STABILIZATION OF THE ELETTRA STORAGE-RING FREE-ELECTRON LASER THROUGH A DELAYED FEEDBACK CONTROL METHOD

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Abstract

We numerically investigate the effect of a delayed control method on the stabilization of the dynamics of the Elettra storage-ring free-electron laser in Trieste (Italy). Simulations give evidence of a significant reduction of the typical large oscillations of the laser intensity. Results are compared with numerical data obtained with a derivative feedback. The possibility of an experimental implementation of the proposed method is also discussed.

INTRODUCTION

Since its discovery, lasers receive a great attention as sources of coherent radiation. Nowadays, the possibility to extend the spectral range of the laser emission towards shorter wavelengths opens new scenarios for research and applications. In this aspect, free-electron lasers (FELs) play a crucial role for their complete tunability in the range between the far infrared to the soft x-rays region [1]. FEL systems, which exploit the radiation emitted by relativistic electrons when passing in a periodic magnetic field generated by an undulator, may rely on single-pass or oscillator configurations [1]. In the first case, the light amplification is obtained during one single pass of the electron beam through the undulator structure. In the second case, the light emitted by the electron beam is stored in an optical cavity and the laser effect is reached by means of successive light-electron interactions. Among oscillators, storage-ring free-electron lasers (SRFELs) present the most intricate dynamics: the electron beam is not renewed (as in the case of linac-based oscillators) and keeps track of previous interactions.

In this paper we focus on the dynamics of the SRFEL which is currently operational at Elettra [2]. The necessity of disposing of a laser cavity with good reflecting mirrors imposes a lower limit to the possible emission wavelength; up to now laser emission has been demonstrated down to $175\ nm$ [3], which is the shortest wavelength ever reached using an oscillator FEL.

Presently, an important limitation to the use of such a system for scientific applications is the rather poor quality of the laser temporal stability. The laser intensity of a SR-FEL is characterized by a sequence of micropulses having a duration of the order of ten picoseconds and separated by intervals of about one microsecond. This feature is associated to the impulsive character of the laser medium (i.e. the electron bunch). Moreover, the evolution of the envelope of such a micropulses on a millisecond temporal scale is strongly related to the temporal overlapping between the photons in the laser cavity and the electron beam circulating into the ring at each pass thorough the interaction region [4, 5]. More precisely, the envelope attaines a “cw” regime for a perfect electron-photon tuning. Similarly to the case of a synchronously pumped mode locked system [6], the laser-electron detuning is at the origin of instabilities which may manifest in periodic/a-periodic oscillations [7] or even induce a chaotic behaviour [8]. The Elettra SR-FEL shows a very high sensitivity to different kind of instabilities which may perturb the electron-beam (and, thus, the laser) dynamics [9]. As a result, the “cw” behavior is generally not observed even when the system is close to the perfect tuning condition. In this paper, we illustrate a possible approach for the stabilization of the laser envelop based on a delay feedback method [10, 11].

The paper is organized as follows. We first introduce the model employed for simulating the longitudinal FEL dynamics. Then, we present numerical results obtained when including the delay control feedback into the model. Finally, results are compared with those obtained making use of a derivative filter, a feedback system successfully implemented on the Elettra [12] and Super-ACO [13] SRFELs.

THE MODEL

The longitudinal dynamics of the SRFEL is studied by considering the coupled evolution of the laser intensity and of the electron bunch. It is modeled by means of an iterative map [5]. The evolution of the laser-intensity profile, $I(\tau)$, at the j^{th} passage is modeled by the following recursive equation:

$$I_j(\tau) = R^2 I_{j-1}(\tau - \epsilon)(1 + g_{j-1}(\tau)) + i_s(\tau) \quad (1)$$

where τ is the temporal position with respect to the centroid of the electron bunch, R is the cavity mirror reflectivity and i_s stands for the spontaneous emission of the optical klystron; The parameter ϵ accounts for the detuning between the period of photons in the optical cavity and the revolution time of the electron bunch. The gain $g_j(\tau)$ is given by

$$g_j(\tau) = g_0 \frac{\sigma_0}{\sigma_j} \exp(-\frac{\sigma_j^2 - \sigma_0^2}{2\sigma_0^2}) \exp(-\frac{\tau^2}{2\sigma_{t,j}}). \qquad (2)$$

In Eq. (2) g_0 and σ_0 account, respectively, for the initial peak gain and energy spread; σ_j and $\sigma_{t,j} \simeq \frac{\alpha}{\Omega}\sigma_j$ [1], where α is the momentum compaction and Ω the synchrotron frequency, are, respectively the energy spread and the bunch length of the j^{th} interaction. The evolution of thr former is given by

$$\sigma_{j+1}^2 = \sigma_j^2 + \frac{2\Delta T}{\tau_s}(\gamma I_n + \sigma_0^2 - \sigma_j^2) \qquad (3)$$

where $\gamma = \sigma_e^2 - \sigma_0^2$ is the difference between the equilibrium value σ_e, and the initial value of the energy spread. $I_j = \int_{-\infty}^{+\infty} I_j(\tau)$ is the normalized laser intensity, ΔT accounts for the revolution period of electrons in the ring and τ_s is the synchrotron damping time. For a more exhaustive description of the model we refer to Ref. [12].

NUMERICAL RESULTS

It has been shown that the SRFEL operating at Elettra is subjected to external perturbation with strong frequency component at $50\ Hz$ [12, 14]. This external noise, together with the residual detuning, is at the origin of the instability of the system and of the large oscillations of the laser intensity [12].

The control method we are presenting is based on a feedback delayed signal [10]. Such a method consists in applying to the system a signal $F(t)$ proportional to $A \cdot (I(t) - I(t - T_d))$ where the loop gain A and the delay time T_d are the parameters to be set to control and stabilize the laser evolution. The signal $F(t)$ is used to change the detuning according to $\epsilon = \epsilon + F(t)$. This signal is used in Eq. (1) by considering that $t = j \cdot \Delta t$.

Detuned SRFEL

We first investigate the laser dynamics in presence of a detuning ϵ of $0.15\ fs$, without additional external perturbations. In these conditions the laser shows large periodic oscillations (Fig. 1), whose frequency, for the case of the Elettra SRFEL, is about $250\ Hz$.

If the gain A of the control signal is strong enough ($A \geq 0.5 \cdot 10^{-6}$) and the delay time is close to the oscillation period ($\sim 4\ ms$) it is possible to stabilize the system on its unstable fixed point characterized by $I \simeq 1$ and a normalized standard deviation of the laser signal $sd = \frac{\sqrt{<I^2> - <I>^2}}{<I>}$ close to zero. However, as shown in Fig. 2, the stabilization is effective only in a quite small range of the delay T_d. Outside such an interval oscillations are still present and no stabilization is obtained.

Using a delay time significantly different from the period of the "natural" oscillations affects the frequency and the amplitude of the oscillations. This fact is evident looking at Fig. 3 where a time T_d shorter than the period of the system has been used. Similar results are obtained for T_d larger than the period.

[1]Neglecting the effect of micro-wave instabilities.

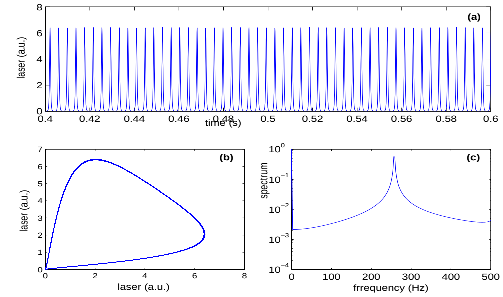

Figure 1: Simulation of the SRFEL dynamics with Eqs (1,2,3), the used parameter values are: $\tau_s = 86\ ms$, $\sigma_0 = 1.12 \cdot 10^{-3}$, $\frac{\sigma_e}{\sigma_0} = 1.3$, $\alpha = 1.6 \cdot 10^{-3}$, $\Omega = 16\ kHz$, $g_0 = 0.145$, $i_s = 4.3 \cdot 10^{-7}$, $\Delta T = 216\ ns$, $R = 0.96$. a) Temporal evolution of the laser intensity in presence of a detuning $\epsilon = 0.15\ fs$. The laser shows large oscillations with a characteristic period of about $4\ ms$. b) Attractor of the SRFEL dynamics reconstructed by means of embedding technique[15] with and embedding delay time $\tau_{emb} = 345\ \mu s$. c) Power spectrum of the laser signal.

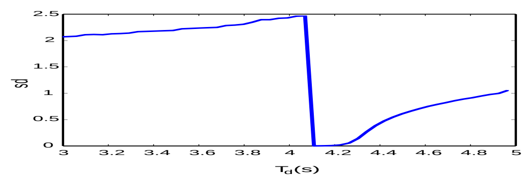

Figure 2: Maxima of the laser signal as a function of the delay time T_d and for $A = 0.5 \cdot 10^{-6}$ showing the small range suitable for stabilization.

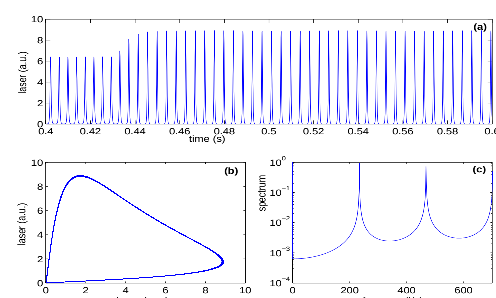

Figure 3: Evidence of the change in the frequency and amplitude of the laser oscillations when the control loop is activated (at $t = 0.43\ s$) with a delay time far from the "natural" oscillation period. Here $T_d = 3.49\ ms$.

The high precision required by the method is a problem when envisaging its experimental implementation. In fact, the system generally shows oscillations with many frequency components and this makes impossible to define a suitable delay time.

However, following the idea of the adaptive control of chaos [16], it has been recently shown that for the stabilization of an unstable fixed point a better method consists in the use of a control loop relying on two incommensurable delay times [11];

$$F(t) = A_1 \cdot (I(t) - I(t - T_{d1})) + A_2 \cdot (I(t) - I(t - T_{d2})). \tag{4}$$

We investigate the effect of such a control on the detuned SRFEL. Here we present results for the case of $A_1 = A_2 = 0.5 \cdot 10^{-6}$. As shown in Fig. 4, the new control term leads to a significant improvement of the feedback robustness.

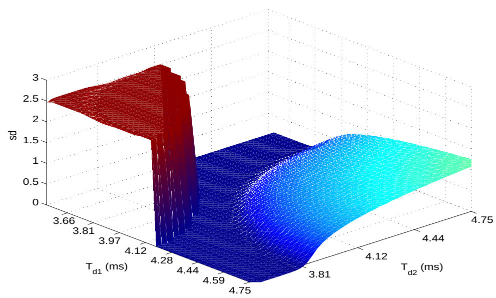

Figure 4: Normalized standard deviation of the laser intensity I of the SRFEL controlled by the signal provided by Eq.(4) where $A_1 = A_2 = 0.5 \cdot 10^{-6}$ and T_{d1}, T_{d2} lie in the interval $3.5 - 4.75\ ms$.

Indeed, stabilization (corresponding to zero standard deviation) is achieved in a quite extended region around the diagonal ($T_{d1} = T_{d2}$).

This result is important when, due to an external noise perturbing the electron-beam dynamics, the laser shows a chaotic or multifrequency regime. In fact, in this case the system is not characterized by an exact periodicity and, as already stressed, the stabilization can not be achieved using a single delay time.

Detuned SRFEL in presence of an external noise

We now investigate the case of a detuned SRFEL in presence of an additional external signal at $50 Hz$ perturbing the evolution of the optical gain (see Eq.(5)). This situation is close to the one currently observed when operating the Elettra SRFEL [12]. As shown in Ref. [12], the model previously described is able to reproduce quite accurately such an experimental condition when the detuning ϵ is modified as follows

$$\epsilon = \epsilon_0 + \delta\epsilon \cdot \sin(2\pi\nu t). \tag{5}$$

In the previous equation ϵ_0, the "unperturbed" detuning, has been set to $0.05\ fs$, the perturbation amplitude $\delta\epsilon$ to $0.18\ fs$ and the perturbation frequency ν to $50\ Hz$.

In this configuration the system shows a chaotic dynamics characterized by a broad spectrum around $250\ Hz$ (see Fig. 5). As expected, simulations carried out using a single delay T_d show that the feedback can reduce the amplitude of the oscillations (see Fig. 6), but is not able to stabilize the system on the fixed point.

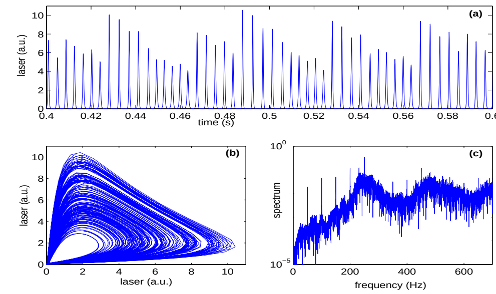

Figure 5: Temporal evolution of the laser signal I in presence of a slight "unperturbed" detuning and with an added external perturbation at $50\ Hz$. b) Reconstructed attractor. c)Spectrum.

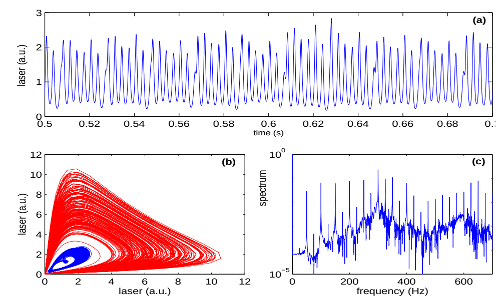

Figure 6: a) Temporal evolution of the laser signal I in presence of a slight "unperturbed" detuning and with an added external perturbation at $50\ Hz$. Stabilization attempted using a single delay time $T_d = 4.38\ ms$ and with $A = 0.5 \cdot 10^{-6}$. b) Reconstructed attractor of the system with and without the control signal (blue and red respectively). c) Spectrum of the laser intensity of the SRFEL with the controlling signal.

The effect of the control feedback is evident from Fig.6(b) where the attractor of the uncontrolled system is reported together with that of the controlled one. The effect of the control feedback can be also estimated by measuring the standard deviation of the signal I with ($sd = 0.76$) and without ($sd = 1.89$) the control.

The use of a control loop with two different delay times leads to a noticeable improvement. The normalized standard deviation is indeed reduced from $sd = 1.89$ to $sd = 0.28$.

The effect is also visible from a comparison between the attractors of the uncontrolled and of the controlled system reported in Fig.7(b). Results are similar when using delay times and coupling strength slightly different from the values used in the presented cases.

COMPARISON WITH THE DERIVATIVE FILTER METHOD

We can now compare the results of the proposed method with those obtained with a derivative feedback, a control

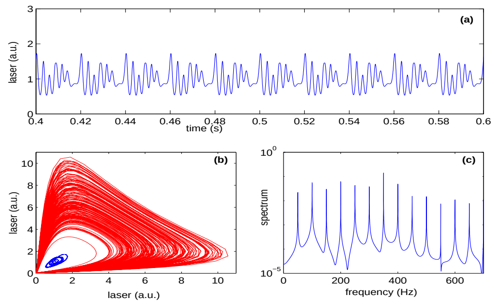

Figure 7: a) Temporal evolution of the laser signal I in presence of a slight detuning and with an added external perturbation at $50\ Hz$. The system is also driven by a control signal (Eq.4) with two different delay times T_{d1} and T_{d2} equal to $4.36\ ms$ and $3.11\ ms$ and with $A_1 = A_2 = 1.95 \cdot 10^{-6}$. b) reconstructed attractors of the controlled system (blue) and of the system without control (red). c) Spectrum of the controlled SRFEL

method that has been already implemented at Elettra [12] that uses as a control signal the derivative of the laser intensity provided by

$$F_{j+1} = \beta \cdot (I_j - I_{j-1}). \tag{6}$$

In Fig. 8 the results of the two control methods applied on the SRFEL in the same conditions are compared between them. Fig 8(a) shows the best results for the derivative feedback ($\beta = 0.03$) applied on the SRFEL with the same values of ϵ_0 and $\delta\epsilon$ used for implementing Eq.5, while Fig. 8(b) shows the temporal evolution of the laser intensity when controlled by the two-delay method.

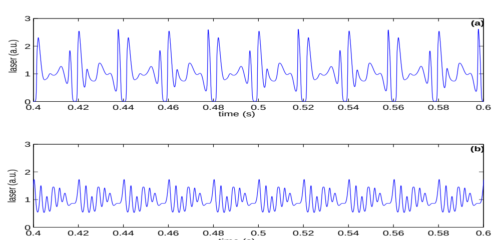

Figure 8: Temporal evolution of the laser signal I in presence of a slight detuning ($\epsilon_0 = 0.05\ fs$) and with an added external perturbation at $50\ Hz$ ($\delta\epsilon = 0.18\ fs$). a) The system is stabilized by a control signal with two different delay times T_{d1} and T_{d2} equal to $4.36\ ms$ and $3.11\ ms$ and with $A_1 = A_2 = 1.95 \cdot 10^{-6}$ (see Eq. (4)). b) The system is stabilized by a control signal provided by a derivative control loop [12] with ($\beta = 0.03$) (see Eq. (6)).

In the studied conditions, characterized by a small detuning and a large noise modulation, the delay method is more effective with respect to the derivative one that presents larger residual oscillations characterized by $sd = 0.55$.

It is important to note that the derivative control method can be considered as a limit case of the delay control method when T_d approaches ΔT. In the case of periodic oscillations, but not in the case of multi-periodicity or chaotic behavior, also the use of a delay time $T_d = T + \Delta T$ (where T is the period of oscillations) corresponds to the derivative case.

CONCLUSIONS

We presented a numerical study for the stabilization of the temporal dynamics of a SRFEL. Simulation have been performed considering the longitudinal dynamics for the Elettra SRFEL where both the detuning and external residual noise signal are present. The dynamics of the system is characterized by oscillations with a broad spectrum around at $250\ Hz$. We showed that a control loop using a single delay is able to reduce these oscillations. The reduction of the oscillation is more evident when using a control feedback with two delay times. Although the method is not able to stabilize the "cw" regime, we consider that the strong reduction of oscillations of the system is an important improvement of the performance of the Elettra SRFEL and that the method should be implemented experimentally. We also plane to consider the possibility of a feedback control signal using more than two delay times.

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