STATISTICAL STUDY OF SPONTANEOUS EMISSION IN THE ISRAELI ELECTROSTATIC ACCELERATOR FREE-ELECTRON LASER

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Abstract

We have measured spontaneous emission in the Israeli Electrostatic Accelerator FEL (EA-FEL). The measurements were repeated numerous times in order to get information about statistical features. The probability density function of the radiation power was formed and its moments were derived to characterize spontaneous emission radiation. The results from analytical and numerical models are compared to those obtained in experimental measurements.

INTRODUCTION

Electron devices such as microwave tubes and free-electron lasers (FELs) utilize distributed interaction between an electron beam and electromagnetic radiation. Random electron distribution in the e-beam cause fluctuations in current density, identified as shot noise in the beam current. Electrons passing through a magnetic undulator emit radiation, which is called undulator synchrotron radiation [1]. The electromagnetic fields excited by each electron add incoherently, resulting in a spontaneous emission having a certain power spectral density [2].

In this work we study the statistical distribution of power measured for spontaneous radiation. Our aim is to extract from those measurements data regarding the electromagnetic properties of our FEL system. We start by describing the phenomena of spontaneous radiation both analytically and numerically. Followed by a section describing the theory of detection and the effects of detector noise on distribution. Finally the experimental results are compared with the theoretical ones and the relevant parameters are extracted.

SPONTANEOUS EMISSION

Analytical Spontaneous Average Power

The power spectral density that results from the sum of electromagnetic fields excited by electrons passing through a magnetic undulator added incoherently is according to [2]:

$$\frac{dP_{sp}(L_W)}{df} = \tau_{sp} P_{sp}(L_W)\,\mathrm{sinc}^2(1/2 \cdot \theta L_W) \tag{1}$$

Where $P_{sp}(L_W)$ is the expected value of the total spontaneous emission power at the end of the undulator with length L_W, $\tau_{sp} = \left| L_W/V_{z0} - L_W/V_g \right|$ is the slippage time and $\theta = \omega/V_{z0} - (k_z + k_w)$ is the detuning parameter. In the above equations V_{z0} is the electron axial velocity, V_g is the group velocity of the radiation wave packet, k_z is the axial wave number, k_w is the wiggler wave number and ω is the angular frequency. The spontaneous emission null-to-null bandwidth is approximately $2/\tau_{sp} \approx 2 f_0/N_W$. In which f_0 corresponds to the tuning frequency for which $\theta = 0$ and N_w is the number of periods in the FEL wiggler.

In an FEL utilizing a magneto-static planar wiggler, the total average power of the spontaneous emission is given by:

$$P_{sp}(L_W) = \frac{1}{8}\frac{eI_0}{\tau_{sp}}\left(\frac{a_w}{\gamma\beta_{z0}}\right)^2 \frac{Z}{A_{em}} L_W^2 \tag{2}$$

The spontaneous emission average power is proportional to eI_0, where I_0 is the DC beam current and e is the electron charge. Additional symbols appearing in this formula are defined in [2]. Those include the wiggler parameter a_w, $\beta_{z0} = \frac{V_{z0}}{c}$ in which c is the velocity of light in vacuum. And the relativistic factor γ is defined as $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$. Further symbols are the mode impedance Z and mode effective area A_{em}. In the low gain limit the spontaneous emission power grows as the square of interaction length L_W. The power spectral density for typical parameters (see table 1) is described in Fig. 1.

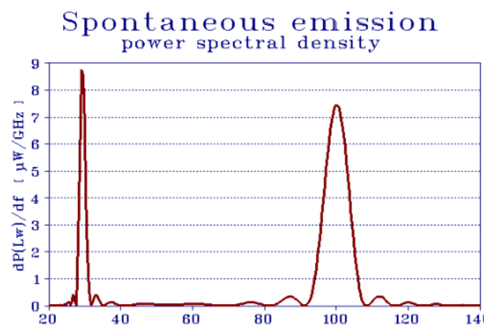

Figure 1: Power spectral density of spontaneous emission for typical parameters.

Resonator Spontaneous Emission

Consider a waveguide resonator, as in Fig. 2. The average power spectral density of the spontaneous emission power coupled out of the cavity is described by:

$$\frac{T}{\left(1-\sqrt{R}\cdot e^{-\alpha L_c}\right)^2+4\sqrt{R}\cdot e^{-\alpha L_c}\sin^2\left(\frac{1}{2}k_z L_c\right)}\cdot\frac{dP_{sp}(L_W)}{df} \tag{3}$$

Where L_c: Resonator (round-trip) length, R: The total power reflectivity of the mirrors, T: Power transmission of the out-coupler, α: Field attenuation factor and $k_z(f)$ the wavenumber.

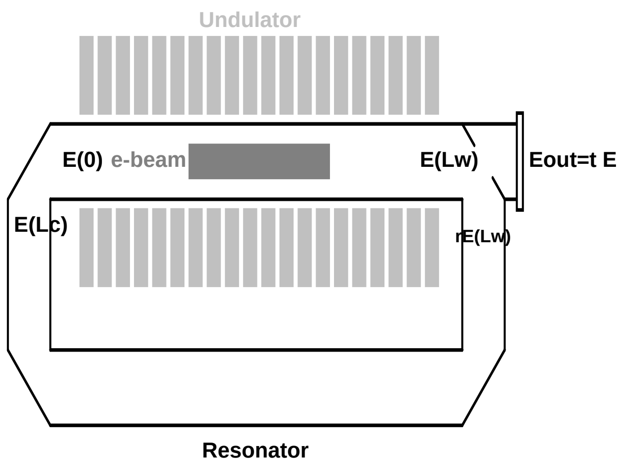

Figure 2: Waveguide resonator

Spontaneous Power Distribution

Spontaneous emission field is a sum of N wave packets E_{wp} emitted by electrons entering the wiggler at times t_i $i \in [1, N]$ with random phases:

$$E(t)=\sum_{i=1}^{N} E_{wp}(t-t_i) \qquad N \to \infty; \quad t \geq t_1 \tag{4}$$

Thus, the fields of the different wave packets are independent random variables. The result is known as thermal light or chaotic light. As $N \to \infty$, according to the central limit theorem, its statistical distribution is Gaussian:

$$f_{E(t)}(E)=\frac{1}{\sqrt{2\pi}\sigma_E}e^{-E^2/2\sigma_E^2} \tag{5}$$

With zero mean and σ_E^2 variance. The field can be presented as:

$$E(t)=A(t)\cdot\cos\left[\omega_0 t+\theta(t)\right] \tag{6}$$

Where A(t) and θ(t) are stochastic processes representing the amplitude envelope and phase, respectively. Note that the amplitude is positive and follows a Rayleigh distribution with mean $\overline{A(t)}=\sqrt{\frac{\pi}{2}}\sigma_E$ and second moment $\overline{A^2(t)}=2\sigma_E^2$ and the phase θ(t) is uniformly distributed between [-π,+π].

The instantaneous RF power is related to the envelope squared: $P_{RF}(t)=1/2Z_0 A^2(t)$ in which Z_0 is the vacuum impedance. This is the result of time averaging over the fast-changing field squared. The power is a stochastic process with the negative-exponential probability density function described by:

$$f_{sp}(P)=\frac{1}{P_{sp}}e^{-\frac{P}{P_{sp}}}u(P) \tag{7}$$

Where $P_{sp}=\frac{\overline{A^2}}{2Z_0}=\frac{\sigma_E^2}{Z_0}$ derived from equation (2) is the expected average power, $u(P)$ is a step function. The variance of the power is P_{sp}^2. Those results are based on classical assumptions and cannot be implemented for very low powers for which quantum mechanics is applicable.

Numerical Analysis

The parameters used for numerical simulation are given in Table 1. Those parameters are typical for the Israeli Free Electron Laser:

Table 1: Parameters used in numerical simulation

Wiggler	
Magnetic induction:	B_w=2 kG
Period:	λ_w=4.444 cm
Number of periods:	N_w=20
Waveguide	
Parallel curved Plates:	R=17.2 mm, b=10.7 mm
Mode:	TE_{01}
Coefficient for the Long (Focusing) Magnets:	α=2.9 Tl/m
Accelerator	
Beam energy:	E_k=1.4 MeV
Beam radius:	r_b=2 mm
Beam current:	I_0=1 A
Spontaneous Emission (theory)	
Slippage time $\tau_{sp}=L_w/v_{z_0}-L_w/v_{gr}$	0.112 nSec

The calculations were done using a Gaussian random longitudinal distributions of electrons with a standard deviation (the beam time duration parameter) T=100 pSec (Compare with slippage time given above). The radial distribution was also a random Gaussian distribution with a standard deviation equal to 2 mm (the beam radius). The synchronism frequency is about f_0= 100 GHz, so the beam duration includes about T f_0 = 10 periods of the radiation. There were 100 quasi-particles (10 particles per radiation period) in the simulations. Average power of spontaneous emission given in Fig. 3 where evaluated as:

$$P_{avrg}=\frac{N_q}{N_e}\frac{W_{tot}}{T}=\frac{N_q e}{I_0 T}\frac{W_{tot}}{T} \tag{8}$$

Here W_{tot} is the total energy of spontaneous emission, N_q=100 is the number of quasi-particles are taken into account in simulations, N_e=I_0T/e is the number of electrons in the beam impulse, N_e/N_q is a number of electrons in each quasi-particle and T=100 pSec is the beam duration. Resulting in a numerical value of $37\mu W$.

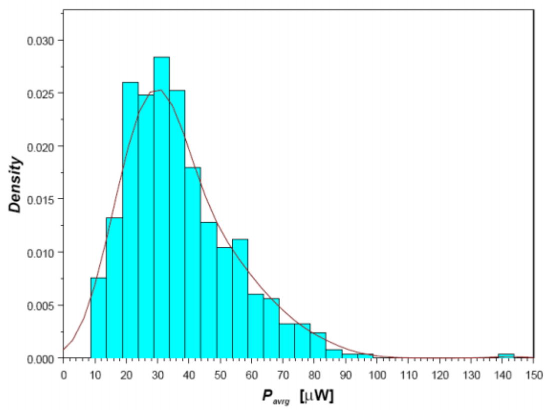

Figure 3: Numerical results for spontaneous power distribution.

Despite the limitations of the numerical simulations (short time duration and limited number of particles) which is due to computer constraints the results is of the same order of magnitude as the analytical one, which is about $60\mu W$ for the same parameters. The standard deviation in the numerical experiment is about $17\mu W$ which is quite different from expected analytical theory that predicts a standard deviation of $60\mu W$.

Notice that the results above do not take into account the effect of the resonator (see equation (3)). We evaluate $R \cdot e^{-2\alpha L_c} = 0.66 \quad T = 6\%$. Thus, the resulting total out-coupled power (out coupling is taken to be frequency independent in a "worst case" scenario) of the spontaneous emission is:

$$P_{sp}^{out} = 0.18 P_{sp}(L_w) \tag{9}$$

This however, does not change the power distribution functional form just its average.

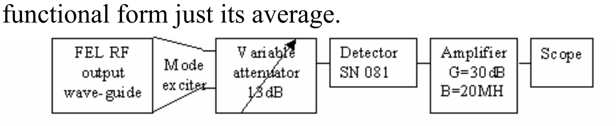

Figure 4: Measurement setup.

THEORY OF DETECTION

Measurement Setup

In order to compare theoretical and numerical results with experiment, additional components of the measurement setup must be taken into account. Fig. 4 illustrates the measurement setup. We see that the radiation power is attenuated by the FEL wave-guide which is a cylindrical corrugated waveguide supporting the EH11 mode. Further attenuation is due to the mode exciter, which is a tapered feed converting the EH11 mode into the TE01 mode of the standard W-band rectangular waveguide. Its insertion loss is 1.8dB. Those two elements do not affect the form of the power distribution function just the average power.

On the other hand the detector will change the distribution function as will be explained in the next section. The detector is a GaAs diode detector. Its video bandwidth is 10MHz. Its responsivity is 1µV/nW.

Fig. 5 shows the model for the diode detector with its equivalent noise sources. No bias voltage is applied. The spreading resistance, R_s, is 50Ω. This is the ohmic resistance associated with the semiconductor outside the junction and the contacts. The dynamic resistance of the junction, R_j, at zero bias is 2.2kΩ. C_j is the junction capacitance and its value is 30fF. R_L, the load resistance, is the amplifier's input impedance. Its value is 20kΩ. Total detection system responsivity is $\Re = v_{out}/P_{RF} = 1\mu V/nW$.

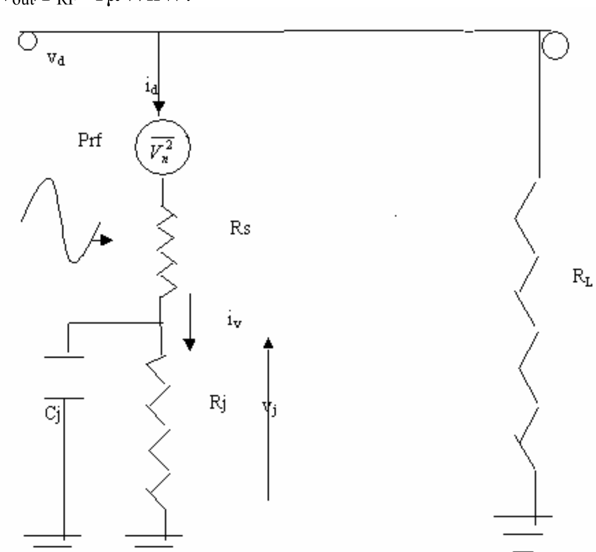

Figure 5: Diode detector with equivalent noise current sources.

Detected Power Distribution

Since the detector's bandwidth is 10MHz, the measured power is integrated in intervals of T=0.1µs. This corresponds to the summation of the instantaneous power of M independent wave packet packets of length τ_{sp} within the integration interval. As a result, a gamma distribution is expected [4,5]:

$$f_{P_D}(P) = \frac{M^M}{\Gamma(M)} \left(\frac{P}{P_{sp}} \right)^{M-1} \frac{1}{P_{sp}} e^{-M\frac{P}{P_{sp}}} \tag{10}$$

With mean P_{sp} and standard deviation $P_{sp}/\sqrt{M}$. Notice that in this distribution the variance is much smaller with respect to the average in the case of large M . The derivation of this distribution assumes a quasi-stationary process, with the mean of the instantaneous power being constant within the integration interval. Within a larger time scale, the process is not stationary due to the power envelope changing along the pulse duration.

As M→1, the negative-exponential distribution is obtained. For very high values of M, this is the sum of a very large number of random variables, and, according to

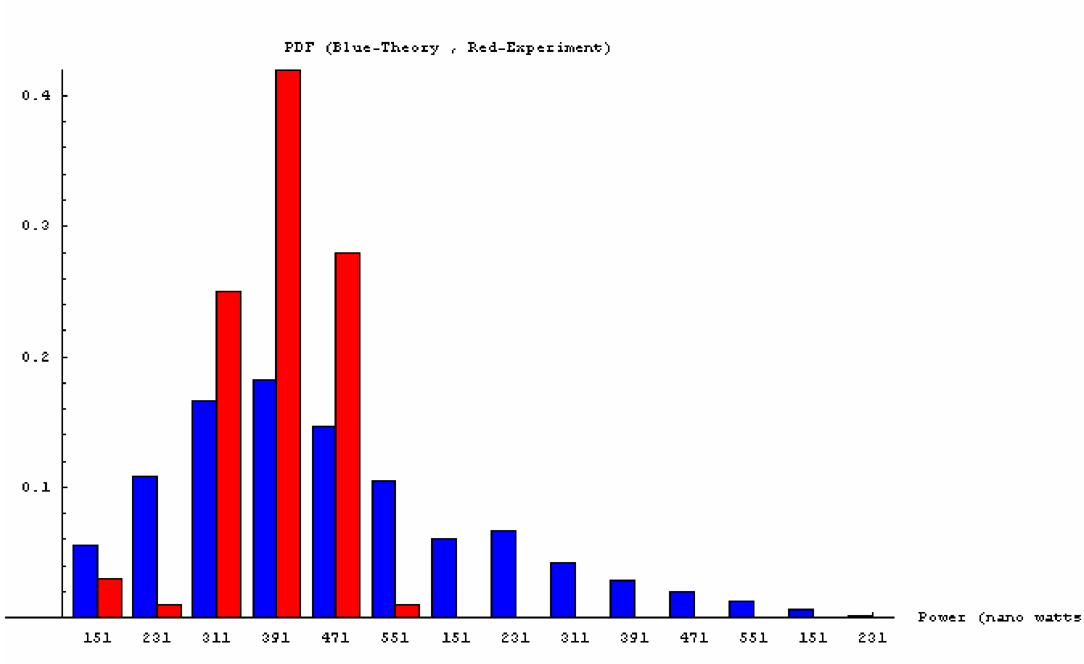

Figure 6: Experimental and theoretical power distributions.

the central limit theorem, the distribution is Gaussian.

Taking τ_{sp} from table 1 we see that $M \cong 1000$, hence a Gaussian distribution of power is expected.

MEASUREMENTS AND RESULTS

We measured the power of one hundred subsequent pulses of spontaneous emission, obtained at the same FEL conditions. The results are depicted in figure 6. The average power obtained was 495 nW, and the standard deviation 74 nW. The distribution is approximately Gaussian as expected. Subtracting from the standard deviation the technical noise which is estimated to 45 nW we obtain: $M = \left(\frac{495}{29}\right)^2 \approx 291$ which is the same order of magnitude smaller than the theoretical prediction. Comparison of the experimental results with the theoretical analysis (and taking into account the parameters of table 1 but with a different current of 2 A that is a better estimate of the experimental current). We observe that the theoretical average was reduced by the total attenuation of the resonator and wave-guide (attenuation by the mode exciter and variable attenuator is balanced by the amplification of the voltage amplifier). Taking into account an initial power double of the $37\mu W$ obtained by the numerical analysis we obtain a reduction of:

$$Attenuation = 10 Log\left(\frac{74\mu W}{495 nW}\right) \approx 22 dB \qquad (12)$$

Since a 7.5 dB attenuation is caused by the resonator (see equation (9)), we estimate the losses of the wave-guiding system to be about 14 dB. This agrees approximately with independent measurements.

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