_0}. \tag{3}$$

The harmful effect of the shifters is an increase in the beam energy spread:

$$\frac{\Delta E_w}{\Delta E} = \left[\frac{\tau_p}{\tau_p^w}\cdot\frac{1}{1+u}\right]^{1/2}. \tag{4}$$

The effectiveness of the described system as applied to CEPC can be judged by its parameters in Table 1. The Eqs. (2-4) are writen in the isomagnetic approximation (the characteristic field in the CEPC magnets is $B_0 \approx 0.013$ T at 45.6 GeV, the averaged-over-aximuth field $< B_0 >\approx 0.01$ T). At the same time, the calculated data presented in the table refer to the detailed design structure.

The reduction of τ_p by an order, down to 30 ÷ 20 hours, means that it becomes possible to polarize the beams up to 10% in a few hours. This degree of polarization is quite sufficient for its observation by a laser polarimeter under the conditions of application of the resonant depolarization technique for determining the energy of the particles [6]. As the analysis below shows, a further increase in the field of wigglers and their number leads to an undesirable increase of depolarizing effects due to the large energy spread.

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Table 1: Parameters of the wiggler system (45.6 GeV)

N_w	B_+ T	L_+ m	B_- T	L_- m	$\frac{\tau_p}{\tau_p^w}$	u	$\frac{\Delta E_w}{\Delta E}$
10	0.5	1	0.125	2	8.3	0.20	2.6
10	0.6	1	0.15	2	13.6	0.29	3.3

DEPOLARIZATION FACTOR

Non-Resonant Spin Diffusion

Quantum fluctuations lead to the scattering of particle trajectories in the beam relative to the equilibrium orbit. In turn, this causes diffusion of the vertical projection of the spins in the presence of the guide field perturbations. The corresponding depolarizing effect is characterized by the depolarization time τ_d. As a result, an actual equilibrium polarization degree $P < P_0$ is established with a relaxation time $\tau_{rel} = (1/\tau_p + 1/\tau_d)^{-1} < \tau_p$. The depolarization factor

$$G = P/P_0 = \tau_{rel}/\tau_p \tag{5}$$

depends on the distributed radial magnetic and vertical electric fields which perturb the trajectories of particles in the vertical plane. The strongest depolarizing effect is produced by the sources that cause vertical distortions $y + 0(\vartheta) = y_0(\vartheta + 2\pi)$ of the closed orbit. Their influence increases with the approach to integer spin resonances $\nu = k$. Here, k is a natural number; $\nu = \gamma a$ is the spin tune averaged over the beam particles. In general, ν is a real number of spin precession cycles per one turn of particle (γ and a are the Lorentz factor and the anomalous part of the gyromagnetic ratio of electron, respectively). Because of synchrotron oscillations with frequency ν_γ, the spin tune is modulated by the law

$$\tilde{\nu} = \nu + \Delta \cdot \cos\psi_\gamma, \tag{6}$$

Δ is the amplitude related to the amplitude of energy oscillations. The distribution function of Δ is $f(\Delta) = (\Delta/\sigma_\nu^2)\exp[-\Delta^2/(2\sigma_\nu^2)]$; $\sigma_\nu = \nu\sigma_\gamma = (\overline{\Delta^2/2})^{1/2}$ is the spin tune spread due to the beam energy spread σ_γ. Modulation Eq. (6) leads to the appearance of the dependence of the factor G on the detuning from the modulation resonances $\nu = k + m\nu_\gamma$ (m is integer).

To estimate an actually achievable polarization degree with accounting the sychrotron modulation one can use the known formula [7]

$$G \approx \left\{1 + \frac{11\nu^2}{18}\sum_{k,m}\frac{|w_k|^2 I_m(\sigma_\nu/\nu_\gamma)\exp(-\sigma_\nu/\nu_\gamma)}{[(|\nu-k| - m\nu_\gamma)^2 - \nu_\gamma^2]^2}\right\}^{-1}. \tag{7}$$

Here $I_m(x)$ is the modified Bessel function; w_k is the kth azimuthal Fourier harmonic amplitude of the spin perturbations related to radial magnetic and vertical electric fields. Approximately, the harmonic amplitude is determined as

$$w_k \approx \left\langle \nu \frac{d^2 y_0}{d\vartheta^2}\exp(-ik\vartheta)\right\rangle, \tag{8}$$

$y_0 = y_0(\vartheta)$ is the vertical closed orbit distortion in units of R. In Eq. (7) the following expansion in series in terms of Bessel functions is used:

$$w_k \exp\left[-i\frac{\Delta}{\nu_\gamma}\sin\psi_\gamma\right] = w_k\sum_{l=-\infty}^{l=\infty} J_l\left(\frac{\Delta}{\nu_\gamma}\right)\exp(-il\psi_\gamma). \tag{9}$$

In the strict sense, the expansion Eq. (9) and then Eq. (7) for G are valid if [8]

$$\sigma_\nu^2\Lambda_\gamma << \nu_\gamma^3, \tag{10}$$

Λ_γ is the radiation decrement of the synchrotron oscillations in units of inverse turns. The diffusion coefficient of the energy can be expressed as

$$D_\gamma = \frac{1}{2}\frac{d}{dt}\overline{\left(\frac{\delta\gamma}{\gamma}\right)^2} = \frac{11}{18\tau_p} \tag{11}$$

The diffusion rate of the spin precession phase is written in the form

$$D_\Psi = \frac{d}{dt}\overline{(\delta\Psi)^2} = \frac{\nu^2}{\nu_\gamma^2}D_\gamma \tag{12}$$

The parameter [1]

$$\Gamma = \frac{11\nu^2}{18\nu_\gamma^3\tau_p f_0} = \frac{D_\Psi}{\nu_\gamma f_0} = \frac{\sigma_\nu^2\Lambda_\gamma}{2\nu_\gamma^3} \tag{13}$$

characterizes the precession phase increment due to diffusion per the period of synchrotron oscillations $(\nu_\gamma f_0)^{-1}$. The condition Eq. (10) is almost equivalent to the condition

$$\Gamma << 1, \tag{14}$$

which means that such an increment is negligibly small. Under the condition

$$\Gamma \gtrsim 1, \tag{15}$$

the spectrum of spin perturbations becomes blurry, i.e. must differ from the strictly linear spectrum Eq. (9). In Table 2, the data of LEP and CEPC on the Z pole energy are presented, which allow them to be compared by the parameter Γ. In the case of LEP, as well as of CEPC without the use of wigglers, this parameter is small, the expansion Eq. (9) is quite applicable. When $\Gamma \gtrsim 1$, the results obtained on the basis of this approximation are of an evaluative nature.

Calculation Examples for LEP and CEPC

In Fig. 1, the polarization at the colliders LEP and CEPC is calculated from Eq. (7) for the magnitude of the resonance harmonic of random perturbations of a closed orbit $w_k = 2\times10^{-3}$ in dependence on the beam energy (and also on the spin tune, in the example for LEP). Only two nearest

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Table 2: Precession phase diffusion increment (Γ) in 45 GeV LEP and CEPC. Star indicates cases with special wigglers

	σ_ν	ν_γ	f_0 kHz	τ_p hr	Γ
LEP	0.061	0.083	11	5	0.054
CEPC	0.039 0.128 0.103	0.028 0.028 0.028	3 3 3	268 19.7* 32.3*	0.103 1.40 0.860

integer spin resonances with $k = 103$ and $k + 1 = 104$ are taken into account in the calculation. For simplicity, we put $|w_k| = |w_{k+1}|$. The red curves respond to the case without consideration of the modulation when (in the approximation $|\nu - k| >>\max(\sigma_\nu, \nu_\gamma)$)

$$G \approx \left[1 + \frac{11\nu^2}{18}\sum_k \frac{|w_k|^2}{(\nu-k)^4}\right]^{-1} \qquad (16)$$

The esimate for LEP for the harmonic amplitude $|w_k| = 2 \cdot 10^{-3}$ is correlated with the real data on the observation of the polarization at that facility. According to [9], the polarization at LEP reached 40% at 44.7 GeV ($\nu = 101.46$). This approximately corresponds to the calculation at the spin tune of 103.46. In the LEP polarization simulations [10], performed long before the LEP activity started, the same polarization degree at the mentioned energy was obtained for the 50 μm random vertical displacements of the LEP quads.

The plots for CEPC in Fig. 1 relate to two cases. First case (a bottom left graph) is that of the current CEPC design in the luminosity mode. It is characterized by $\nu_\gamma = 0.028$ and the moderate spreads of beam energy and spin tune. The maximal equilibrium polarization degree is close to 50%. The modulation resonances can be neglected. With that a time to build up polarizaion is too large: $\tau_{rel} \approx 0.5\tau_p = 134$ hrs. In the point of view of obtaining polarization, this mode is relevant if only using injection of polarized beams from the CEPC booster. In this case, the relaxation of the polarization occurs to a level of 50%, which guarantees the preservation of a high degree of polarization for the whole life time of the injected beam.

Another case is an example of CEPC using the special wigglers to speed up the polarization process more than 8 times in accordance with Table 1 (see a bottom right graph in Fig. 1). The energy and spin tune spreads are increased approximately 3 times. This reduces approximately twice the degree of polarization at the maximum.

In order to increase the equilibrium degree of radiation polarization in the wiggler mode, it is necessary to decrease the harmonic amplitudes of the two nearest integer spin

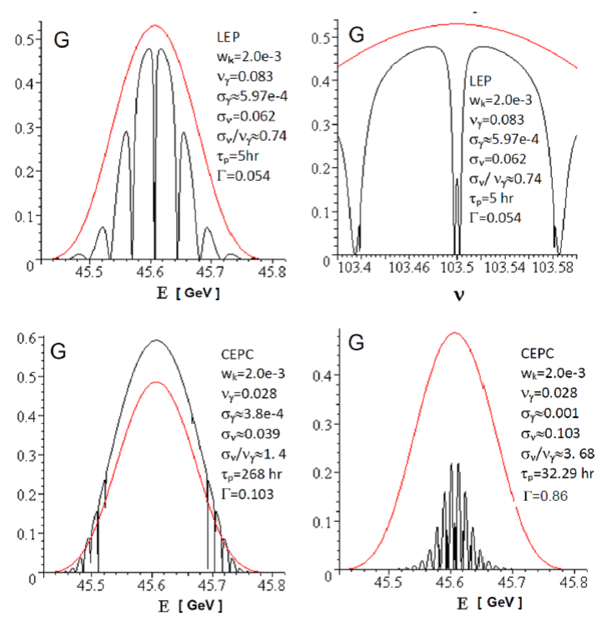

Figure 1: Polarization at LEP and CEPC at the same spin harmonic amplitude $|w_k| = 2 \cdot 10^{-3}$ vs. energy and spin tune. The red curves correspond to the case of no consideration of synchrotron modulation, see Eq. (16).

resonances (in this case, k = 103 and k = 104) due to a special correction of the vertical closed orbit. Estimates show that in the case of CEPC, one should reduce the harmonic to a level of $|w_k| = 10^{-3}$ or even less. It is also necessary to limit the field of wigglers, since the increase in the spin tune spread caused by them leads to a drop in the degree of polarization. As can be seen from Fig. 2, the field of wigglers 0.5 Tesla is preferable in comparison with the field of 0.6 Tesla, so it gives a 7-fold greater degree of polarization.

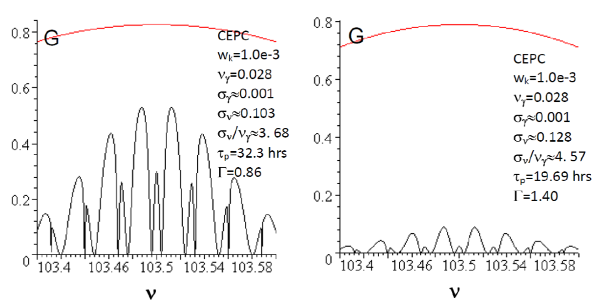

Figure 2: Polarization at 45 GeV CEPC at $|w_k| = 10^{-3}$ vs. the spin tune in two cases of the special wiggler parameters from Table 1: $B+ = 0.5$ T (left) and $B+ = 0.6$ T (right).

With increasing beam energy, the depolarizing effects of the guiding field imperfections intensify. At the threshold energy of the W pair production, in order to obtain polarization, an even more thorough correction of the spin harmonics associated with the distortions of the vertical closed orbit is needed. But, as will be shown below, the wigglers will not be needed to speed up the polarization process. The dependence of the polarization on the energy/spin tune in

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the region of the W-threshold is plotted in Fig. 3 for the harmonic amplitude $|w_k| = 5 \cdot 10^{-4}$.

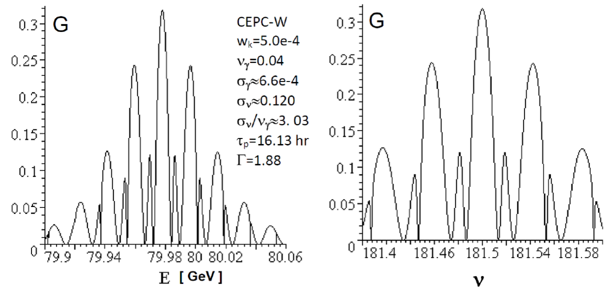

Figure 3: CEPC polarization near the W pair production threshold vs. the beam energy (left) and the spin tune (right).

SPIN RESPONSE TO MISALIGNMENTS

Spin Response Function

In electron-positron storage rings, as well as in proton rings with flat torsion-free orbit, the power of the depolarizing resonances associated with the action of transverse perturbations on the particle is determined by the so-called spin response function ($\nu >> 1$) [7]:

$$F^{\nu}(\theta) = \frac{\nu e^{i\nu\theta}}{2}\left[f_y \int_{-\infty}^{\theta} \overline{f_y'} K e^{-i\nu\phi} d\theta' - \overline{f_y} \int_{-\infty}^{\theta} f_y' K e^{-i\nu\phi} d\theta' \right], \quad (17)$$

where $f_y e^{-i\nu_y \vartheta}$ is the vertical Floquet function; ν_y is the vertical betatron tune; the bar means a complexial conjugation; $\phi = \int_0^{\vartheta} K d\vartheta$. This function has a period of $2\pi/m_p$ with m_p, the number of the magnetic structure super-periods (at CEPC, $m_p = 2$). It varies nonmonotonically with the beam energy and increases indefinitely in magnitude with approach to the resonances $\nu \pm \nu_y = k$.

Estimate of Resonance Harmonics

The amplitudes of integer spin resonance harmonics can be estimated using the calculated values of F^k ($\nu = k$!) at the azimuths of the field error location. For the random vertical offsets of the orbit in N_q quads with the spread δy:

$$|w_k^{(1)}|^2 = (\delta y)^2 \left(\frac{\nu}{2\pi}\right)^2 \sum_{i=1}^{N_q} \left(\frac{\partial H_y}{\partial x} \cdot \frac{l}{HR}\right)_i^2 |F_i^{\nu}|^2. \quad (18)$$

For the tilts of the N_b bending magnets around theis axis with the angular spread $\delta\chi$:

$$|w_k^{(2)}|^2 = (\delta\chi)^2 \left(\frac{\nu}{2\pi}\right)^2 \sum_{i=1}^{N_b} \left(\frac{H_y l}{HR}^2\right)_i |F_i^{\nu}|^2. \quad (19)$$

For the tilts of the N_q quads (η_x is the horizontal dispersion):

$$|w_k^{(3)}|^2 = (\delta\chi)^2 \left(\frac{\nu}{2\pi}\right)^2 \sum_{i=1}^{N_q} \left(\frac{\partial H_y}{\partial x} \cdot \frac{l\eta_x}{HR}\right)_i^2 |F_i^{\nu}|^2. \quad (20)$$

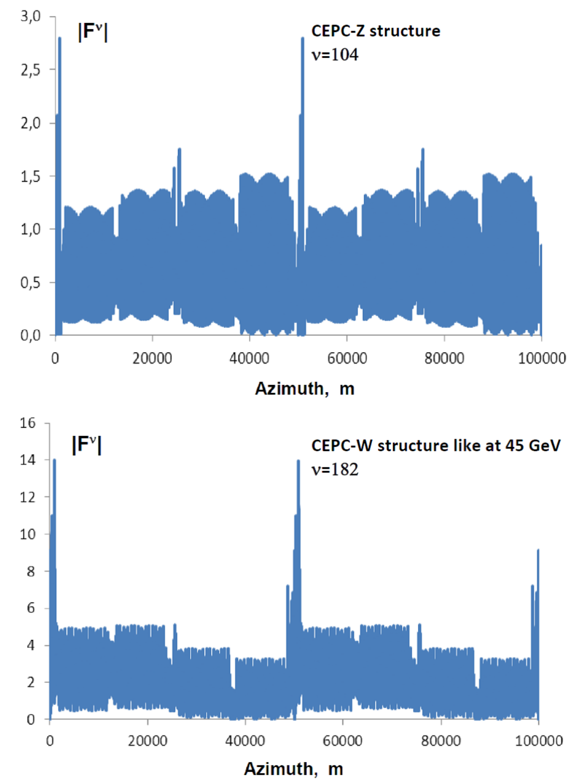

Figure 4: Spin response function for the integer resonance harmonics from the regions of Z-pole and W threshold.

In Eqs. (18-20), l_i is a length of the ith magnetic element, H_y is a vertical magnetic field, $HR =< B_0 > R$ is the machine magnetic rigidity. The generalized amplitude is

$$|w_k|^2 = |w_k^{(1)}|^2 + |w_k^{(2)}|^2 + |w_k^{(3)}|^2 \cdot (k-\nu)^2. \quad (21)$$

In Eq. (21), the factor $(k-\nu)^2$ takes into account the difference of the latter case from the first two in the power-law dependence of the depolarization effect on the resonance detuning. The beam energy in Z-pole peak corresponds to the detuning of approximately 0.5 from two nearest spin resonances with $k = 103$ and $k = 104$. Fig. 4 and Fig. 5 shows the azimuthal distributions of the spin reponse at the CEPC main ring calculated using Eq. (17) for these two values of the harmonic number [2]. In Table 3, the expected values of the resonance harmonic amplitudes, estimated from Eq. (18-21), are given for the typical magnitudes of the misalignments mentioned above. The resulting amplitudes are about 3 and 1.5 times larger than desirable one (10^{-3}), which indicates the need for correction of the harmonics.

At the energy of the W pair production threshold, $|F^{\nu}|$ is noticeably larger than that at Z pole energy (Fig. 4) This means higher requirements to the quality of correction for obtaining polarization at 80 GeV.

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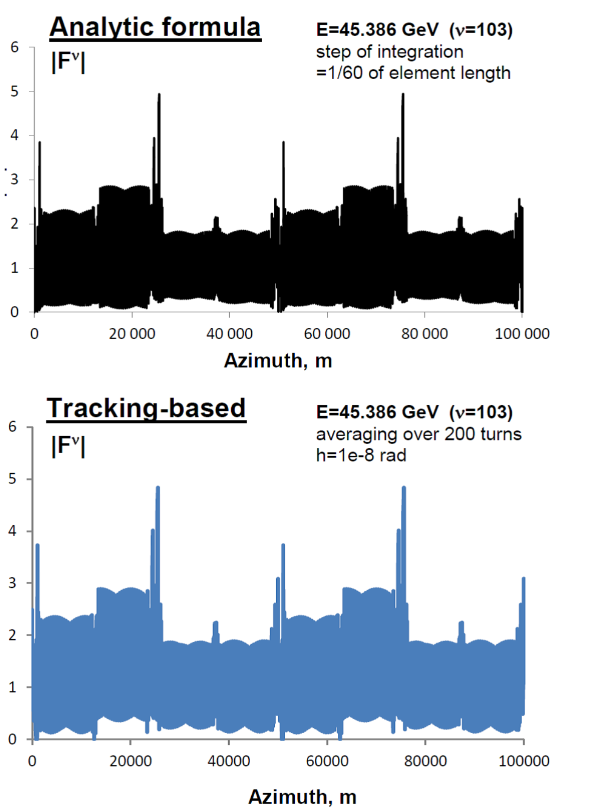

Figure 5: Comparison of the analytic-based and tracking-based calculations of F^ν.

Table 3: Spin integer resonance harmonic amplitudes at $N_q = 5368$, $N_b = 2458$, $\delta y = 50\mu$m, $\delta\chi = 3 \cdot 10^{-4}$

k	103	104
$\lvert w_k^{(1)}\rvert$	2.7×10^{-3}	1.2×10^{-3}
$\lvert w_k^{(2)}\rvert$	7.4×10^{-4}	4.1×10^{-4}
$\lvert w_k^{(3)}\rvert$	2.8×10^{-3}	2.9×10^{-3}
$\lvert w_k\rvert$	2.9×10^{-3}	1.5×10^{-3}

Comparison of Spin Response Calculations by Analytic- and Tracking-Based Methods

In view of the importance of these results, a calculation [2] of the spin response function was carried out using an alternative method [11] basing on the particle tracking simulation. In this method, the probe particle experiences a vertical kick $h << 1$ at an arbitrary azimuth θ_0, where the spin response is calculated, and begins to oscillate in the fields of a storage ring. The increment $\delta S_\perp^{(j)} = \delta S_x^{(j)} + i\delta S_z^{(j)}$ of the transverse component of the spin vector $\vec{S}$ ($|\vec{S}| = 1$) acquired to the j-th turn is found by multiplying the spinor matrices corresponding to the perturbed spin motion. In a conventional storage ring, this motion is described by the equations (the orth system $\vec{e}_x \times \vec{e}_z = \vec{e}_y$ is used):

$$\begin{aligned} d\vec{S}/d\vartheta &= \vec{W} \times \vec{S} \\ W_x &= (1+\nu)y'' \\ W_z &= (1+a)K'y + (a-\nu)Ky' \\ W_y &= \nu K - (1+\nu)x''. \end{aligned} \tag{22}$$

It can be shown that

$$F^\nu(\theta_0) = \left\langle -\frac{ie^{-i2\pi j}}{\nu h} \cdot \delta S_\perp^{(j)} \right\rangle - 1 \tag{23}$$

where averaging is done over a large number of turns. Comparison of the graphs in Fig. 5 demonstrates the practically complete agreement between the two methods.

Influence of Betatron Oscillations

Because of the very small emittance of the beam, the contribution of betatron oscillations to the kinetics of the radiation polarization is expected to be negligible. In particular, this is indicated by the calculation of the depolarizing factor for the case of random tilts of quadrupoles (Fig. 6). The calculation is based on the approach [10] in which $F^\nu(\vartheta)$

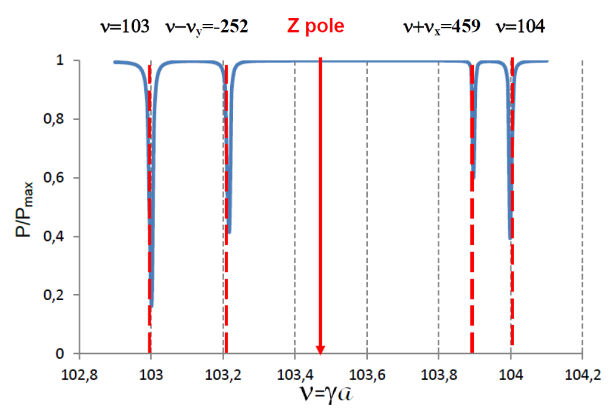

Figure 6: Depolarizing effect of the CEPC quadrupole tilts with the spread $\delta\chi = 6 \times 10^{-4}$ rad ($\nu_x/\nu_y = 355.1/355.21$).

is used. Three types of resonances appear in the linear approximation. Integer resonances are associated with harmonics whose amplitude $w_k^{(3)}$ is estimated like in Eq. (20). Resonances $\nu \pm \nu_x = k$ arise from the betatron coupling. Resonances $\nu \pm \nu_y = m_p k$ are owing to the feature of the spin responce function. Synchrotron oscillations are not taken into account, since the corresponding modulation resonances are even narrower than the main ones.

TIME TO REACH 10% POLARIZATION

The polarization increases in time according to the law $P(t) = GP_0[1 - \exp(-t/\tau_{rel})]$. Let us define the operation time we should spent to reach the certain polarization degree of $\eta = P(t_\eta)$ percentages in the conditions under consideration:

$$t_\eta = -\tau_{rel} \ln\left(1 - \frac{\eta}{92G}\right). \tag{24}$$

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Here, $\tau_{rel} = G\tau_p$ or $\tau_{rel} = G\tau_p^w$ depending on whether special wigglers are used or not. Based on the maximum values of factor G in Fig. 2 and Fig. 3, a table of parameters for obtaining polarization at Z-pole and W pair production threshold is compiled, including the time t_η, which in the considered cases considered ranges from 2 to 4 hours (see Table 4).

Table 4: Parameters of obtaining polarization at CEPC. * and ** indicate the cases $B_+ = 0.5$ T and $B_+ = 0.6$ T.

E GeV	$\|w_k\|$	G_{max}	ν_γ	τ_{rel} hr	η %	t_η hr
45.602	10^{-3}	0.53	0.028	17.1*	10	3.93
45.602	10^{-3}	0.09	0.028	1.8**	6	2.28
79.978	.0005	0.32	0.040	4.8	10	2.14

POLARIZATION SCENARIO AT Z- POLE

About 100 pilot electron/positron bunches of relatively small total current I_p are stored to be partially polarized up to 10% in 2.5 hrs using 10 shifter magnets with the field of 0.6 T. The SR power form each shifter magnet is 3 kW at $I_p = 2$ mA (of the order of 1% of the main train) When the polarization process ends the shifter magnets turn off. Then the main bunch train is stored. The pilot bunches are not in collision. Their lifetime is about 10^5 s due to scattering of particles on thermal radiation photons [12]. The polarized bunches are used one by another for the RD calibration of beam energy every 15 min. So, a single polarized bunch train is spent per day while taking data in detector occurs. A qualitatively similar scenario was proposed some time ago for the FCCee project.

ALTERNATIVE APPROACH

Acceleration of Polarized Beam in Booster

In [1] we drew attention to one more way to obtain the polarization at the CEPC collider. It seems reasonable to accelerate in the CEPC booster the electrons of $(60 \div 80)\%$ polarization degree coming from the 10 GeV linac with a photo-gun source, taking measures to preserve the polarization (Fig. 7). Obviously, in the case of positrons, this will require the creation of a 10 GeV damping ring. In general, preservation of the particle polarization during acceleration in the booster saves a time spent on the process of radiative polarization, and can also be decisive for obtaining the longitudinal polarization in the collider.

Without special measures, such an acceleration is possible if the total depolarization effect at fast crossing a system of spin resonances, estimated with the help of the Froissart-Stora formula, is small:

$$\varepsilon' = \frac{d\varepsilon}{d\vartheta} \gg \pi \sum_k |w_k|^2 \sim \pi N_{res} |w_{char}|^2. \qquad (25)$$

Here, ε' is a rate of change in the detuning $\varepsilon = \nu - \nu_k$ from the k-th spin resonance (integer or spin-betatron one) with the harmonic amplitude w_k. The particle energy in the booster rises from 10 to 45 GeV in 2 seconds. This corresponds to a rather high rate of change in the detuning: $\varepsilon' \approx 2 \cdot 10^{-3}$. With $N_{res} = 80$, the number of intersected

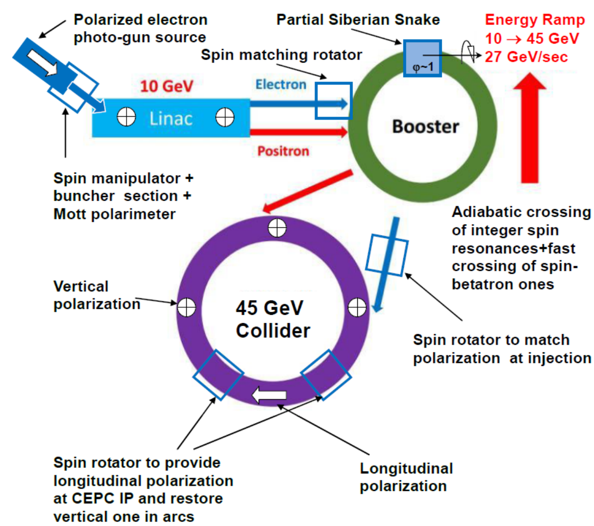

Figure 7: Sketch of obtaining longitudinal polarization using acceleration of polarized electrons in booster .

integer resonances in the mentioned range, we obtain from Eq. (25) an estimate for the admissible characteristic magnitude of the harmonics : $|w_{char}| \ll 0.003$. At the same time, the typical amplitude for integer spin resonances, due to magnet alignment errors, is of the order of at least 10^{-3}. The amplitudes for the intrinsic resonances $\nu_k = k \pm \nu_{x,y}$ are much smaller but the amount of such reonances is 4 times larger ($N_{res} \approx 320$).

In view of the unreliability of the maximally simplified approach, one should apply the well-known Partial Siberian Snake (PSS) method [4] to maintain the polarization in the booster. Let us consider an example with a helix snake [8,13] rotating the spin through an angle $\varphi = 0.4$ rad around the particle velocity. In this case, there will be an adiabatically slow intersection of the energy levels related to the integer resonances , since $\varphi^2 >> 4\pi^2\varepsilon'$. In the presence of PSS, the equilibrium polarization direction is not vertical and changes with the booster energy. By this reason, it is required to match the beam polarization at injection as well as at ejection using appropriate spin rotators in the beamlines. In the case of ejection, the rotator should have two modes - one for the transverse and the other for longitudinal polarization in the collider.

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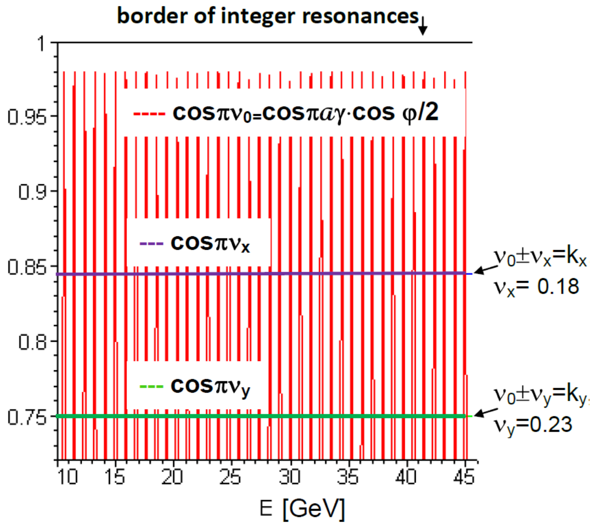

Figure 8: Diagram of the main spin resonance intersections as function of energy at $\varphi = 0.4$ rad.

The adiabatic mode mentioned above ensures to keep the particle polarization oriented along the equilibrium polarization direction during acceleration. The effective frequency of spin precession (ν_0) is determined from the equation $\cos\pi\nu_0 = \cos\pi a\gamma \cdot \cos\frac{\varphi}{2}$. This function, oscillating with energy during acceleration, is shown in Fig. 8. Its module does not equal 1 anywhere, which means that the spin tune averaged over the beam does not intersect any integer spin resonance. At the same time, this function curve intersects at certain points the horizontal lines of the intrinsic spin resonances $\nu_0 \pm \nu_{x,y} = k_{x,y}$. The rate of the effective detuning change at these points is

$$\varepsilon_0' = \varepsilon' \frac{\sqrt{\cos^2\varphi/2 - \cos^2\pi\nu_{x,y}}}{\sin\pi\nu_{x,y}} \approx \varepsilon'. \qquad (26)$$

Fast crossing these resonances will occur with a small loss of the polarization if the characteristic resonant harmonic amplitude of the corresponding perturbations satisfies the condition Eq. (25) at $N_{res} = 320$: $w_{char} \ll 10^{-3}$. From a practical point of view, this is easily done for beams with a small emittance and an appropriately compensated betatron coupling.

With $\varphi = 0.4$ rad, the helix snake can have the following parameters: transverse field=0.9 T, length=8.6 m, number of twists=4. The closed orbit makes the transverse excursions inside the snake and restores at exit. The maximum orbital deviations from the axis $Y_{max} = 5$ mm, $X_{max} = 3.7$ mm at 10 GeV and $Y_{max} = 1.1$ mm, $X_{max} = 0.8$ mm at 45 GeV. In cardinal case $\varphi = \pi$, absolutely all spin resonances are avoided. But this leads to an unacceptable increase of radiation losses. For instance, at the helix snake field of 1.78 T, its length of 8.6 m ($\varphi = \pi/2$) the losses due to the snake at 120 GeV are comparable with that from the booster main field. At 10 GeV, they exceed the nominal losses by 25 times. For comparison, the solenoid-based snakes do not increase radiation losses. But an advantage of helix snakes is compact arrangment. The required integral of the solenoid field in the full Siberian snake ($\varphi = \pi$) is 104 T·m at 10 GeV and 474 T·m at 45.6 GeV. At the same time, the necessary helix snake field integral does not depend on energy at φ =const.

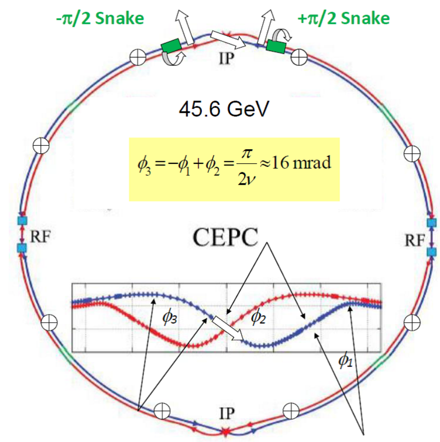

Figure 9: Kinematic scheme of longitudinal polarization.

On Longitudinal Polarization

In one of the ways [5] to organize longitudinal polarization, the equilibrium direction of polarization in the arcs remains vertical, and in a certain region evolves due to the use of spin rotators, taking the direction along the velocity at IP (Fig. 7). As rotators, for example, solenoids can be used in combination with magnets with a vertical field. The examples of a detailed calculation of such systems including the radiative kinetics of polarization are given in [14, 15].

In [1] attention is drawn to the fact that the S-shaped twist of the orbit in median plane in the CEPC section with IP (Fig. 9) can be a base to design a kinematic scheme of longitudinal polarization with minimization of the depolarization effect of quantum fluctuations. For this, two conditions must be met. First, the rotators should be distinguished by signs of rotation of the spin around the velocity vector by an angle of $\pi/2$. With a field of 8 T, the length of corresponding solenoid should be about 30 m at 45 GeV. Second, the angles of the orbit twist to the left and right of IP (between the solenoids) are equal in magnitude to $\pi/(2\nu)$ and opposite in sign (Fig. 9). As a result of this antisymmetry of the system of rotators as a whole, the main contributions to the spin-orbit coupling which are due to chromaticity of spin rotation in the solenoids as well as in the magnets between them cancel each other out [16]. This sharply reduces the depolarizing effect of quantum fluctuations in the arcs. In the top injection mode, the average in time degree of polarization of electrons in the collider will be close in magnitude to the degree of polarization of injected particles.

MORE ISSUES

There are two more issues that need to be studied. First of them concerns a depolarizing influence of SR in the inter-

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action area where beams intersect the 3T detector solenoid axis at a large angle (of about 15 mrad). Another is the resonance spin diffusion at a large spin tune spread in framework of a model basing on the radiative excitation and damping. In principle, the latter can be actual in the mode using the strong wigglers to speed up polarization. For particles from the distribution function tail, the amplitude of spin tune modulation by synchrotron oscillations can overlap the distance to a closest integer spin resonance.

CONCLUSION

- Depolarization effect of the different misalignments of the CEPC magnets has been estimated in comparision with the LEP case taking into account modulation of the spin precession frequency by synchrotron oscillations. With a large spin tune spread, this modulation significantly enhances the depolarization effect.
- The proposed parameters of the special wigglers in the number of 10 pieces to speed up polarization at 45 GeV are quite moderate.
- Sensitivity of spin motion to transverse field imperfections is determined through Spin Response Function. In order to self-test, this function has been calculated in two ways: in the analytic approach and by partical tracking simulation. The results are in full agreement.
- At Z-pole, the spin harmonic related to the sources of vertical closed orbit distortions should be corrected to the levels of $\lesssim 10^{-3}$. Similar problem was successfully solved in past at LEP. At the W pair production threshold energy, requirements for the spin harmonic matching are significantly tightened because of the increased spin response in the existing version of the magnetic structure.
- If the proper spin harmonic matching is done, it is possible to reach polarization in range $(6 \div 10)\%$ in time of $2 \div 4$ hours at 45 GeV CEPC, and 10% polarization in 2 hours at the $W^{\pm}$ threshold energy.
- Because of very small beam emittance, accounting for betatron oscillations in the radiative kinetics of spins leads to very narrow depolarization resonances. This is demonstrated by the calculation of the effect of random tilts of the CEPC quadrupoles.
- There is an alternative possibility of obtaining polarization by accelerating the polarized particles in the CEPC booster and then injecting them into the main ring. To ensure non-zero detuning from the integer resonances in the booster, we suggest to apply the 0.9 T 4-fold twist helix snake of about 9 m in length with a small angle of spin rotation around velocity. Intersection of the main spin-betatron resonances will occur in the fast crossing mode. This option saves time spent on the polarization process, and can also be crucial for obtaining longitudinal polarization.
- The design twist of the CEPC orbit at the IP section allows one to try one of the known methods of organizing longitudinal polarization with the restoration of vertical polarizatioon in the arcs while minimizing the depolarizing effect of the spin rotators.

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