MODE COUPLINGS IN A RAMAN FREE-ELECTRON LASER WITH ION-CHANNEL GUIDING

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Abstract

The free-electron laser (FEL) theory in the collective or Raman regime relies on the unstable coupling between the radiation and the negative-energy space-charge wave. Due to the high density and low energy of electron beam a focusing mechanism like an axial magnetic field is usually required to guide the beam. It is found that the wiggler has direct effect on the right and left waves and the wiggler effect on their dispersion relations are of the second order in the wiggler amplitude. Due to the fully relativistic treatment the dispersion relation is to fourth order in wiggler amplitude and it can be used to study new couplings between the negative and positive-energy space-charge waves as well as between the right and left circularly polarized electromagnetic waves.

INTRODUCTION

Relativistic electron bean injected into an ionized plasma channel ejects plasma electrons leaving a positive ion core which attracts and confines the beam electron. There are important applications in this subject such as advanced accelerators [1] and free-electron lasers (FELs) [2,3]. Ion-channel guiding as an alternative to the conventional axial magnetic-filed guiding, was first proposed for use in FELs by Takayama and Hiramatsu [4]. Experimental results of a FEL with ion-channel guiding have been reported by Ozaki at al. [5] Also, Yu et al. [6] have reported that the combination of ion focusing and beam conditioning would lead to high gain FEL operation in the soft x-ray regime. Jha and Wurtele [7] developed a three-dimensional code for FEL simulation that allows for the effects of an ion channel. The theoretical studies of this problem with a helical wiggler are carried out in the low-gain [8] and high-gain [9,10] regimes. In Ref 10, the relativistic Raman backscattering theory is used to find the FEL dispersion relation with ion-channel guiding, in the beam frame of electrons, with the left circularly polarized backscattered wave neglected. This DR was used to find the growth rate of the FEL resonance due to the coupling of radiation with the slow space-charge wave.

The purpose of the present investigation is to obtain the dispersion relation (DR) for the interaction of all possible waves in a relativistic electron bean that passes through a one-dimensional helical wiggler magnetic field with ionchannel guiding. The motion of a relativistic electron

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through the wiggler is analyzed. Three coupled equations are derived and a formula for the general DR is obtained.

ELECTRON MOTION

Consider a relativistic electron moving along the z axis of an idealized helical wiggler magnetic field described by

$$\mathbf{B} = B_w (\hat{\mathbf{x}} \cos k_w z + \hat{\mathbf{y}} \sin k_w z), \tag{1}$$

where B_w denotes the wiggler amplitude, and $k_w (= 2\pi/\lambda_w)$ is wiggler wave number. In the presence of an ion channel, with its axis coincident with the wiggler axis, the following transverse electrostatic field is acted on the electron beam

$$\mathbf{E}_{i} = 2\pi e n_{i} \left(x \,\hat{\mathbf{x}} + y \,\hat{\mathbf{y}} \right), \tag{2}$$

where n_i is the number density of positive ions with charge *e*. The steady-state motion of an electron in the above field consists of an axis centred helical motion, with radius $R_0 = v_w / k_w v_{\parallel}$, given by Eq. (16) with [8,11]

$$v_{w} = \frac{\Omega_{w} k_{w} v_{\parallel}^{2}}{\omega_{i}^{2} - k_{w}^{2} v_{\parallel}^{2}},$$
(3)

where $\Omega_w \equiv eB_w / \gamma mc$, $\omega_i^2 = 2\pi n_i e^2 / \gamma m$, *m* is the electron rest mass, *e* is the magnitude of the charge of an electron, and *c* is the speed of light in vacuum. This velocity is related to the axial velocity v_{\parallel} through

$$\gamma_0^{-2} = 1 - \frac{v_w^2}{c^2} - \frac{v_{\parallel}^2}{c^2}.$$
(4)

Equation (4) is cubic in v_{\parallel}^2/c^2 and describes two classes of trajectories propagating along the positive *z* axis of the FEL.

DISPERSION RELATION

An analysis of the propagation of electromagnetic/electrostatic waves in the electron beam may be based on the continuity equation,

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0, \qquad (5)$$

the relativistic momentum equation

$$\frac{d\mathbf{v}}{dt} = -\frac{e}{\gamma m_0} \left(\mathbf{E} - \frac{1}{c^2} \mathbf{v} \, \mathbf{v} \cdot \mathbf{E} + \frac{1}{c} \, \mathbf{v} \times \mathbf{B} \right),\tag{6}$$

and the wave equation

$$\nabla \times \left(\nabla \times \mathbf{E}\right) + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{\partial}{\partial t} \frac{4\pi \, e \, n \, \mathbf{v}}{c^2}.$$
(7)

Here *n* is the electron density, **v** is the electron velocity, γ is the Lorentz factor corresponding to **v**, **E** is the electric field, and **B** is the magnetic field. With the unperturbed electron density n_0 taken to be independent of position and time and the self-fields of the unperturbed state neglected, the electron and field variables may be expressed in the form

$$n = n_0 + \delta n, \tag{8}$$

$$\mathbf{v} = \mathbf{v}_0 + \delta \mathbf{v},\tag{9}$$

$$\mathbf{E} = \mathbf{E}_i + \delta \mathbf{E},\tag{10}$$

$$\mathbf{B} = \mathbf{B}_{w} + \delta \mathbf{B},\tag{11}$$

$$\mathbf{R} = \mathbf{R}_0 + \delta \mathbf{R} \tag{12}$$

The linearized equations for the continuity equation, the relativistic momentum equation, and the wave equation may be derived as $\partial \delta n$

$$\frac{\partial \delta n}{\partial t} + n_0 \nabla \cdot \delta \mathbf{v} + \mathbf{v}_0 \cdot \nabla \delta n = 0, \qquad (13)$$

$$\frac{\partial \delta \mathbf{v}}{\partial t} + \mathbf{v}_0 \cdot \nabla \delta \mathbf{v} + \delta \mathbf{v} \cdot \nabla \mathbf{v}_0 = -\frac{e}{\gamma_0 m_0} \left[\delta \mathbf{E} - \frac{1}{c^2} \mathbf{v}_0 \mathbf{v}_0 \cdot \delta \mathbf{E} - \frac{1}{c^2} \mathbf{v}_0 \mathbf{v}_0 \cdot \delta \mathbf{E}_i + \frac{1}{c} \delta \mathbf{v} \times \mathbf{B}_w + \frac{1}{c} \mathbf{v}_0 \times \delta \mathbf{B} - \frac{\gamma_0^2}{c^2} \left(\mathbf{E}_i + \frac{1}{c} \mathbf{v}_0 \times \mathbf{B}_w - \frac{1}{c^2} \mathbf{v}_0 \mathbf{v}_0 \cdot \mathbf{E}_i \right) (\mathbf{v}_0 \cdot \delta \mathbf{v}) \right], \quad (14)$$

$$\nabla \times \left(\nabla \times \delta \mathbf{E}\right) + \frac{1}{c^2} \frac{\partial^2 \delta \mathbf{E}}{\partial t^2} = \frac{\partial}{\partial t} \frac{4\pi e}{c^2} \left(\delta n \mathbf{v}_0 + n_0 \,\delta \mathbf{v}\right). \tag{15}$$

By introducing a new set of basis vectors $\hat{\mathbf{e}} = (\hat{\mathbf{x}} + i\,\hat{\mathbf{y}})/\sqrt{2}$, $\hat{\mathbf{e}}^* = (\hat{\mathbf{x}} - i\,\hat{\mathbf{y}})/\sqrt{2}$, and $\hat{\mathbf{e}} = \hat{\mathbf{z}}$, the unperturbed magnetic field, electron density, and transverse electrostatic field can be written as

$$\mathbf{B}_{w} = \left(B_{w} / \sqrt{2} \right) \exp(-ik_{w}z) \hat{\mathbf{e}} + \left(B_{w} / \sqrt{2} \right) \exp(ik_{w}z) \hat{\mathbf{e}}^{*}, \quad (16)$$
$$\mathbf{v}_{0} = \left(v_{w} / \sqrt{2} \right) \exp(-ik_{w}z) \hat{\mathbf{e}} + \left(v_{w} / \sqrt{2} \right) \exp(ik_{w}z) \hat{\mathbf{e}}^{*} + v_{\parallel} \hat{\mathbf{e}}_{z}, \quad (17)$$

$$\mathbf{E}_{i} = i\sqrt{2} \pi e n_{i} R_{0} \left(\exp(-ik_{w}z)\hat{\mathbf{e}} + \exp(ik_{w}z)\hat{\mathbf{e}}^{*} \right), \quad (18)$$

The perturbed state is assumed to consist of a longitudinal space-charge wave and right and left circularly polarized electromagnetic waves, referred here as radiation, with all perturbed waves propagating in the positive z direction. Accordingly, solution of the system of equations (13)-(15) may be assumed as

$$\delta \mathbf{v} = \delta v_R \hat{\mathbf{e}} + \delta v_L \hat{\mathbf{e}}^* + \delta v_Z \hat{\mathbf{e}}_z , \qquad (19)$$

$$\delta \mathbf{E} = (2 \pi e n_i \, \delta R_R + \delta E_R) \hat{\mathbf{e}} + (2 \pi e n_i \, \delta R_L + \delta E_L) \hat{\mathbf{e}}^* + \delta E_Z \hat{\mathbf{e}}_z , \qquad (20)$$

$$\delta \mathbf{B} = \delta B_R \hat{\mathbf{e}} + \delta B_L \hat{\mathbf{e}}^*, \qquad (21)$$

$$\delta \mathbf{R} = \delta R_R \mathbf{e} + \delta R_L \mathbf{e} \tag{22}$$

$$\delta n = n \exp[i(kz - \omega t)], \qquad (23)$$

$$\delta v_R = v_R \exp[i(k_R z - \omega t)], \qquad (24)$$

$$\delta v_L = v_L \exp[i(k_L z - \omega t)], \qquad (25)$$

 δv_z and δE_z are analogous to δn ; δE_R , δR_R , and δB_R are analogous to δv_R ; δE_L , δR_L , and δB_L are analogous to δv_L ; the wave numbers are related to by

$$k_R = k - k_w, \tag{26}$$

$$k_L = k + k_w. (27)$$

The linearized wave equation yield

$$\left(k_R^2 c^2 - \omega^2\right) \widetilde{E}_R = -4\pi i e \omega \left(\widetilde{n} \frac{v_w}{\sqrt{2}} + n_0 \widetilde{v}_R\right), \qquad (28)$$

$$\left(k_L^2 c^2 - \omega^2\right) \widetilde{E}_L = -4\pi i e \omega \left(\widetilde{n} \frac{v_w}{\sqrt{2}} + n_0 \widetilde{v}_L\right), \qquad (29)$$

$$-\omega^{2} \widetilde{E}_{z} = -4\pi i e \omega \left(\widetilde{n} v_{\parallel} + n_{0} \widetilde{v}_{z} \right).$$
(30)

Similarly, the linearized continuity and momentum equations yield

$$\widetilde{n}(\omega - kv_{\parallel}) = n_0 k \widetilde{v}_z , \qquad (31)$$

$$\begin{bmatrix} \omega - k_{R}v_{\parallel} - \frac{\omega_{i}^{2}}{\omega - k_{R}v_{\parallel}} \left(1 - \frac{v_{w}^{2}}{2c^{2}} \right) - \frac{\omega_{i}^{2}}{k_{w}v_{\parallel}} \frac{v_{w}^{2}}{2c^{2}} + A_{1} \end{bmatrix} \widetilde{v}_{R} \\ + \left[\frac{\omega_{i}^{2}}{k_{w}v_{\parallel}} \frac{v_{w}^{2}}{2c^{2}} + \frac{\omega_{i}^{2}}{\omega - k_{L}v_{\parallel}} \frac{v_{w}^{2}}{2c^{2}} + A_{1} \right] \widetilde{v}_{L} + \frac{ie}{\gamma_{0}m_{0}} \left[\left(1 - \frac{v_{w}^{2}}{2c^{2}} \right) - \frac{k_{R}v_{\parallel}}{\omega} \right] \widetilde{E}_{R} - \frac{ie}{\gamma_{0}m_{0}} \frac{v_{w}^{2}}{2c^{2}} \widetilde{E}_{L} - \frac{ie}{\gamma_{0}m_{0}} \frac{v_{w}v_{\parallel}}{\sqrt{2c^{2}}} \widetilde{E}_{z} \\ + A_{2}\widetilde{v}_{z} = 0 , \qquad (32)$$

$$\begin{split} & \left[\omega - k_L v_{\parallel} - \frac{\omega_i^2}{\omega - k_L v_{\parallel}} \left(1 - \frac{v_w^2}{2c^2} \right) + \frac{\omega_i^2}{k_w v_{\parallel}} \frac{v_w^2}{2c^2} - A_1 \right] \widetilde{v}_L \\ & + \left[- \frac{\omega_i^2}{k_w v_{\parallel}} \frac{v_w^2}{2c^2} + \frac{\omega_i^2}{\omega - k_R v_{\parallel}} \frac{v_w^2}{2c^2} - A_1 \right] \widetilde{v}_R + \frac{ie}{\gamma_0 m_0} \left[\left(1 - \frac{v_w^2}{2c^2} \right) - \frac{k_L v_{\parallel}}{\omega} \right] \widetilde{E}_L - \frac{ie}{\gamma_0 m_0} \frac{v_w^2}{2c^2} \widetilde{E}_R - \frac{ie}{\gamma_0 m_0} \frac{v_w v_{\parallel}}{\sqrt{2c^2}} \widetilde{E}_z \\ - A_2 \widetilde{v}_z = 0 \,, \end{split}$$
(33)

$$-i\left(\omega-kv_{\parallel}\right)\widetilde{v}_{z} + \frac{e}{\gamma_{0}m_{0}}\left(1-\frac{v_{\parallel}^{2}}{c^{2}}\right)\widetilde{E}_{z} + \frac{e}{\gamma_{0}m_{0}}\left[-\frac{v_{w}v_{\parallel}}{\sqrt{2}c^{2}}\right]$$
$$+ \frac{k_{R}c}{\omega}\frac{v_{w}}{\sqrt{2}c}\widetilde{E}_{R} + \frac{e}{\gamma_{0}m_{0}}\left[-\frac{v_{w}v_{\parallel}}{\sqrt{2}c^{2}} + \frac{k_{L}c}{\omega}\frac{v_{w}}{\sqrt{2}c}\right]\widetilde{E}_{L}$$
$$+ i\left[\left(\frac{\omega_{i}^{2}}{k_{w}v_{\parallel}} - \frac{\omega_{i}^{2}}{\omega-k_{R}v_{\parallel}}\right)\frac{v_{w}v_{\parallel}}{\sqrt{2}c^{2}} - \frac{\Omega_{w}}{\sqrt{2}}\right]\widetilde{v}_{R}$$
$$+ i\left[\left(-\frac{\omega_{i}^{2}}{k_{w}v_{\parallel}} - \frac{\omega_{i}^{2}}{\omega-k_{L}v_{\parallel}}\right)\frac{v_{w}v_{\parallel}}{\sqrt{2}c^{2}} + \frac{\Omega_{w}}{\sqrt{2}}\right]\widetilde{v}_{L} = 0, \qquad (34)$$
where

$$\gamma_{\parallel}^{-2} = 1 - \frac{v_{\parallel}^2}{c^2} \,, \tag{35}$$

$$A_{L} = k_{L}^{2}c^{2} - \omega^{2}, \qquad (36)$$

$$A_{R} = k_{R}^{2}c^{2} - \omega^{2}. \qquad (37)$$

$$A_{1} = \left(\frac{1}{2}\right) \gamma_{0}^{2} \frac{v_{w}}{c} \left(\frac{v_{w}}{c} \frac{\omega_{i}^{2}}{k_{w} v_{\parallel}} - \frac{v_{\parallel}}{c} \Omega_{w}\right), \qquad (38)$$

$$A_{2} = \left(\frac{1}{\sqrt{2}}\right) \left(k_{w}v_{w} + \Omega_{w}\right) + \left(\frac{1}{\sqrt{2}}\right) \gamma_{0}^{2} \frac{v_{\parallel}}{c} \left(\frac{v_{w}}{c} \frac{\omega_{i}^{2}}{k_{w}v_{\parallel}} - \frac{v_{\parallel}}{c}\Omega_{w}\right).$$

$$(39)$$

The electron density \tilde{n} may be eliminated in (30) by use of (29) to obtain

$$\widetilde{v}_z = -i \frac{\omega - k v_{\parallel}}{4\pi \, e n_0} \widetilde{E}_z \,. \tag{40}$$

with the use of Eqs. (28)-(31) and (40), \tilde{v}_R , \tilde{v}_L and \tilde{v}_z may be eliminated in the three components of the momentum equation (32)-(34) to obtain

$$\begin{split} & \left[D_{R}^{0} + \psi_{R}^{+} v_{w}^{2} / c^{2} \right] \widetilde{E}_{R} + \left[\psi_{L}^{-} v_{w}^{2} / c^{2} \right] \widetilde{E}_{L} + \left[\xi_{R1} v_{w} / c \right. \\ & \left. + M^{+} v_{w}^{3} / c^{3} \right] \widetilde{E}_{z} = 0 , \qquad (41) \\ & \psi_{R}^{-} \left(v_{w}^{2} / c^{2} \right) \widetilde{E}_{R} + \left[D_{L}^{0} + \psi_{L}^{+} v_{w}^{2} / c^{2} \right] \widetilde{E}_{L} + \left[\xi_{L1} v_{w} / c \right. \\ & \left. + M^{-} v_{w}^{3} / c^{3} \right] \widetilde{E}_{z} = 0 , \qquad (42) \end{split}$$

$$\begin{aligned} \xi_{R2} \left(\frac{v_w}{c} \right) \widetilde{E}_R + \xi_{L2} \left(\frac{v_w}{c} \right) \widetilde{E}_L - \omega \left[\varepsilon^0 + k v_{\parallel} \left(\frac{\omega_i^2}{\omega - k_L v_{\parallel}} + \frac{\omega_i^2}{\omega - k_R v_{\parallel}} \right) \frac{v_w^2}{2c^2} \right] \widetilde{E}_z = 0, \end{aligned}$$
(43)

where D_R^0 , D_L^0 , ψ_R^{\pm} , ψ_L^{\pm} , M^{\pm} , ξ_{R1} , ξ_{R2} , ξ_{L1} , ξ_{L2} , ε^0 are defined in the appendix. Here, D_R^0 , D_L^0 , and ε^0 are the uncoupled dispersion relations, i.e., in the absence of the wiggler, for the right and left circularly polarized electromagnetic waves, and the space-charge wave, respectively. Equations (41) and (42) show that the DR for the right and left waves, alone, in the absence of the other two waves are

$$D_R = D_R^0 + \psi_R^+ (v_w^2/c^2) = 0, \qquad (44)$$

$$D_L = D_L^0 + \psi_L^+ (v_w^2 / c^2) = 0, \qquad (45)$$

which indicate that the wiggler has direct effect on the right and left waves and the wiggler effect on their DRs are of the second order in the wiggler amplitude. On the other hand, Eq. (43) Shows that the DR for the space-charge wave in the absence of the right and left wave is $\varepsilon^0 = 0$, which indicates that the wiggler has no direct effect on the space-charge wave. The reason is that the transverse helical motion of electrons, due to the wiggler, has no effect on the longitudinal oscillations of the space-charge wave. Therefore, if the electromagnetic waves are removed the wiggler effect on the space-charge wave will

also be removed and the space-charge wave will be unaffected by the wiggler in the absence of the transverse electromagnetic waves.

The necessary and sufficient condition for a nontrivial solution consists of the determinant of coefficients in Eqs. (41)-(43) equated to zero. Imposing this condition yields the dispersion relation

$$\begin{split} \omega \Bigg[\varepsilon^{0} + \omega k v_{\parallel} \Bigg(\frac{\omega_{i}^{2}}{\omega - k_{L} v_{\parallel}} + \frac{\omega_{i}^{2}}{\omega - k_{R} v_{\parallel}} \Bigg) \Bigg] D_{R} D_{L} &= - \Bigg[D_{R} \xi_{L2} \\ \left(\xi_{L1} + M^{-} \frac{v_{w}^{2}}{c^{2}} \right) + D_{L} \xi_{R2} \Bigg(\xi_{R1} + M^{+} \frac{v_{w}^{2}}{c^{2}} \Bigg) \Bigg] \frac{v_{w}^{2}}{c^{2}} + \Bigg[\psi_{L}^{-} \xi_{R2} \\ \left(\xi_{L1} + M^{-} \frac{v_{w}^{2}}{c^{2}} \right) + \psi_{R}^{-} \xi_{L2} \Bigg(\xi_{R1} + M^{+} \frac{v_{w}^{2}}{c^{2}} \Bigg) + \omega \varepsilon^{0} \psi_{L}^{-} \psi_{R}^{-} \\ &+ \psi_{L}^{-} \psi_{R}^{-} \omega k v_{\parallel} \Bigg(\frac{\omega_{i}^{2}}{\omega - k_{L} v_{\parallel}} + \frac{\omega_{i}^{2}}{\omega - k_{R} v_{\parallel}} \Bigg) \Bigg] \frac{v_{w}^{4}}{c^{4}} . \end{split}$$
(46)

Equation (46) is the DR for coupled electrostatic and electromagnetic waves propagating along a relativistic electron beam in the presence of a wiggler magnetic field and an axial guide magnetic field. A numerical analysis of the general dispersion relation can be used to study interactions among all possible waves. In group II orbits, with relatively large wiggler induced velocities, new couplings between the negative and positive-energy space-charge waves as well as between the right and left circularly polarized electromagnetic waves are expected to be found. These instabilities are distinct from the usual FEL resonance.

APPENDIX: DEFINITION OF QUANTITIES

The following quantities are used in equations (41)-(43)

$$\begin{split} D_R^0 &= \left(\omega - k_R v_{\parallel} - \frac{\omega_i^2}{\omega - k_R v_{\parallel}} \right) A_R + \omega_p^2 (\omega - k_R v_{\parallel}) ,\\ D_L^0 &= \left(\omega - k_L v_{\parallel} - \frac{\omega_i^2}{\omega - k_L v_{\parallel}} \right) A_L + \omega_p^2 (\omega - k_L v_{\parallel}) ,\\ \psi_R^\pm &= \left(\frac{1}{2} \right) \left[\left(\frac{\omega_i^2}{\omega - k_R v_{\parallel}} - \frac{\omega_i^2}{k_w v_{\parallel}} \pm \gamma_0^2 k_w v_{\parallel} \right) A_R - \omega \omega_p^2 \right] ,\\ \psi_L^\pm &= \left(\frac{1}{2} \right) \left[\left(\frac{\omega_i^2}{\omega - k_L v_{\parallel}} + \frac{\omega_i^2}{k_w v_{\parallel}} \frac{\omega_i^2}{\omega - k_L v_{\parallel}} \mp \gamma_0^2 k_w v_{\parallel} \right) A_L \right. \\ &- \omega \omega_p^2 \right] \\ M^\pm &= \left(\frac{\omega k c}{2\sqrt{2}} \right) \left(\frac{\omega_i^2}{\omega - k_R v_{\parallel}} + \frac{\omega_i^2}{\omega - k_L v_{\parallel}} \pm 2 \gamma_0^2 k_w v_{\parallel} \right) ,\\ \xi_{R1} &= \left(\frac{\omega}{\sqrt{2}} \right) \left[k c \left(\omega - k_R v_{\parallel} - \frac{\omega_i^2}{\omega - k_R v_{\parallel}} \right) - \omega_p^2 \frac{v_{\parallel}}{c} \end{split}$$

$$\begin{split} &-\left(\frac{\omega_i^2 c}{k_w v_{\parallel}^2} + \frac{\gamma_0^2 k_w v_{\parallel}}{c}\right) (\omega - k v_{\parallel}) \right], \\ \xi_{L1} = \left(\frac{\omega}{\sqrt{2}}\right) \left[k c \left(\omega - k_L v_{\parallel} - \frac{\omega_i^2}{\omega - k_L v_{\parallel}}\right) - \omega_p^2 \frac{v_{\parallel}}{c} \\ &+ \left(\frac{\omega_i^2 c}{k_w v_{\parallel}^2} + \frac{\gamma_0^2 k_w v_{\parallel}}{c}\right) (\omega - k v_{\parallel}) \right], \\ \xi_{R2} = \left(\frac{1}{\sqrt{2}}\right) \left[- A_R \left(\frac{\omega_i^2}{k_w c} - \frac{\omega_i^2 c}{k_w v_{\parallel}^2} + k_w c - \frac{\omega_i^2}{\omega - k_R v_{\parallel}}\right) \\ &+ \omega_p^2 (k_R c - \omega v_{\parallel}/c) \right], \\ \xi_{L2} = \left(\frac{1}{2}\right) \left[- A_L \left(-\frac{\omega_i^2}{k_w c} + \frac{\omega_i^2 c}{k_w v_{\parallel}^2} - k_w c - \frac{\omega_i^2}{\omega - k_L v_{\parallel}}\right) \\ &+ \omega_p^2 (k_L c - \omega v_{\parallel}/c) \right], \\ \varepsilon^0 = (\omega - k v_{\parallel})^2 - \omega_p^2 / \gamma_{\parallel}^2. \end{split}$$

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