ENHANCEMENT OF A COHERENT (SUPER-RADIANT) EMISSION IN FEL BY MEANS OF ENERGY MODULATION OF AN EMITTING SHORT ELECTRON BUNCH

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Abstract

Super-radiant emission from a wiggling short electron bunch, is strongly suppressed at high frequencies in comparison with radiation, obtained in a FEL if an ultra-short e-beam pulse of the same total charge is available. However, radiation intensity of a wiggling short electron bunch can be greatly enhanced by means of energy modulation. In this way, a super-radiant FEL source driven by short electron bunches and operating in the Tera-Hertz regime can be realized. Analytical evaluations and numerical simulations utilizing a space-frequency 3D model show, that a linear energy modulation enables one to increase the power of super-radiant emission by few orders of magnitude. Possible limitations in application of this method are also discussed, as well as a spectral purity of enhanced radiation.

INTRODUCTION

Development of experimental set-ups utilizing mechanism of short bunching of relativistic electron beams enables construction of high-power free-electron lasers (FELs) operated by a single short pulse or by a train of such pulses. Optimal efficiency of such radiation sources may be achieved with ultra-short beam pulses, when a super-radiant (SR) emission occurs (see [1, 2] and references therein).

Unfortunately, a beam pulse duration obtained in practice is still relatively long, leading to a dramatic reduction in SR emission at high frequencies. For example, in a case of Gaussian electron pulse, the super-radiant to spontaneous emission ratio at a synchronism frequency f_s is decreases with the beam duration as fast as the Gauss function [1]:

$$\text{SNR} \equiv \frac{dP_q^{sr}(z)/df}{dP_q^{sp}(z)/df} = \overline{k} \cdot e^{-(2\pi f_s T)^2} \tag{1}$$

(here $\overline{k}$ is a number of electrons in the bunch and T is the standard deviation of the electron Gauss distribution, which corresponds to the bunch duration $\Delta T \approx 2T$). It means that to provide an intense high frequency SR emission, ultra-short electron bunches of time duration $\Delta T << 1/f_s$ have only to be applied. For example, with ultra-short femtoseconds bunches, intense SR emission may be obtained at radiation frequencies up to approximately 100 THz only.

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It was suggested that emission of a short electron bunch of some finite duration may be greatly enhanced by means of its proper energy modulation [3]. In this work, the effect of energy modulation on SR emission of a wiggling short electron bunch is analyzed in a space-frequency 3D approach and demonstrated by numerical simulations carried out using WB3D code [4].

ANALYTICAL EVALUATIONS

The total electromagnetic field emitted by a wiggling electron bunch may be found in the frequency domain in terms of expansion over transverse eigenmodes of the medium (free-space or waveguide) in which the radiation is excited and propagates [5]:

$$\vec{E}(\vec{r},t) = \sum_q \Re\left\{ \int_0^\infty \widetilde{C}_q(z,f)\, \widetilde{\mathcal{E}}_q(x,y)\, e^{+jk_{zq}(f)z}\, df \right\} \tag{2}$$

where $\widetilde{\mathcal{E}}_q(x,y)$ is the transverse profile (Hermitte-Gaussian free-space mode or waveguide mode) of the mode q and $k_{zq}(f)$ is its wavenumber. (Although the form of mode presentation given in Eq. (2) is not valid in the far-field free-space propagation, it is still applicable to most electron devices in which the interaction takes place within a Rayliegh length of the Hermitte-Gaussian modes, where the diffraction is small). $\widetilde{C}_q(z,f)$ is the propagating mode amplitude, which may be found from the excitation equation (see [4, 5] for details).

Neglecting the influence of the emitted electromagnetic field on the emitting electrons, the mode expansion coefficients $\widetilde{C}_q(z,f)$ of electromagnetic field emitted by *a single wiggling electron* may be found in the following simple analytical form:

$$\widetilde{C}_q(L_w,f) = \mathcal{A}_q \,\text{sinc}\left(\tfrac{1}{2}\,\theta_q\, L_w\right)\; e^{j\left(2\pi f t_0 + \frac{1}{2}\theta_q L_w\right)} \tag{3}$$

where $\mathcal{A}_q$ is some normalization coefficient, sinc$(\alpha) \equiv \sin(\alpha)/\alpha$, t_0 is the time that the electron enters into the interaction region, L_w is the length of the interaction region, and

$$\theta_q \equiv \frac{2\pi f}{v_z} - (k_{zq} + k_w) \tag{4}$$

is the detuning parameter (k_w is the wiggler wave number and v_z is the electron longitudinal velocity in z-direction).

In the low-gain regime, electromagnetic field emitted by a bunch of N electrons may be given in the terms of a summation of (3) over all the electrons in the bunch:

$$\widetilde{C}_q(L_w, f) = \mathcal{A}_q \sum_{p=1}^{N} \text{sinc}\left(\frac{1}{2}\,\theta_{q_p}\,L_w\right)\, e^{j\left(2\pi f t_{0_p} + \frac{1}{2}\theta_{q_p} L_w\right)} \tag{5}$$

here the coefficient $\mathcal{A}_q$ is supposed to be approximately a constant for all the electrons in the bunch. At some synchronism frequency f_s, a coherent summation in (5) may only be obtained when sinc $\left(\frac{1}{2}\,\theta_{q_p}\,L_w\right) \approx 1$ (or $\theta_{q_p} \approx 0$, what defines the synchronism frequency f_s), and the phase matching condition $2\pi f t_{0_p} + \frac{1}{2}\theta_{q_p} L_w \approx 0$ is satisfied. A SR emission from a single ultra-short ($\Delta T << 1/f_s$) bunch takes place in this situation, when the radiated field is simply proportional to a number of the electrons in the bunch (to the total charge of the driving bunch): $\widetilde{C}_q \approx \mathcal{A}_q \cdot N$, and energy flux spectral density of the emitted radiation may be given as

$$\frac{dW}{df} \sim \left|\widetilde{C}_q\right|^2 \sim \mathcal{A}_q^2 \cdot N^2$$

As mentioned above, this energy flux is drastically reduced if the driving bunch duration is increasing compared with period $1/f_s$ of the emitted radiation. But also in this case of "finite-duration" beam bunches, the summation in (5) may still remain a partly coherent in the vicinity of synchronism frequency f_s, if the previous conditions are fulfilled in a bit modified form:

- The most part of sinc-functions in (5) save their sign: sinc $\left(\frac{1}{2}\,\theta_{q_p}\,L_w\right) > 0$, to prevent the terms compensation in the summation; and
- The phase shifts $2\pi f_s t_{0_p} + \frac{1}{2}\theta_{q_p} L_w$ remains constant.

The first condition means, that:

$$\left|\frac{2\pi f_s}{v_{z_p}} - \left(k_{z_q} + k_w\right)\right| \lesssim \frac{2\pi}{L_w}$$

Introducing

$$k_{z_q} + k_w \equiv \frac{2\pi f_s}{\overline{\beta_z}\ c} \tag{6}$$

where $\overline{\beta_z}\ c$ is some bunch mean longitudinal velocity in z-direction, the previous equation may be rewritten as:

$$\left|\frac{1}{\beta_{z_p}} - \frac{1}{\overline{\beta_z}}\right| < \frac{\lambda_s}{L_w} \tag{7}$$

here λ_s is the radiation wave length and $\beta_{z_p} = v_{z_p}/c$. The phase-matching condition may only be satisfied, if the driving electron bunch is *pre-modulated* so that the longitudinal velocities of the electrons are distributed as follows:

$$\beta_z(t) = \overline{\beta_z}\,\frac{L_w}{L_w - 2\overline{\beta_z}ct} \tag{8}$$

where we assume that $\beta_z(t=0) = \overline{\beta_z}$. Substitution of (8) into (7) provides the following limitation on the maximal bunch duration:

$$|t| < \frac{1}{2f_s} \tag{9}$$

Therefore the considered enhancement of SR emission may be obtained with as long as $\Delta T \approx 1/f_s$ beam bunches.

Condition (8) actually requires an *energy modulation* of the driving electron bunch as follows:

$$E_k(t) = m_0 c^2 \left\{\sqrt{\frac{1+a_w^2}{1-\overline{\beta_z}^2/\left(1-2\overline{\beta_z}\,c\,t/L_w\right)^2}} - 1\right\} \tag{10}$$

here $a_w = eB_w/(k_w m_e c)$ is the wiggler parameter. Because

$$\left(\frac{2\overline{\beta_z}c}{L_w}\right)|t| \ < \ \left(\frac{2\overline{\beta_z}c}{L_w}\right)\frac{1}{2f_s} = \overline{\beta_z}\frac{\lambda_s}{L_w} \ << 1$$

the distribution (10) may be linearized:

$$E_k(t) \approx E_k(t=0) + \left(\frac{dE_k}{dt}\right)\bigg|_{t=0} \cdot t \tag{11}$$

where

$$E_k(t=0) \equiv m_e c^2 \left\{\sqrt{\frac{1+a_w^2}{1-\overline{\beta_z}^2}} - 1\right\} \tag{12}$$

and the modulation rate is

$$\left(\frac{dE_k}{dt}\right)\bigg|_{t=0} \approx m_e c^2 \left(\frac{2\overline{\gamma}\,\overline{\beta_z}^3}{1-\overline{\beta_z}^2}\right)\frac{c}{L_w} \tag{13}$$

here

$$\overline{\gamma} = \frac{m_e c^2 + E_k(t=0)}{m_e c^2} = \sqrt{1 - \frac{1+a_w^2}{\overline{\beta_z}^2}} \tag{14}$$

So, energy modulation of a wiggling electron bunch according (10) or (11) may considerably enhance its SR emission, even if the driving bunch is as long as $\Delta T \approx 1/f_s$.

Obviously, energy modulation of the beam reduces the spectral purity of the emitted radiation in comparison with that of SR emission, emitted by an unmodulated ultra-short electron bunch. The reason is that the synchronism frequencies of radiation emitted by individual electrons are different due to difference in there kinetic energies. Therefore a bandwidth of radiation, emitted by energy modulated bunch is supposed to be more wide by a factor of $\Delta f \approx |f_s(+T) - f_s(-T)|$, so that the radiation band width may be evaluated by

$$BW \approx \frac{f_s}{N_w} + \underbrace{\frac{df_s}{dE_k}\left(\frac{dE_k}{dt}\right)\Delta T}_{\Delta f} \tag{15}$$

revealing a linear growth with the energy chirp rate dE_k/dt.

NUMERICAL SIMULATIONS

To demonstrate the effect, a number of numerical simulations with the code WB3D [4] were carried out for a THz-regime free-electron laser with operational parameters given in the table 1. Simulations demonstrate, that extremely short electron bunches ($f_s\Delta T \lesssim 2\%$) have to be applied in order to produce a strong super-radiant emission. With such ultrashort bunches, the total energy flux of super-radiant emission may be as large as $W_{SR} = 22.5$ nJ. With a longer unmodulated bunches, the total energy flux of emitted radiation is drastically reduced and it is just about $W = 4.9$ pJ when $\Delta T \approx 1.0$ pSec unmodulated bunches are applied. In accordance with analytical evaluations, a linear energy modulation (11) of the electron bunch enables one to enhance this radiation by more than a three orders of magnitude, as demonstrated in the figure 1. It is evident, that the energy modulation causes the emitted total energy flux to increase rapidly, saturating at the energy chirp rate dE_k/dt of about 0.12 MeV/pSec (resulting in the energy flux of $W_{SR} \approx 9$ nJ) and slowly reducing for a higher values of the energy chirp rates. Note, that equation (13) provides the value of $dE_k/dt \approx 0.17$ MeV/pSec. With a longer bunches ($\Delta T \approx 2.0$ pSec, $f_sT \approx 1.0$), the obtained effect is even more strong (circle symbols at the picture).

Energy spectrums of the radiation emitted by energy modulated ΔT=1.0 pSec bunches are given in the figure 2, comparing to that emitted by an unmodulated ultrashort bunch (solid line) and by unmodulated ΔT=1.0 pSec bunch. A spectral purity of the emitted radiation is obviously reduced if the driving electron bunch is energy modulated. At high energy chirp rates, the radiation band width grows linearly with dE_k/dt, as demonstrated at the figure 3, in accordance with above evaluation (15). It's interesting to note some non-linear dependence near the "optimal" value of the energy chirp rate dE_k/dt=0.12 MeV/pSec, when a high-power radiation is emitted.

To explain the effect, a microscopic analysis of trajectories of emitting electrons was carried out (see figure 4). As easily seen from the picture, a non-modulated bunch propagates through the undulator saving its initial temporal duration. Initial energy modulation of the driving bunch develops while propagating in the wiggler to a density compression and the e-beam pulse becomes an ultra-short one, generating a strong super-radiant emission. At this moment, the most intensive radiation is emitted, as may be seen from the time-domain dependence of electric component of the emitted field given in figure 5.

Table 1: Operational parameters for THz FEL.

Accelerator	
Electron beam energy:	E_k=2.8 MeV
Total charge:	$Q = I_0 \cdot \Delta T = 10$ pCl
Wiggler	
Magnetic induction:	B_w=3 kGauss ($a_w \approx 0.56$)
Period:	λ_w=20 mm
Number of periods:	N_w=20
Waveguide	
Rectangular waveguide:	5×5 mm

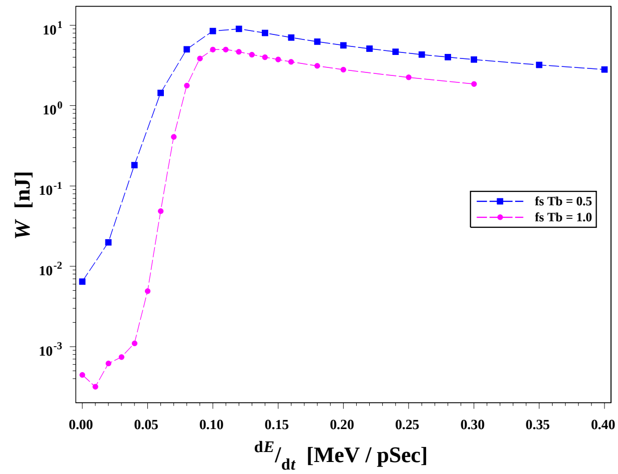

Figure 1: Total energy flux emitted by $\Delta T = 1.0$ pSec (box symbols) and $\Delta T = 2.0$ pSec (circle symbols) bunches as a function of the energy chirp rate dE_k/dt.

CONCLUSIONS

In a practical case of FEL driven by a single short bunch or by a train of such bunches, a super-radiant emission is strongly suppressed at high frequencies. It was shown that it can be greatly enhanced by means of a proper energy modulation of the driving beam. The effect is studied in the framework of space-frequency model and is explained in a

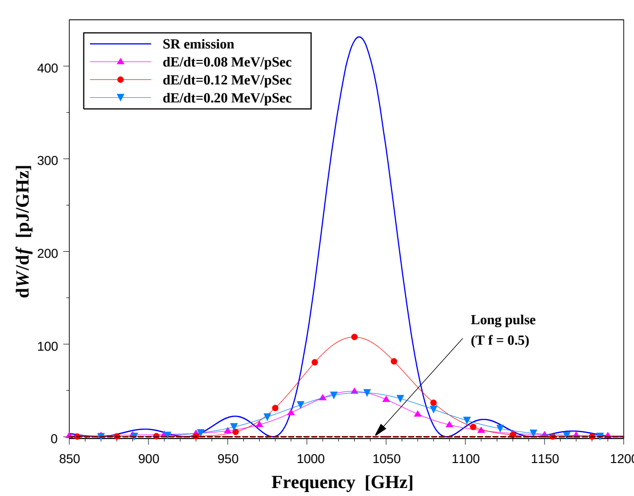

Figure 2: Total energy flux emitted by $\Delta T = 1.0$ pSec bunch as a function of the energy chirp rate dE_k/dt.

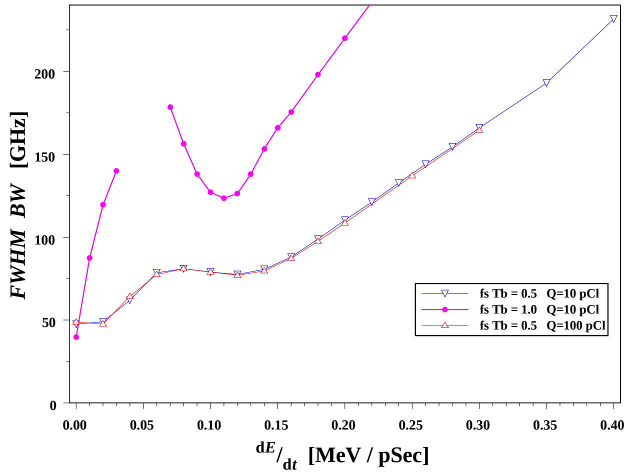

Figure 3: Full-width half-maximum radiation bandwidth as function of the energy chirp rate dE_k/dt.

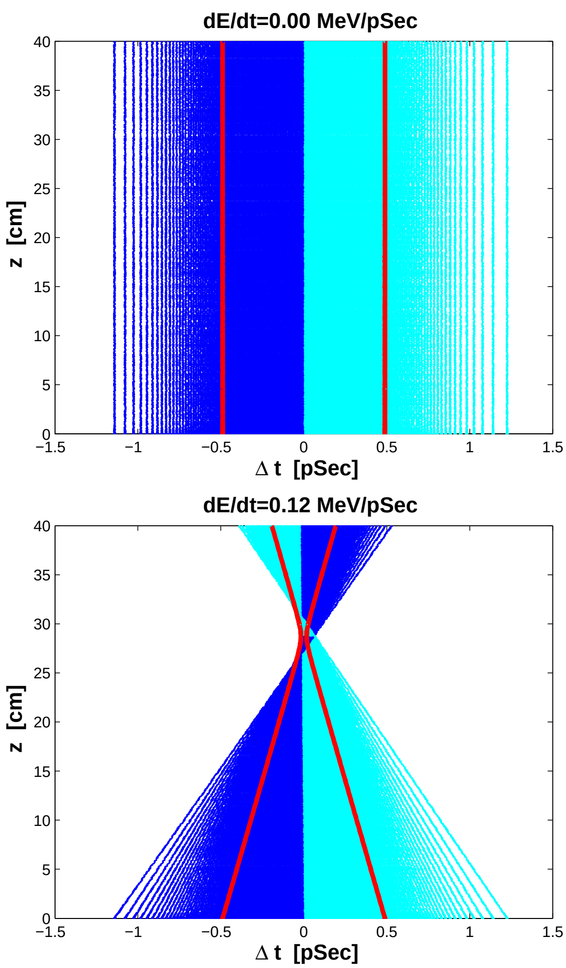

Figure 4: Trajectories of the emitting electrons in unmodulated (top) and energy modulated with dE_k/dt=0.12 MeV/pSec (bottom) bunches.

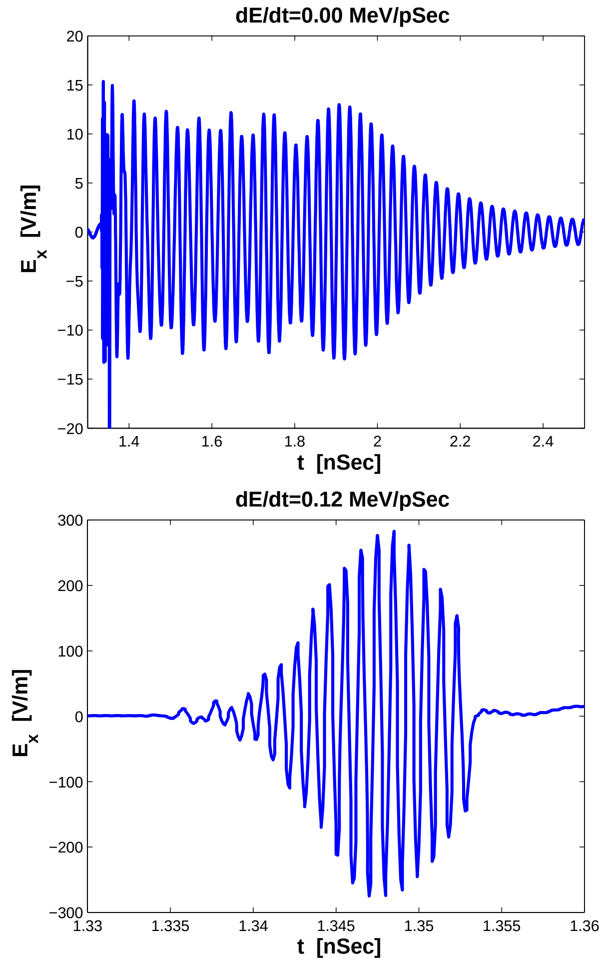

Figure 5: Time-domain electromagnetic field, emitted by unmodulated (top) and by energy modulated (dE_k/dt=0.12 MeV/pSec, bottom) bunches.

simple analytical approach as well as by numerical simulations with WB3D code. The considered principal scheme may be realized in a construction of a pulsed FEL, which enables to provide intensive high-frequency radiation with a reasonable spectral quality.

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