PARTICLE IN CELL SIMULATION ON GRATING RADIATION

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Abstract

Radiation emitted from an open grating driven by a medium-energy electron beam is simulated through a particle-in-cell simulation code. A short cylindrical electron bunch and a rectangle-grooved grating are both dealt with three-dimensional model. The signal of Smith-Purcell radiation is clearly observed in simulation, and an evanescent wave at a frequency lower than the Smith-Purcell radiation is also distinguished, which is predicated theoretically and observed in two-dimensional simulation model. The simulation results also include the dependence of the amplitude of the radiation field on emitting angle.

INTRODUCTION

A renewed interest of grating radiation has been raised in recent years since J. Urata *et al.* experimentally demonstrated the superradiant Smith-Purcell emission at THz region of the spectrum, by using a high brightness electron beam and a diffraction grating at Dartmouth college [1,2]. The emission in a grating-based scheme is very attractive, for it may be a promising means in the development of a convenient, easily tunable, tabletop and high power THz device, which is an active research area at present. To improve on the performance of such kind of device, it is necessary to better understand how a grating radiates.

The well-known radiation from a grating is called Smith-Purcell radiation, which is formed when an electron passes close to the surface of a periodic grating. The wavelength λ of the radiation observed at the angle θ measured from a direction normal to the surface of the grating is [3,4]

$$\frac{\lambda}{d}=\frac{1}{|n|}\left(\frac{1}{\beta}-\sin\theta\right), \qquad (1)$$

where d is the grating period, βc the electron velocity, c the speed of light, and n the order of the reflection from the grating. This equation has been amply confirmed by experiments [1,5]. The incoherent Smith-Purcell radiation has been analysed in many ways, such as diffraction theory, integral equation method and induced surface current model [6-12]. Also, some theories to address the coherent Smith-Purcell radiation have been proposed though they are not in agreement with each other [13-16].

Recently, Kesar *et a.l* presented a two-dimensional simulation on Smith-Purcell radiation, using finit-difference, time-domain approach [17]. They researched the characters of the radiation frequency and the radiation energy, and achieved good agreements to theory. However, less information was provided on the electron-wave interaction, especially, the interesting superradiant emission. More recently, Donohue and Gardelle also carried out a two-dimensional simulation with a particle-in-cell code [18]. They carefully investigate the generation of coherent Smith-Purcell radiation and their results strongly support the theory of Andrews *et al.*. They also indicated in their paper that three-dimensional simulation should be performed for the deep understanding of the grating radiation

In this paper, we present the investigation of the grating-based radiation by using a three-dimensional particle-in-cell simulation code, MAGIC [19]. This code is developed by Mission Research Corporation, which is a finite-difference, time-domain code for simulating plasma physics processes, i.e., those processes that involve interactions between space charge and electromagnetic fields.

SIMULATION MODEL

A three-dimensional model of the grating system considered here is shown in Fig.1, where d is the periodic

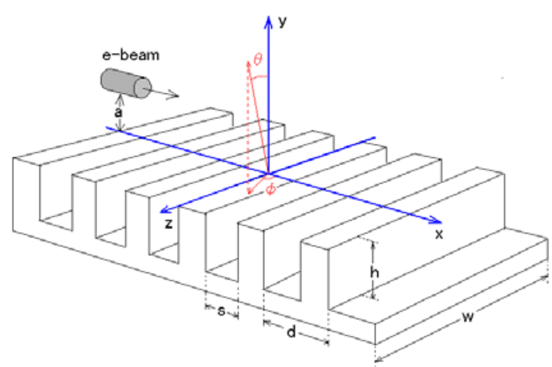

Figure 1: The schematic of a three-dimensional grating system.

Table 1: Parameters used in our simulation.

Electron beam energy (injection)	E=100 keV
Current	I=1 A
Beam radius	r=2.5 mm
Bunch length	τ =10 ps
Beam-grating distance	a=2 mm
Grating period	d=2 cm
Grating groove depth	h=1 cm
Grating groove width	s=1 cm
Grating width	w=10 cm
Number of periods	N=10
External magnetic field	Bx=0 T

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length, *s* the rectangle groove width, *h* the groove depth and *w* the width of grating. A cylindrical electron beam bunch with radius *r* moves along the grating to the *x*-axis. The distance of the bottom edge of the electron beam and the top of the grating surface is denoted by *a*. The parameters of the grating used in this paper are almost same as those employed in the two-dimensional simulation [18], as indicated in table 1.

DISCRIPTION OF SIMULATION GEOMETRY

In Fig.2, we display the x-y plot of the geometry for our three-dimensional simulation. The setup includes a perfectly conducting grating in the center at the bottom, a small cathode emitting mono-energy electron beam, and a vacuum box in which radiation propagates. The boundary is set to be "free space" by MAGIC language, on which the electromagnetic fields are absorbed without reflection.

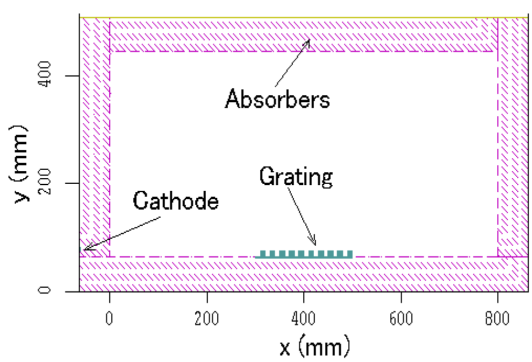

Figure 2: Geometry used in simulation.

The box is divided into a mesh consisting of rectangle cells. The size of the cells is defined as such: small in the region of grating and beam propagating and large in the reminder of the box. Electron beam of mono-energy is emitted from the cathode governed by the MAGIC algorithm. External magnetic field is not necessary since the beam current we used is not large.

RESULTS

In order to clearly observe the radiation process, we use a very short electron bunch to drive the grating. The length of the bunch is 0.3 cm, which is shorter than one quarter of the grating period. Fig. 3 (a) demonstrates the contour of magnetic field component in z direction when the bunch covered six periods. The length of the intervals between the two adjacent contours indicates the wavelength of the radiation. It is seen that those interval lengths are different as the radiation direction varies. The backward radiation has large wavelength while the forward small, and this fact corresponds to the Smith-Purcell radiation predicted by Eq.1. When the beam bunch goes out of the grating region as shown in Fig. 3 (b), the Smith-Purcell radiation is not generated any more, hence we cannot see more magnetic field contour rising from the grating. But, Fig. 3 (b) shows the evidence that radiation from the two ends of the grating begins, and the further evidence can be seen in Fig. 3 (c). The radiation from the grating ends is the diffraction part of the evanescent wave. Actually, grating could be regarded as a kind of slow-wave structure. The components of evanescent wave running slower than the light can interact with the electron beam. The evanescent wave cannot radiate until they come to the ends of grating, and they undergo partial reflection and partial diffraction. The reflection part will oscillate between the two ends, and the diffraction part is radiated and can go anywhere in the

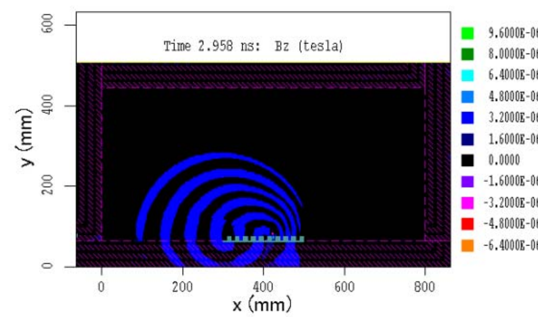

Figure 3 (a): x-y contour map of *Bz* at 2.958 ns.

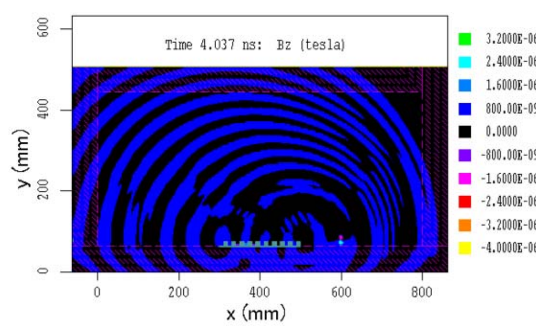

Figure 3 (b): x-y contour map of *Bz* at 4.037 ns.

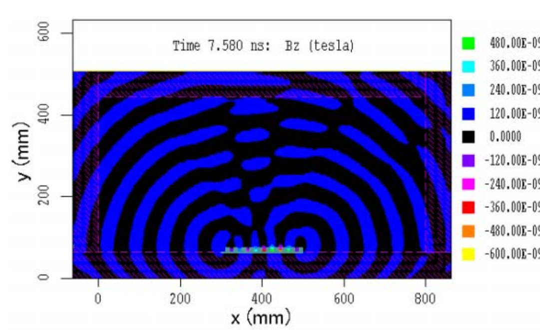

Figure 3 (c): x-y contour map of *Bz* at 7.58 ns.

simulation box. In Fig. 3 (c), we find that the radiation from the two grating ends interfere in the space. Also, we can deduce that the radiation wavelength does not rely on the direction, because the intervals between those adjacent contours are almost same in any direction. The y-z plot corresponding to Fig. 3 (c) is as shown in Fig. 4.

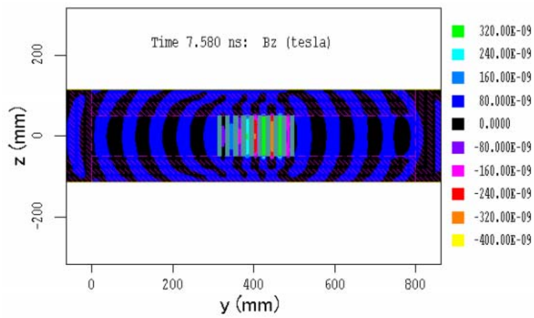

Figure 4: y-z contour map of Bz.

The MAGIC provides the utilities to detect the electromagnetic field at any points in the simulation box. Fig. 5 illustrates the temporal behaviour of Bz, detected at the point of θ =-30°, ϕ =0°, and 37.8 cm away from the

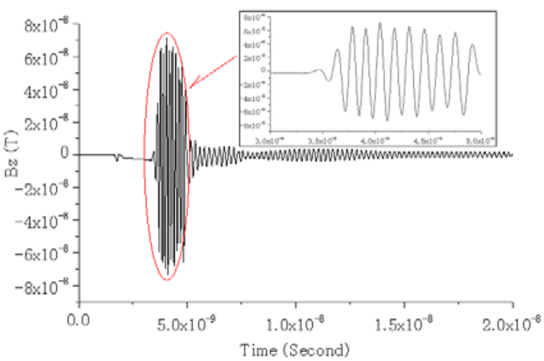

Figure 5: Time signal of Bz.

center of the grating. The Smith-Purcell radiation pulse consists of 10 periods, the same number as the grating periods. Fig. 5 also indicates the fact that the Smith-Purcell radiation finishes after the electron bunch leaves the grating, and after that the evanescent wave oscillates within the grating and diffracts at the two ends of the grating. The corresponding FFT of Fig. 5 is given in Fig. 6, where one can find two clear peaks, one is for the Smith-Purcell radiation and the other is for the evanescent wave. The frequency of the Smith-Purcell signal is 7.11 GHz, matching the theoretical result calculated from Eq.1. The dependences of the radiation frequencies on angle θ are shown as in Fig. 7, under the condition of ϕ =0. One can understand that the simulation results for the Smith-Purcell radiation are in agreement with the theory, and the

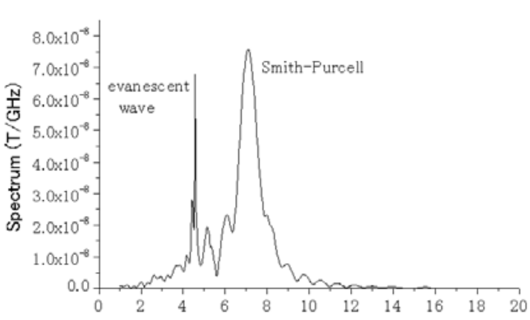

Figure 6: FFT of time signal of Bz

evanescent wave frequency is a constant. Not only the SP frequency changes with the angle θ, but also the amplitude of radiation field depends on the angle. Fig. 8

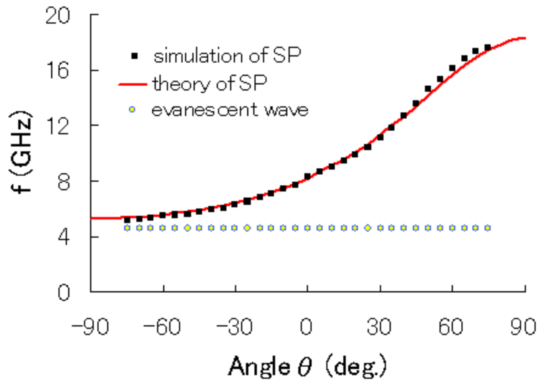

Figure 7: frequency dependence on radiation angle.

illustrates the amplitude distribution. The maximum value occurs at the point of $\theta \simeq$-25°, and we also understand that the forward radiation is weaker than the backward one.

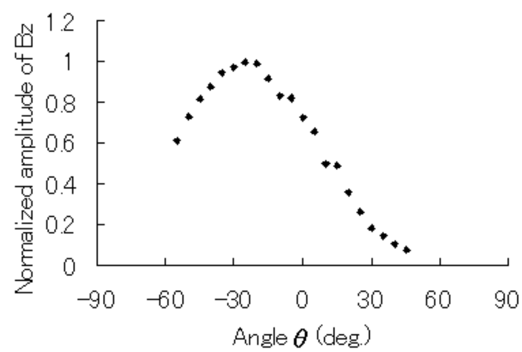

Figure 8: Amplitude of SP radiation field of Bz varies with the angle θ.

We also studied the dependence of field amplitude on the angle ϕ. To do this, the position of the detectors are chosen with same angle θ =45° and same distance 35 cm to the center of the grating but varying ϕ from –8° to 8°.

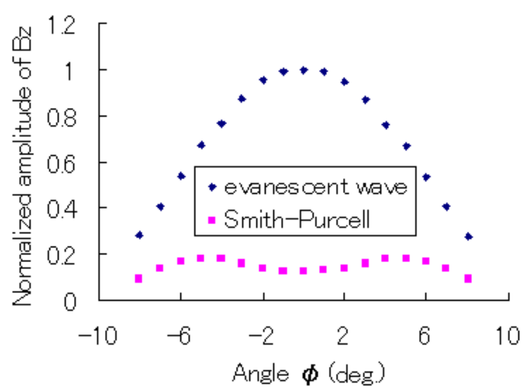

Figure 9: Amplitude of radiation field of Bz varies varies with the angle ϕ.

The results are as shown in Fig. 9. For the evanescent wave, the field is strongest at $\phi = 0^o$, while the Smith-Purcell radiation is at the valley.

CONCLUSION

In conclusion, we performed three-dimensional particle-in-cell simulation for an electron-beam driven grating system and find that, the Smith-Purcell radiation is different from the evanescent wave. The Smith-Purcell radiation is generated only when the electron bunch travelling along the grating, while the induced evanescent wave can oscillate between the two ends of the grating and part of them undergoes diffraction at the ends to radiate. The frequency of evanescent wave is lower than the SP radiation. The distribution of field amplitude along angle θ and ϕ are presented, which implies that the backward SP radiation is stronger than the forward one.

Up to now, the evanescent wave has not been detected in experiments of Smith-Purcell radiation though it is predicated in theory and two-dimensional simulation. In our three-dimensional simulation, this signal still appears, and we conclude that it surely exists in practice.

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