NON LINEAR PULSE EVOLUTION IN SEEDED AND CASCADED FELS

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Abstract

The dynamics of a free-electron laser with short radiation pulses having intensities higher than the steady-state saturation threshold is analysed. This regime, characterized by a superradiant scaling of the power with the e-beam current, may be induced by seeding an FEL amplifier with a high power laser, and is dominated by the interplay between slippage and the non-linear effects associated to saturation. The propagation of superradiant pulses in single pass free electron lasers has been extensively studied with 1D time dependent simulation codes. The superradiant regime is observed also in a three dimensional model and presents a peculiar behaviour in the fact that the radiation transverse size does not blow up when the FEL saturates as it happens in the steady-state regime. The mode is quasi-guided keeping very high the intensity and allowing the non linear effects to dominate the propagation of the wave. In this communication we analyze these effects in the framework of a superradiant free electron laser experiment proposed for the SPARC facility.

INTRODUCTION

When a FEL amplifier is seeded with a short and intense pulse, the coupled system of particles and fields can enter a regime where the radiation evolution is simultaneously dominated by the non-linear effects associated to saturation and by time-dependent effects associated to slippage. This regime is characterized by the propagation of a solitary wave-like pulse with peak power scaling as N^2, where N is the number of electrons, typical of superradiance phenomena [1]. This regime has been observed for the first time in simulation results of the FEL equations in [2], where the effects of the propagation of the radiation have been taken into account. It was pointed out the difference between the observed behaviour where the electrons and the radiation are *self-organizing* in a state in which the radiation power scales like N^2 and the classical superradiance "a la Dicke", spontaneous emission from a coherently prepared system (i.e. prebunched electron beam). In [3], Bonifacio et al. studied the case of a single pass FEL amplifier seeded with a square seed pulse shorter than the e-beam. At the trailing and leading edge of the pulse a higher intensity spike with the typical characteristics of superradiance was observed. Since short and intense laser pulses have become experimentally available as seeds for FEL amplifiers, the situation in which a seed pulse shorter than the e-beam is injected into an high-gain FEL amplifier became recently the subject of a renovate interest [4].

The dynamics of a short pulse injected in a FEL amplifier is summarized by the contour plot shown in Figure 1. The picture represents the normalized laser intensity as a function of the coordinate ζ along the electron bunch as it evolves along the undulator coordinate z (vertical axis). The electron beam current is assumed uniform in the simulation window. Note that the laser intensity actually grows as the pulse evolves along the undulator, an effect which is not shown in this graph because of the normalization at each z. The slope of the trace is an indication of the velocity of propagation of the pulse along the electron bunch.

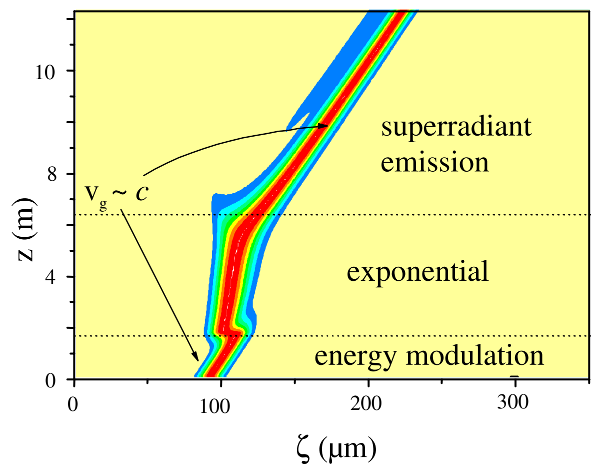

Figure 1: Evolution of a seed pulse shorter than the e-beam in a single pass FEL amplifier vs. the position along the undulator z and vs. the microbunch coordinate ζ. Different colours represent different intensities.

We may distinguish three different regions where the dynamics of the FEL is substantially different. In the first region the pulse velocity corresponds to the velocity of light c. In this region the seed pulse is modulating in energy the electrons. In the second region the energy modulation is converted into a density modulation and the exponential gain regime takes place. In the exponential gain regime the pulse length grows and the pulse velocity becomes significantly slower than c. The exponential regime stops at saturation and a different dynamical regime occurs. In this regime the group velocity of the pulse is again approximately equal to c, and the typical pulse power profile in the e-beam frame is characterized by a self-similar shape shown in Figure 2.

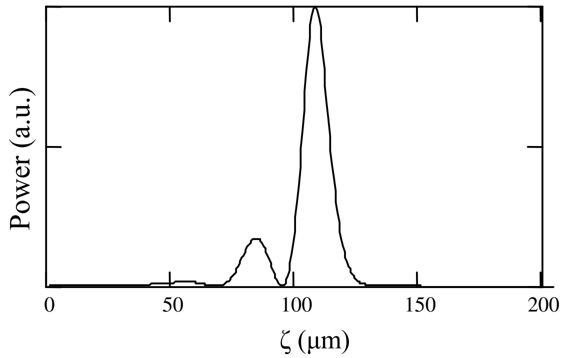

Figure 2: Superradiant pulse power as a function of the longitudinal coordinate along the electron bunch ζ

An approximated analytical expression of the power dependence as a function the longitudinal coordinate along the e-bunch ζ, is given in [5]. The connection between the particles dynamics in phase space at saturation and the slippage phenomena leading to this stable non linear superradiant excitation has been investigated in [4] where it has been pointed out that a simple relation links the radiation pulse length and the period of the synchrotron motion of the trapped particles in phase space. The relation may be expressed as

$$\tau \approx \frac{c\tau_s}{2}\frac{\lambda}{\lambda_u}, \tag{1}$$

where τ is the FEL pulse length, λ is the optical wavelength, λ_u is the undulator period and τ_s is the synchrotron oscillation period. The latter can be expressed as a function of the system parameters as

$$\tau_s = \frac{2\pi}{\omega_s} = \frac{\lambda_u}{c}\frac{\sqrt{1+K^2/2}}{\sqrt{2\cdot JJ\cdot K\cdot K_r}}, \tag{2}$$

where K is the normalized undulator strength and the planar undulator Bessel function factor is given by $JJ(\xi) = J_1(\xi) - J_1(\xi)$, with $\xi = K^2/4(1+K^2/2)$. The parameter $K_r = eE/m_e c^2 k \propto P^{1/2}$ represents the dimensionless normalized vector potential associated with the radiation wave.

The link between the pulse length and the particles synchrotron motion may be better understood looking at the particle phase space evolution along the superradiant pulse, as it is shown in Figure 3. In Figure 3(a) the typical phase space of the beam in its initial state is shown, with a small energy spread and negligible energy modulation. The beam is in this state before the arrival of the wave. In Figure 3(b) a large fraction of the current is trapped in the separatrix and strong energy modulation appears. In Figure 3(c) corresponding to the position along the bunch at the peak of the wave, the phase space density modulation is maximized. The trapped electrons after this position begin to gain energy from the wave, causing a reduction of the energy in the radiation field. In Figure 3(d), the particles gaining energy have reduced the amplitude of the separatrix and are de-trapped with a final energy lower than the initial one. The energy of the wave has grown by a quantity proportional to the product between the energy lost by each electron, proportional to the depth of the separatrix, and the number of particles interacting with the wave, linear with the position of the superradiant pulse along the undulator.

From these simple considerations it is possible to deduce simple scaling relations for the evolution of the wave. The pulse length is indeed related to the pulse peak power by equations (1) and (2)

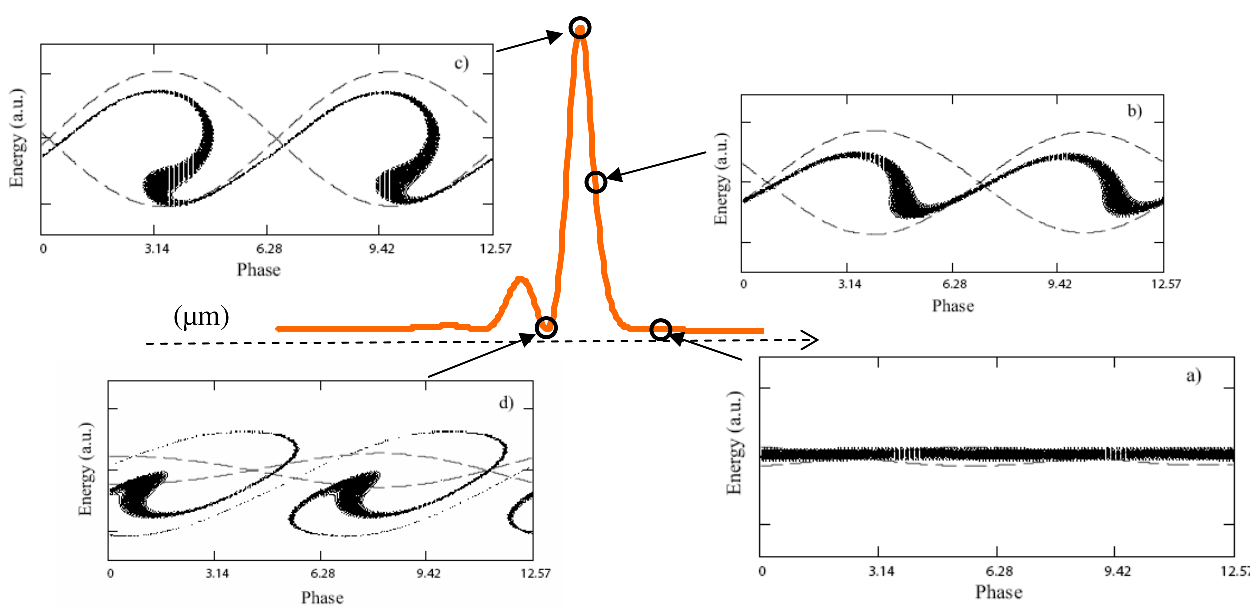

Figure 3: Phase space evolution along the optical pulse

$$\tau \propto P^{-1/4}, \quad (3)$$

For the reasons explained above the energy scales as

$$E \propto zP^{1/2}, \quad (4)$$

which, compared with the relation $E \propto \tau P \propto P^{3/4}$ allows us to write

$$\begin{aligned} P &\propto z^2 \\ E &\propto z^{3/2} \\ \tau &\propto z^{1/2} \end{aligned} \quad (5)$$

The scaling relations (5) have been extensively verified with 1D time dependent simulation codes. In Figure 4 the behaviour of the power vs. z has been calculated by solving the Maxwell-Lorentz equations describing the FEL dynamics with the 1D code Perseo [6]. A fitting function scaling with z^2 is compared to the numerical data. Similar comparisons have been represented in figs. 5 and 6 where the pulse energy and the pulse length are respectively represented.

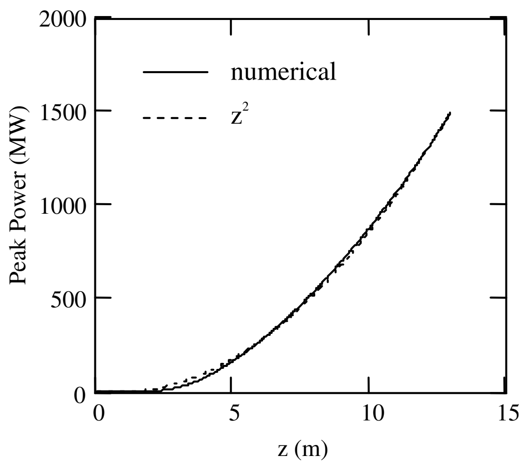

Figure 4: Peak power vs. longitudinal coordinate along the undulator. The dashed line is a fitting function scaling as z^2.

The superradiant regime is observed also in a three dimensional model and presents a peculiar behaviour in the fact that the radiation transverse size does not blow up when the FEL saturates as it happens in the steady-state regime. This condition of a quasi-guided transverse mode allows to preserve a high intensity so that the condition (1) is fulfilled during the evolution. In the next section we will analyse a single pass FEL configuration which is affected by diffraction effects in order to analyse the impact of diffraction on the condition (1).

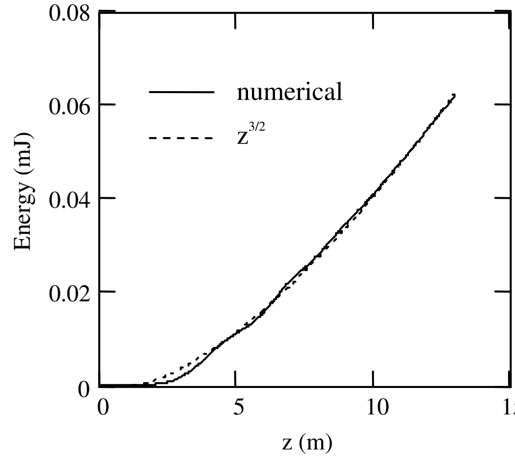

Figure 5: Pulse energy vs. longitudinal coordinate along the undulator. The dashed line is a fitting function proportional to $z^{3/2}$.

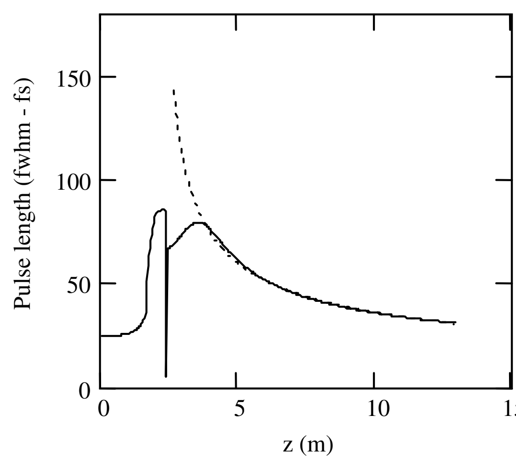

Figure 6: Pulse length vs. longitudinal coordinate along the undulator. The dashed line is a fitting function proportional to $z^{-1/2}$.

THREE DIMENSIONAL EFFECTS

It is well known that in the classical steady-state exponential regime, because of the bunching and the strong gain, the beam acts effectively as a medium with an index of refraction greater than unity. This is the basis for the optical guiding effect [7,8] for which stable modes with transverse profile independent of the longitudinal coordinate can propagate in the FEL. This mechanism is interrupted at saturation when there is no more gain to compensate the natural free space propagation behaviour and the radiation transverse size starts to increase due to diffraction. Because of these considerations, it is not obvious that the stability of the superradiant solution in an FEL model with three dimensional effects taken into account would be preserved.

To understand the dynamics of the superradiant pulse in the transverse dimensions, we performed the simulation with the three dimensional code GENESIS 1.3[9].

The parameters of the simulation are reported in Table 1. The undulator is over 40 3D gain lengths long, and the seed power is such that after few meters the steady-state saturation power level is reached. In this case, the pulse propagates in the superradiant regime for over 15 m.

Table 1: Parameters of three dimensional simulation.

Input seed power	1 MW
Input seed λ	800 nm
Input pulse length (FWHM)	150 fs
Undulator period	3.9 cm
Number of periods	512
Undulator K	1.125
Beam energy	100 MeV
Beam current	350 Amp
Emittance	3 mm-mrad

In Figure 7 the behaviour of the power (a) and of the beam size (b) in the superradiant regime and in steady state regime are compared. The optical beam size is guided by the e-beam when the gain is exponential in both cases. At saturation, in the steady state regime the radiation size expands approximately as it would do in free space by natural diffraction. In the superradiant regime there is no exponential gain but the radiation is still absorbing energy from the beam. As it was pointed out in [8], the guiding effect can occur even if there is negligible growth in light intensity. This keeps the transverse dimensions of the superradiant pulse small compensating the diffraction effects and preserves the interplay between the non-linear effects and the slippage that is at the basis of the superradiant spike propagation.

In Figure 8 it is shown the r.m.s. beam size as a function of the longitudinal coordinate in the e-bunch frame. The power profile is also reported for reference. At the front of the pulse the beam size is large since in this region the radiation is mostly spontaneous emission which is emitted at large angles. At the peak of the pulse the beam size is minimum, showing the quasi-guided condition. Note that in the simulations for this case the wavelength of the seed is at 800 nm, and the electron beam size is 150 μm. In other words, the wavelength is long and the diffraction effects are relatively strong. At shorter wavelengths the role played by diffraction is less significant and the beam size even in the superradiant regime can remain smaller for a longer distance.

Even though once we reach the saturation power level, the mode size does not grow with the same rate of the steady-state case, in the absence of strong gain-guiding, it does get larger. The consequence of this fact is that the scaling laws are modified in 3D when diffraction effects are present. In Figure 9 a comparison between the pulse length and the peak on axis intensity is shown. The 1D simulation (black line) and the 3D simulations (red line) show a difference in the saturation position. The main reason for this, is the different coupling between the seed pulse and the electron beam at the beginning of the undulator. The seed is indeed assumed as "perfectly matched" in the 1D simulation while a convolution factor appears in the 3D case where the seed pulse is affected by natural diffraction before the exponential gain-guiding regime takes place.

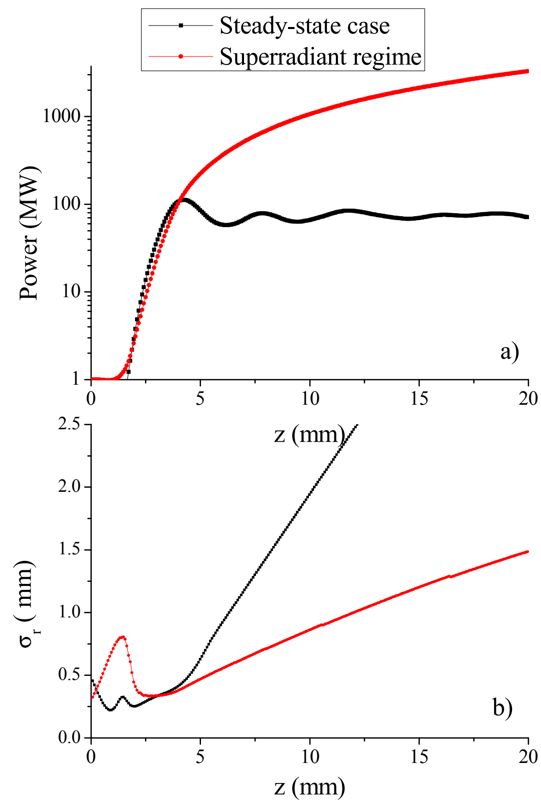

Figure 7: Peak power (a) and beam size (b) in the superradiant (red) and steady-state (black) cases.

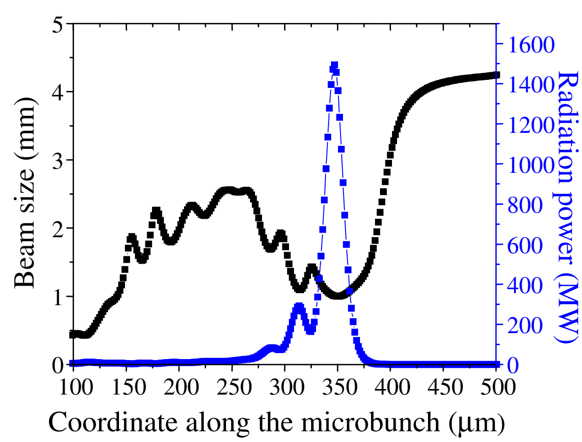

Figure 8: Beamsize along the superradiant pulse. The power profile is shown on the same scale for reference.

A significant difference between the two simulations appears in the behaviour of the on axis intensity at large *z*, where a different scaling behaviour with the longitudinal coordinate is observed. In the three dimensional simulation the peak power grows slower than z^2 and the pulse length does decrease slower than $z^{-1/2}$ as a consequence of the fact that the mode size is getting larger. The on axis

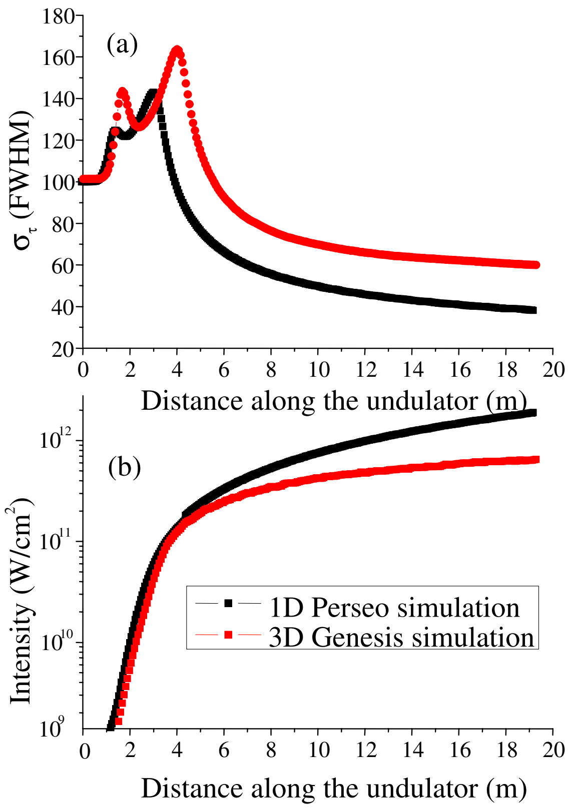

Figure 9: Pulse length (a) and on axis peak intensity (b) vs. the longitudinal coordinate along the undulator. The data corresponding to 1D simulations (Perseo) are in black, the 3D simulations (GENESIS) are indicated in red.

intensity is however a monotonic growing function and for this reason the superradiant state is preserved and is stable also in the three dimensional case.

It must be also observed that in practical applications, as the example of the cascaded FEL configuration analysed in [4] it is not necessary to propagate the pulse for a very long distance in saturated conditions. A single radiation pulse, propagating in this non linear slippage-dominated regime, with a peak power higher than the saturation value, is indeed able to propagate with a self similar shape in a FEL cascade, up-converting itself in frequency while it passes from a cascade stage to another. In the next section an example of a test experiment of the superradiant cascade that may be implemented at the R&D SPARC facility is presented.

SUPERRADIANT CASCADE EXPERIMENTAL TEST AT SPARC

SPARC is a dedicated R&D facility for research in FELs and high brightness electron beams [10]. The electron source is a photoinjector driven by a Ti:Sa laser system. Using a fraction of the optical power available from this laser system, it is relatively easy to obtain a short and intense laser pulse synchronized with the electron beam that can be used as a seed for the FEL amplifier [11]. Moreover, because of the adjustability of the gap between the poles, the six undulator sections can be tuned to resonate at different wavelengths. These considerations make the SPARC facility a valid candidate to test and study the short pulse superradiant regime in the FEL cascaded configuration.

In particular, with a 175 MeV e-beam provided by the SPARC linac, the first two sections can be tuned to resonate at λ_r=400 nm, the second harmonic of Ti:Sa where a suitable seed is available by non linear second harmonic generation in crystals.

The gap is then increased for the remaining four sections to reduce the undulator K and bring the resonance wavelength down to λ_r=200 nm. A small adjustment of the quadrupoles between the sections is sufficient to ensure the control of the transverse electron beam size. A limited transverse mismatch due to the different undulator focusing induces betatron oscillations with a beam size always remaining smaller than 150 μm.

Seeding with a short (100 fs FWHM) and intense (> 1 MW) pulse at 400 nm , we can enter the non linear regime of propagation already at the end of the first stage and then observe how the superradiant spike propagates itself at the shorter wavelength.

We studied the feasibility of this experiment, making simulations with the code GENESIS 1.3 using the parameters reported in Table 2.

Table 2: Parameters of the SPARC superradiant experiment.

Input seed power	1 MW
Input seed λ	400 nm
Input pulse length (FWHM)	100 fs
E-beam energy	177 MeV
E-beam current	110 Amp
Emittance	1 mm-mrad

The results of this simulation are reported in Figure 10 where we show the power along the electron bunch after the first stage (a) and after the second stage (b). The power in the seed is sufficient to induce the formation of the self-similar solitary wave-like superradiant pulse right at the end of the first stage. In Figure 11 the contour plot of the spike in the second undulator (notation as in Figure 1) is shown.

At the beginning, a very short radiation pulse is formed in correspondence of the spike of the second harmonic bunching coefficient created by the superradiant evolution in the first stage. The radiation pulse at 200 nm at first increases its length due to the finite FEL bandwidth. Once it reaches intensity comparable with the steady-state saturation value, the system re-enters the superradiant regime and the pulse-shortening starts again. It can be observed that the pulse velocity is always approximately *c* and that the pulse never enter in the exponential gain regime, passing from coherent harmonic emission directly to superradiance.

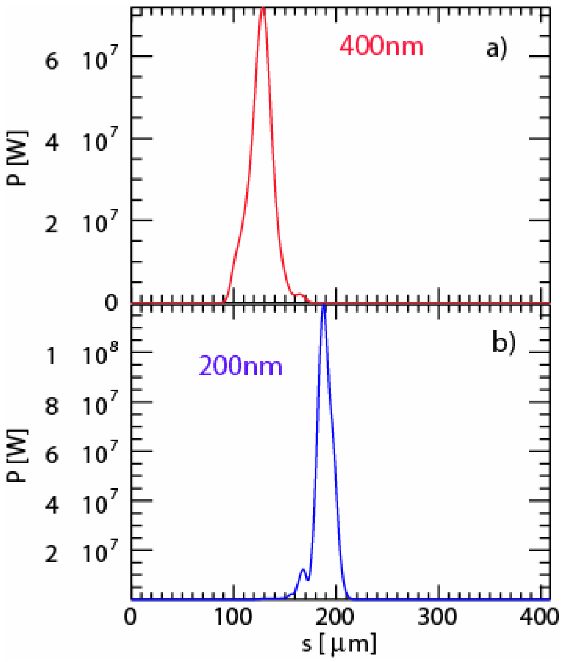

Figure 10: Radiation pulse at the end of the first (a) and the second (b) stage in the superradiant cascade test at SPARC.

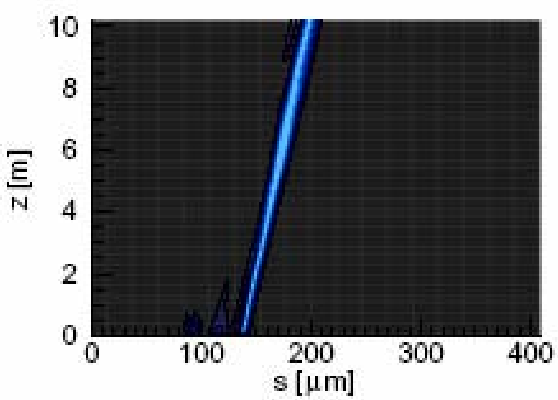

Figure 11: History of evolution of radiation pulse in the second stage of the SPARC superradiant experiment.

CONCLUSIONS

We have reviewed the dynamical properties of the nonlinear pulse propagation in saturated conditions in FEL amplifiers. We have re-derived the main scaling laws governing the pulse dynamics according to the procedure shown in [5]. The scaling laws have been tested in a GENESIS 1.3 simulation, based on a three spatial dimensions model. In this case diffraction effects were not negligible and a deviation from the 1D simulation behaviour of the pulse length and the peak intensity has been observed.

We have finally proposed an experiment aimed at testing the idea of the superradiant cascade at SPARC with the harmonic conversion from 400nm to 200 nm.

Once confirmed by experiments, the concept of the superradiant cascade, which has many interesting properties ranging from its simplicity to the reduced sensitivity to perturbations, will have an impact in the design of future FEL facilities aimed at reaching ultra-short pulses at short wavelengths.

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