SIMULATION OF SMITH-PURCELL FELS AT TERAHERTZ FREQUENCIES

J.T. Donohue, Centre d'Etudes Nucléaires de Bordeaux-Gradignan, BP 120, 33175 Gradignan, France
J. Gardelle, CEA CESTA, BP 2, 33114 Le Barp, France

Abstract

Our previous work on the 2D simulation of a coherent Smith-Purcell FEL operating in the terahertz domain is extended to a systematic study of the dependence on various parameters. The important question of the starting current required to produce coherent radiation is addressed, and our new results are presented. As in our previous work we concentrate on two configurations, one similar to the Dartmouth S-P FEL, with a low energy continuous beam, and the other similar to the MIT experiment that used a pre-bunched 15 MeV beam.

INTRODUCTION

At FEL 2005[1]we reported on simulations of coherent Smith-Purcell (S-P) radiation at terahertz (THz) frequencies using a two-dimensional (2D) particle-in-cell code, MAGIC. Two quite different experimental set-ups were considered; one similar to the long-running Dartmouth College experiments initiated by Walsh[2], while the other was similar to the MIT experiment that used a pre-bunched 15 MeV electron beam[3].The results we presented were rather preliminary and a fuller account has recently appeared[4].We remind the reader of the S-P relation, $\lambda = L(1/\beta - \cos\phi)/n$, where λ is the wavelength, L the grating period, β the relative velocity (in a plane parallel to the grating and perpendicular to the direction of the grooves), ϕ the angle of emission (with respect to the beam direction), and the integer n denotes the order. One of our aims was to verify the analytical model proposed by Andrews and Brau[5](AB) and subsequently extended by Andrews, Boulware, Brau and Jarvis[6]to explain coherent Smith-Purcell radiation. Our results did indeed support the viewpoint of Brau and co-workers, that the mechanism for coherent radiation is the bunching of the initially continuous beam by an evanescent wave that is significant only in the vicinity of the grating. The frequency of this wave is always less than the minimum allowed S-P frequency. The process is unstable in the sense that the wave bunches the beam, the beam drives the wave and growth occurs, both in time and along the grating. Our simulation of the Dartmouth set-up found that this is indeed what happens. In particular, the frequency and axial wave number (in the first Brillouin zone) of the simulated grating wave were very close to what the model predicts. Since the bunching is inherently non-linear, once it becomes significant the current is modulated at harmonics of the fundamental frequency, and these may correspond to allowed S-P frequencies. The radiation emitted then shows both intrabunch (since the bunches are small compared to a wavelength) and interbunch (since the fields of all bunches over the grating add up) coherence. Consequently, the coherent radiation occurs only at integer multiples of the fundamental, and only at the corresponding S-P angles. However, the simulation also revealed a major unexpected effect, namely the copious emission of radiation at the fundamental frequency. Indeed, this unforeseen radiation made the direct observation of the S-P radiation quite difficult. Our simulated grating has a finite length. When the evanescent wave reaches the end of the grating, part of its energy is emitted as free radiation of the same frequency, and part of it is reflected back in the opposite direction. The result is that there are two evanescent waves on the grating, which propagate with equal and opposite wave numbers. Only one of these waves is resonant with the electron beam, and since the beam-wave instability is absolute in the Dartmouth configuration, it displays growth both in space and time. The other Floquet wave grows in time only (through reflections), but not in space. In a companion paper we show how both of these Floquet waves may be extracted from the simulation data.

For the MIT experiment, since the beam was already bunched, the bunching mechanism referred to played no role, and the simulation was expected to be straightforward. However, the simulation failed to display unambiguous S-P radiation. In fact, there were two distinct problems, first the emission of considerable radiation even when no grating was present, and secondly, the expected frequency-angle correlation was not apparent. The former has been understood to be a consequence of standard electrodynamics. It is associated with the appearance and disappearance of a relativistic short bunch of electrons. The latter was caused by the fact that the S-P relation is valid only at distances large compared to the grating size. Since the MIT grating was 10 cm long, S-P radiation can be seen clearly only at distances of order 40 cm or more from the center of the grating. Since we are dealing with radiation whose wavelength is a few hundred μm, the mesh size in the simulation must be kept on the order of tens of μm. Under such circumstances, the computing time and memory needed become unreasonable, at least for a PC-based simulation. In the companion paper we outline a simple method we call the Small Box→Big Box transformation, which enables us to circumvent this difficulty. While techniques for passing from the near zone fields to the far zone based on Green's theorem are well known[7],our method appears to work quite well. It makes use of Finite Fourier Transforms (FFT) and fitting tools available with symbolic manipulation programs such as *Mathematica* or MAPLE. Once this transformation is performed, we see clearly the expected coherent S-P radiation at a large number of angles and frequencies.

Since the last FEL conference, other work on coherent S-P radiation has appeared, notably by Dashi Li and collaborators[8],who also use MAGIC for simulations, and by Kumar and Kwang-Je Kim[9]. In their simulations Li and co-workers studied the effects both of a single short bunch and a periodic train of such bunches passing over a grating of the Dartmouth type. The former showed intrabunch coherent S-P radiation, followed by the emptying of the evanescent wave from the grating after the passage of the bunch. The latter illustrated interbunch coherence, since the second harmonic of the imposed bunch frequency was S-P allowed and indeed emerged at the expected angle. The extensive analysis of Kumar and Kim, based on the traditional approach of diffraction by the grating of the incident electron's field, yielded a dispersion relation quite similar to that obtained by Andrews and Brau. The analysis included a treatment of coherent S-P radiation similar to that of a Backward Wave Oscillator (BWO), for which a start current is known to exist. Using both numerical and analytical methods, they estimated a start current (minimum current needed to produce the instability) of 36.5 A/m for a sheet beam 10 μm above the grating. This is similar to the estimate of 50 A/m found in reference 6.

Our attempt to find the start current for the Dartmouth configuration is discussed in the second Section, and some results on our simulation of the MIT set-up are presented in the third Section. Simulation parameters are shown in the Table 1.

Table 1. Parameters of the Simulations

Parameters	Dartmouth	MIT
beam energy	35 keV	15 MeV
Current (peak for MIT)	variable	25 kA/m
Beam thickness	$\delta = 10\ \mu$m	1 mm
Beam-grating distance	$e = 10\ \mu$m	0.7 mm
Grating period	$L = 173\ \mu$m	1 cm
Max. wave number	$K = 363$ cm^{-1}	2π cm^{-1}
Grating groove depth	$H = 100\ \mu$m	echelette
Grating groove width	$A = 62\ \mu$m	
Number of periods	$N = 74$	10
External magnetic field	$B_x = 2$ T	0
Mesh size	$8.65 \times 8\ \mu$m^2	$(100\mu$m$)^2$

START CURRENT FOR DARTMOUTH

In Figure (1) we show a contour map of the magnetic field B_z in the area of our MAGIC simulation. The beam appears as a red line, and one sees cylindrical waves of wavelength ≈ 700 μm radiating from the grating ends. This is the evanescent wave, predicted by AB.

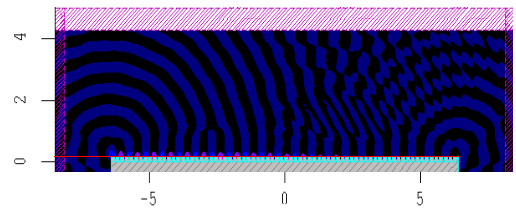

Figure 1. Contour map of B_z.

In contrast to our earlier simulations, we have raised the grating slightly. This is more consistent with the physical situation, where the grating is not flush with a ground plane. Upon doing so, we found that the height of the grating plays a significant role in the reflection at the ends of the grating. In the Dartmouth set-up the system behaves like a BWO, with two Floquet waves proceeding in opposite directions. The backward wave has a component in resonance with the beam, and it displays an absolute instability, with growth both in time and in space. The forward wave shows no growth in space, but since it is fed by reflection at the upstream end, it does grow in time. We found this to be the case in our earliest simulations, with a very large current. In fact, more sophisticated analyses, such as those of references 6 and 9 point out that the boundary conditions at the grating ends play an important role in determining the start current. In our companion paper[10],we separate the two Floquet waves, and we see that the forward wave is essentially constant across the grating, while the backward wave grows strongly in the negative *x*-direction. However, this was for a current of 175 A/m, which reached saturation in about 1 ns. In order for the BWO mechanism to work, the gain must be sufficient to allow the ratio of the backward wave to forward wave be small at the downstream end and large at the upstream end. The greater the reflection the less is the gain needed to close the loop, and in principle the smaller the starting current.

In order to estimate the start current, we monitor the variation with time of the current at the middle of the grating. We filter the signal, and then estimate the gain by fitting a logarithmic plot of the summits. We also note the time required to reach saturation, t_{sat}, defined by the first maximum of the bunched current. Finally we note the ratio of the peak current at saturation to the direct current, I_{sat}/I. Once the details of the simulation have been fixed, such as mesh size, beam width, beam height above grating, kinetic energy, we then repeat the simulations for various currents. To our surprise, we obtained the curves shown below.

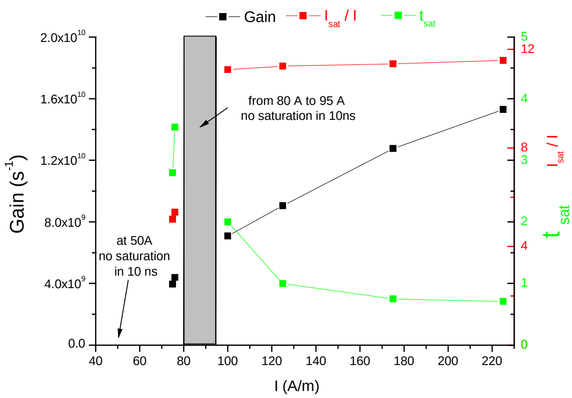

Figure 2. Gain, t_{sat} and I_{sat}/I *vs*. current *I*.

In the shaded region running from 80 to 95 A/m, and including three runs at 80, 90 and 95 A/m, no instability was observed, even for times of 10 ns. In contrast for currents > 100 A/m, the system always saturated in just a few ns. More astonishingly, at 75 and 76 A/m, the system reached saturation. However, at 50 A/m, it again failed. We are unable to understand these results, but clearly more work is needed. As additional evidence, we show the MAGIC phase space densities (kinetic energy-x) for the 76, 95 and 100 A/m runs.

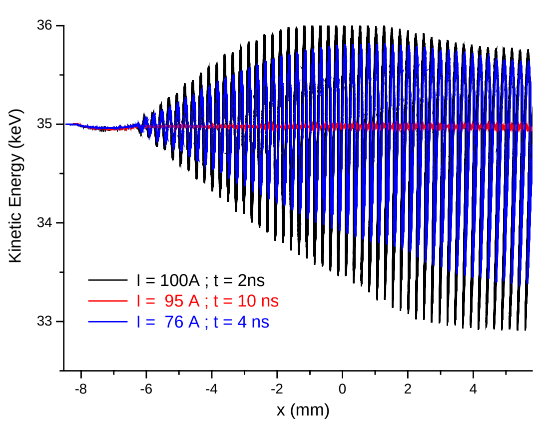

Figure 3. Phase space density plots in the Kinetic Energy-*x* plane, for 76 (blue), 95 red and 100 (black) A/m.

The oscillations correspond to an axial wave number of 260 cm^{-1}, which is what the AB theory suggests. For the 76 and 100 A/m runs, the average energy losses are roughly 0.5 and 0.8 keV, respectively, while the red curve shows neither oscillation nor energy loss. Again, we can only express our surprise at this result.

MIT SIMULATION

In our first simulation of the MIT experiment, we encountered a major difficulty in trying to identify the coherent S-P radiation that was produced. The beam consisted of short pulses (1 ps) produced by a linear accelerator functioning at 17.14 GHz. Consequently, the only frequencies allowed are integer multiples of that frequency. Since the beam energy was 15 MeV, and the grating period was 1 cm, it is straightforward to compute the frequencies predicted by the standard S-P relation. If we keep only the first six orders, and concentrate on angles in the forward direction, we find the curves shown in Figure (4). The harmonics are shown as horizontal lines, and each intersection of any of them with any of the curves marked "order" corresponds to coherent S-P radiation at the angle shown. Clearly there are a great number of possibilities, and for some angles like 49, 65 and 82 degrees, several frequencies may occur. The problem of observing S-P radiation is thus quite complex.

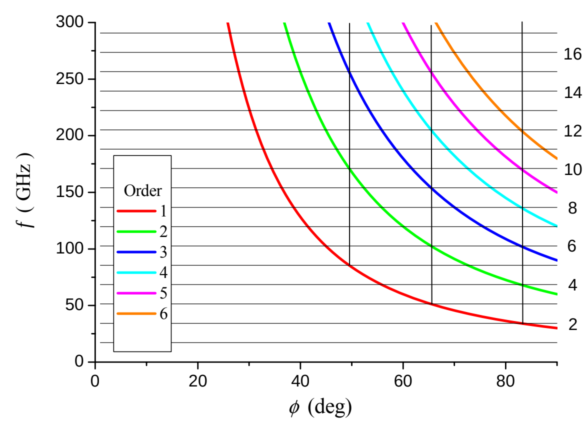

Figure 4. Intersections of the harmonics of 17.14 GHz with the first six S-P orders.

The situation was complicated further when we found that even if we removed the grating, we still observed substantial radiation. This was clearly not S-P radiation but it was present in our simulation. As we indicated in reference 4, the radiation we observed without grating may be calculated exactly using classical electrodynamics, at least for a sheet beam of infinitesimal width. We show in Figure (5) the results of such a calculation. The contour map of B_z in a 120 mm ×100 mm region of the x-y plane bears a strong resemblance to the results of our simulation without grating. A «plume» in which the magnetic field is mainly concentrated accompanies each bunch. The tiny red dots visible beneath each bunch indicate a large positive magnetic field between the beam and the ground plane. This component is an important contribution even when a grating is present, and no attempt to understand our simulation can avoid taking it into account.

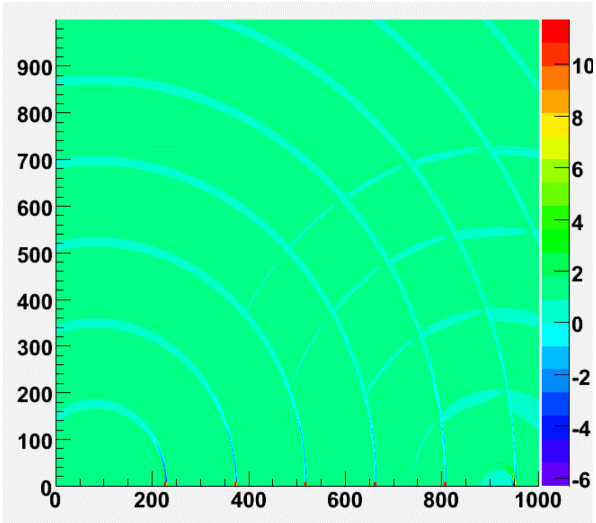

Figure 5. Contour map of B_z following the predictions of classical electrodynamics. The area shown corresponds to 12 cm in x, and 10 cm in the y direction.

In order to illustrate the difficulty of interpreting the simulation directly, we show in Figure (6) the Finite Fourier Transform (FFT) of $B_z(t)$ as observed directly in our simulation, which was made in a relatively small area, 12×6 cm^2. The observation point is at 5.5 cm from the center of the grating, and placed at 65°. The most prominent lines in the spectrum are the forth, sixth and eighth harmonics, but only the sixth is an S-P frequency at this angle.

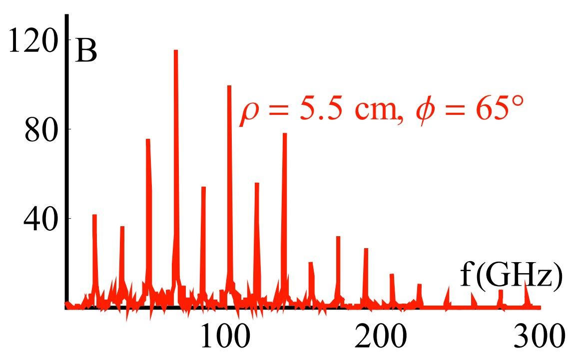

Figure 6. FFT of B_z at position indicated.

As outlined in reference 10, we have developed a procedure for extrapolating the results of our simulation to larger distances from the grating. Shown in Figure 7 is the result of applying our procedure to the data obtained in the small area simulation, after extrapolation to a larger distance from the center, but at the same angle.

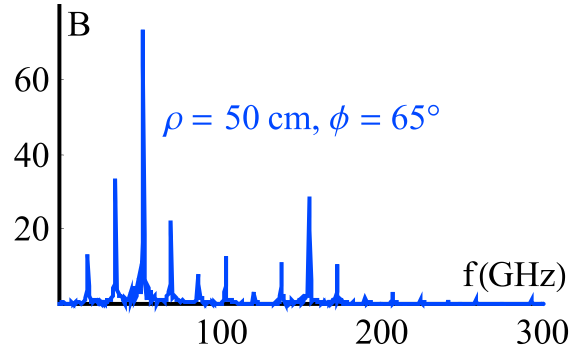

Figure 7. FFT of B_z following extrapolation. Third, sixth and ninth harmonics are more apparent.

It is clear that the relative importance of the third, sixth, ninth and twelfth harmonics, all of which are S-P allowed at this angle, has increased.

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