ISBN: 978-3-95450-217-2 doi:10.18429/JACoW-ERL2019-TUCOXBS05

BEAM TIMING AND CAVITY PHASING

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Abstract

In a multi-pass Energy Recovery Linac (ERL), each cavity must regain all energy expended from beam acceleration during beam deceleration. The beam should also achieve specific energy targets during each loop that returns it to the linac. To satisfy the energy recovery and loop requirements, one must specify the phase and voltage of cavity fields, and one must control the beam flight times through the return loops. Adequate values for these parameters can be found by using a full scale numerical optimization program. If symmetry is imposed in beam time and energy during acceleration and deceleration, the number of parameters needed decreases, simplifying the optimization. As an example, symmetric models of the Cornell BNL ERL Test Accelerator (CBETA) are considered. Energy recovery results from recent CBETA single-turn tests are presented, as well as multi-turn solutions that satisfy CBETA optimization targets of loop energy and zero cavity loading.

INTRODUCTION

The Energy Recovery Linac (ERL) is designed to create high-quality, high-current beams at a lower energy cost than conventional linacs. Energy transferred to the beam during acceleration is later recovered by the system. In an ERL where the beam accelerates and decelerates through the same linac, full energy recovery is achieved when each radio-frequency (RF) cavity in the linac recovers the energy that it originally expended: a beam ideally causes zero net power load on the system. In multiturn ERLs, the beam enters the linac at different speeds during each accelerating or decelerating pass. As a result, the beam may experience phases slipped away from the ultrarelativistic case, and this phase slippage can result in incomplete energy recovery.

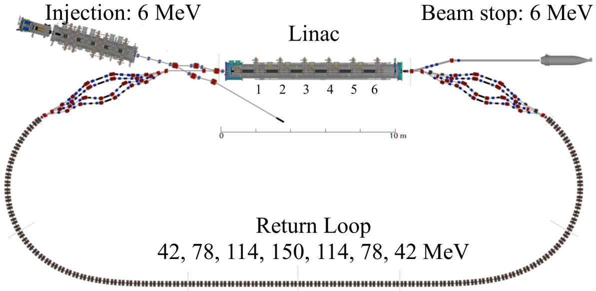

Figure 1: CBETA layout [1]. CBETA has a single linac with 6 RF cavities. The return loop has 4 independent beam paths shared by accelerating and decelerating beams of corresponding energy.

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The Cornell BNL ERL Test Accelerator (CBETA) is a single-linac ERL with four independent loops that return the beam to the linac (Fig. 1). A 6 MeV beam accelerates to 150 MeV over four passes of the main linac, where it may be used for experiments. The beam then decelerates to 6 MeV over four more passes, using the same set of loops to return to the linac as during acceleration. The intended beam will have a current of 40 mA [1]. In Summer 2019, CBETA was tested in a 1-turn configuration. A beam of under 0.1 μA accelerated from 6 MeV to 42 MeV, then decelerated back to 6 MeV. Better than 99.8% energy recovery was achieved in each cavity.

CBETA has not yet been tested in multiturn operation, where energy recovery may be more challenging to achieve due to the increased complexity of the system. If a 4-turn CBETA model is simulated with RF phase and loop length settings that give energy recovery when $v = c$ everywhere, then a 40 mA beam of expected non-ultrarelativistic speed incurs up to 46 kW power load in a single cavity. However, the CBETA cavities only have 2 kW power allotted for beam acceleration; assuming a beam speed of $v = c$ would result in unfeasible power consumption.

Optimization of RF phase and loop timing is needed to reduce the simulated beam load during multi-turn operation. Direct optimization would require a large system of variables and constraints, but the system size can be greatly reduced if RF phases are chosen for a symmetric accelerating and decelerating energy configuration. The ERL symmetry strategy presented here is further discussed in [2] and [3].

OPTIMIZATION SYSTEM

Suppose a single-linac ERL with shared accelerating and decelerating return loops (*e.g.* CBETA) has M linac passes and N cavities. For CBETA, $M = 8$ and $N = 6$. The optimization system must have N constraints to minimize each cavity load. An additional ($M - 1 = 7$) constraints are needed to ensure that the beam has the correct energy during return loops, such that the shared loops can direct both accelerating and decelerating beams identically, and the central loop can achieve the correct maximum energy for experiments. To achieve these goals, one can vary the length of the $\frac{M}{2} = 4$ independent return loops, or the RF phase and voltage of the N cavities. This optimization system will have a total of ($N + M - 1 = 13$) constraints and ($2N + \frac{M}{2} = 16$) possible variables.

If the ERL is made symmetric, then the accelerating time-energy profile of a single-particle ideal beam is experienced in exactly reverse order during deceleration. This mirrored energy profile causes the load on each pair of cavities equidistant from the linac center, *e.g.* the first and last, to be correlated: only $\frac{N}{2} = 3$ load constraints are needed. The symmet-

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ric energy also causes identical beam energy during each pair of return loops, *e.g.* the first and last; only 1 energy constraint is needed for the central loop, as the maximum energy must be accurate for experiments. The phase and voltage variables are halved by the cavity pairing effect, but the return loops remain free to vary. The symmetric ERL yields a total of ($\frac{N}{2}+1=4$) constraints and ($N+1=7$) variables.

SYMMETRIC ERL CONDITIONS

Consider a small ERL with $N=1$ cavity of length L, and $M=2$ linac passes. The accelerating first encounter is designated A, and the decelerating second encounter is D. Suppose the cavity has an electric field $\mathcal{E}_A$ with spatial symmetry about the center,

$$\mathcal{E}_A(s,t) = \mathcal{E}_{A0}(s)\sin\left(\omega(t-t_{\text{in,A}})+\phi_{\text{in},A}\right), \tag{1}$$

where $\phi_{\text{in},A}$ is the RF phase when the beam enters the cavity, ω is the RF frequency, s is the spatial coordinate, t is time, and $t_{\text{in},A}$ is the time when the beam enters A. For a deceleration profile that reverses the beam energy increase of acceleration, encounter D must have a field satisfying,

$$\mathcal{E}_D(L-s, t_{\text{total}}-t) = \mathcal{E}_A(s,t), \tag{2}$$

where t_{total} is the time between entering A and exiting D. This is only satisfied when the input phase of D follows the condition,

$$\begin{aligned} \phi_{\text{in},D} &= -\phi_{\text{in},A} - \omega T_A = -\phi_{\text{out},A} && \text{[odd]} \\ \phi_{\text{in},D} &= \pi - \phi_{\text{in},A} - \omega T_A = \pi - \phi_{\text{out},A}. && \text{[even]} \end{aligned} \tag{3}$$

where T_A is the time between entering and exiting A. The equations for cavities with even or odd cells differ by π due to the symmetry or antisymmetry of the spatial electric fields.

In a larger ERL, let n and m be cavity or pass indices, where $1 \le n \le N$ and $1 \le m \le M$. Each accelerating cavity of encounter (m,n) has a decelerating pair $(M-m, N-n+1) = (m', n')$ that delivers an equal, opposite energy change to the beam. Since the pair could be chosen from any pass pair m and m', it is useful to write RF phases ϕ_0 at time of injection $t=0$,

$$\phi_{0,n} = \phi_{\text{in},mn} - \omega t_{\text{in},mn} = \phi_{\text{out},mn} - \omega t_{\text{out},mn}. \tag{4}$$

Hence Eq. (3) for a multiturn ERL becomes,

$$\begin{aligned} \phi_{0,n} &= -\phi_{0,n'} - \omega t_{\text{total}} && \text{[odd]} \\ \phi_{0,n} &= \pi - \phi_{0,n'} - \omega t_{\text{total}}. && \text{[even]} \end{aligned} \tag{5}$$

When these phase conditions are applied to all $\frac{N}{2}=3$ cavity pairs, symmetric electric fields should occur during beam-cavity encounters. However, the selected value of t_{total} must first match the total time over which the beam traverses the ERL. The return loop that switches from acceleration to deceleration must have the time of flight,

$$t_{\text{loop},\frac{M}{2}} = t_{\text{total}} - 2t_{\text{out},\frac{M}{2}N}, \tag{6}$$

If Eq. (5) and Eq. (6) are satisfied, the ERL will have symmetric time-energy profiles.

Table 1: Objectives after Optimization of Symmetric CBETA Models. Ranges indicate the closest and furthest values from targets across all model solutions. Loads assume a 40 mA beam current. Note, beam energies in loops 1-3 are not used as optimization parameters.

ERL Output (Objective)	Optimized Value	Target
Power load	28 pW - 32 μW	0 W
Peak energy offset	37 μeV - 0.9 eV	0 eV
Loop 1 (MeV)	42.00 - 42.18	42.00
Loop 2 (MeV)	78.01 - 78.20	78.00
Loop 3 (MeV)	114.00 - 114.23	114.00
Loop 4 (MeV)	150.00	150.00

SYMMETRIC CBETA MODELS

Optimization of energy recovery in a symmetric system is tested in models of CBETA. For the purpose of speed, Newton's Method and Levenberg-Marquardt optimization algorithms are used in the *Mathematica* or *Bmad* softwares. Since RF field modeling can be computationally intensive, the CBETA optimizations use simulated cavities of varying complexity.

(i) Thin Lens (TL) cavities. The simulated cavity delivers an instantaneous delta-function energy kick to a particle passing the center of the physical cavity location. This simulation optimizes the most quickly, although it is also the least physical model.

(ii) Ultrarelativistic (UR) cavities. Inside the cavity region, the particle experiences a time of flight and energy change consistent with the $v=c$ case, regardless of actual energy.

(iii) Finite Time-tracked (FT) cavities. Inside the cavity region, time evolution is estimated by an average velocity, and energy is delivered based on this time of flight.

(iv) Runge Kutta (RK) cavities. The particle is integrated through a grid of time-varying field intensities. This is the most realistic model, but it also optimizes the most slowly.

Energy recovery and load values are optimized to within machine precision for all models, although the RK model is slightly farther from target due to the higher numerical noise of its optimization. Solution ranges are located in Table 1, where a 40 mA beam is assumed for load calculations. Optimization of the symmetric CBETA models indeed converges upon phase and loop length settings that minimize load and achieve the correct maximum energy.

LONGITUDINAL BEAM TILT

The conditions from Eq. (5) and Eq. (6) indeed result in an energy-symmetric system for a beam with phase space coordinates of the ideal particle. However, for a beam with nonzero spread in time and energy, the injected and final distributions do not display the same energy symmetry as the ideal particle experiences (Fig. 2, Left). If extension of ERL symmetry to non-ideal beams is desired, the input

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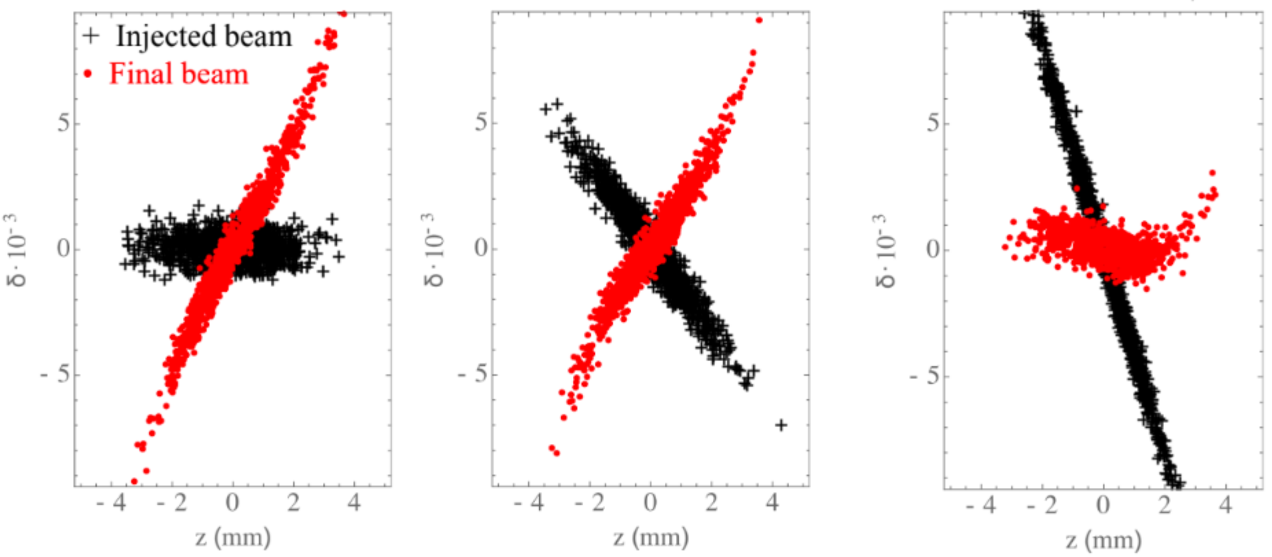

Figure 2: Longitudinal Phase Space at Injection and Beam Stop. Left: an injected Gaussian distribution ends with a positive slope. Center: a Gaussian superposed with a linear injection tilt ends with an equal energy spread. Right: a Gaussian superposed with a large linear injection tilt gives a minimized energy spread.

distribution can be tilted in longitudinal phase space using off-crest injector phases; the appropriate injected beam will result in visually symmetric input and output distributions with equal energy spreads (Fig. 2, Center).

In some ERL systems, it may be preferred to minimize the energy spread at beam stop (beam dump). This may be the case if, for instance, beam stop has a narrow range of energy acceptance. In this case, an even more extreme tilt can minimize the energy spread of the final distribution (Fig. 2). Appropriate tilt patterns can be identified by scanning through a range of simulated injector phases, or by calculating the transport matrix for the full ERL; further details on the analytical calculation of tilt angle can be found in [2].

CONCLUSION

Phase slippage poses one major challenge to achieving energy recovery in multiturn ERLs. This effect can be reduced by optimizing the cavity RF phases, voltages, and loop lengths to minimize power load on each cavity. However, this results in a large and computationally intensive optimization system. The method of ERL symmetry reduces the size of the required optimization system by guaranteeing identical time and energy steps during acceleration and deceleration. In this way, the power loads of cavity pairs are correlated, reducing the total number of independent optimization constraints. By applying ERL symmetry to models of CBETA, phase and loop length settings have been identified to minimize power load and achieve the correct maximum beam energy. Symmetry optimization and beam tilting appear effective in CBETA simulations, but these strategies can also be extended to the optimization of other single-linac, shared return loop ERL systems with N cavities and M passes.

ACKNOWLEDGEMENTS

We thank Dr. David Sagan for assistance with the Bmad simulation software. We thank Dr. Christopher Mayes for assistance with the RF cavity fields. This work was supported by Brookhaven National Labs and the New York State Energy Research and Development Agency (NYSERDA).

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