# TWO-STREAM SMITH-PURCELL FREE-ELECTRON LASER USING A DUAL-GRATING: LINEAR ANALYSIS*

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*Abstract*

A linear theory of two-stream Smith-Purcell Free-Electron Laser (SP-FEL) using a dual-grating has been given in this paper. A rectangular dual-grating is considered to be driven by two admixed electron beams with velocity separation. The linear dispersion equations for even and odd modes are derived with the help of fluid theory and the beam-wave interaction is analyzed through the numerical solutions. The considerable enhancement of growth rate is demonstrated due to the presence of two-stream instability. An example of THz-band two-stream SP-FEL is discussed.

## INTROUCTION

It is well known that Smith-Purcell (SP) radiation is emitted when an electron passes near the surface of grating [1].At present, an intense interest has been raised in SP radiation since J.Urata *et al* observed the super-radiance in the THz regime from the experiment at Dartmouth college [2,3]. The superradiance is regarded as the result of periodic electron bunching, which is produced by the interaction between the electron beam and the fields above the grating. Some theories [4,5] have been proposed to understand the physical mechanism of beam-wave interaction. Recently, D.Li and Z.Yang [6,7], J.T.Donohue and J.Gardelle[8] have performed the simulations of SP-FEL in the THz regime with PIC code.

The THz sources, a currently flourish research area, are of importance in varieties of applications in far-infrared spectroscopy, imaging, ranging and biomedical [9]. In order to improve the performance of such kind of device, it is necessary to find an efficient mechanism for the beam-wave interaction. The two-stream instability is an important physical mechanism, which has been successfully applied to some high power microwave sources [10-12].

In this paper, we present a two-stream Smith-Purcell Free-Electron Laser using a dual-grating, as shown in Fig.1. The two admixed electron beams are symmetric about $z$ axial and guided by an infinite magnetic field. For simplicity, we assume that the system is uniform in the $y$ direction which is parallel to the slots of grating.

## DISPERSION EQUATION

The system of beam-wave interaction is illustrated in Fig.1. Two admixed beams with thickness $2b$ are located above the rectangular grating, which drift in the $z$ direction at a constant velocity $v_1$ and $v_2$, respectively. The quantities *D, H, d, a* denote the rectangular grating period, groove depth, slot width, the distance between $z$ axial and grating surface, respectively. For convenience, the operation area is divided into five regions: $-a \le x < -b$ is region Ⅰ, $b < x \le a$ is region Ⅱ, $-b \le x \le b$ is region Ⅲ, Region Ⅳ and Ⅴ is the lower groove $-a-h \le x < -a$ and upper groove $a < x \le a+h$. In the following analyses, we focus on the TM waves.

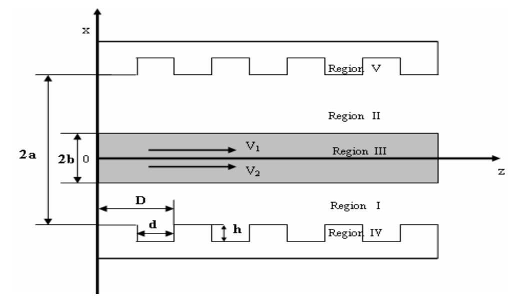


**Fig.1** Schematic illustration of two admixed beams propagating over the dual-grating

We assume that (i)the externally applied magnetic field is so strong that perturbed electron motions are restricted in the $z$ direction;（ii)the effect of beams self static fields are negligible; and (iii)the perturbations are uniform along the $y$ direction. Maxwell's equations and relativistic hydrodynamic equations for two cold electron beams are employed to describe this system as follows:

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \quad (1)$$

$$\nabla \times \vec{B} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\sum_{i=1}^{2} J_i \quad (2)$$

$$(\frac{\partial}{\partial t} + v_{i0}\frac{\partial}{\partial z})\delta v_{i1} = -\frac{e}{\gamma_i^3 m_0} E_z \quad (3)$$

$$(\frac{\partial}{\partial t} + v_{i0}\frac{\partial}{\partial z})\delta n_{i1} = -n_{i0}\frac{\partial}{\partial z}\delta v_{i1} \quad (4)$$

Here, subscript $i = 1,2$ represents the first and second electron beam, respectively. $\vec{E}$ and $\vec{B}$ are the electric and

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magnetic field vectors. $\gamma_i = 1 + eV_i / m_0 c^2$ is the relativistic mass factor, $c$ is the speed of light ,$V_i$ is the $i$th beam voltage, the voltage ratio of two electron beams is $v_r = V_2 / V_1$ ,and $v_{i0} = c \cdot [(\gamma_i^2 - 1)^2 / \gamma_i^2]^{1/2}$ is the unperturbed velocity of $i$th beam. $-e$, $m_0$, $n_{i0}$, $\delta n_{i1}$, $\delta v_{i1}$ are the electron charge, the electron rest mass , the unperturbed density , perturbed density and perturbed velocity of $i$th beam, respectively. $J_i = -e(n_{i0}\delta v_{i1} + \delta n_{i1} v_{i0})$ is perturbed current density. The relative density factor $\alpha$ is $n_{20} / n_{10}$ .To obtain the dispersion equation, we expand $n_i = n_{i0} + \delta n_{i1}$ , $v_i = v_{i0} + \delta v_{i1}$ , $\vec{E} = \delta \vec{E}_z$ , $\vec{B} = \delta \vec{B}_y$ and solve for the perturbed variables. Using Floquet's theorem, all the fields and beam parameters can be written as a sum of space harmonics which have the periodicity of the grating [13],

$$f(x,z) = \sum_{n=-\infty}^{\infty} f_n(x) \exp(jk_n z - j\omega t) \qquad (5)$$

where $k_n = k_0 + 2\pi n / D$ , $k_0$ is wave number, $\omega$ is the angular frequency, $n$ is an integer.

From Eqs.1, 2, 3, 4 and expression (5), the wave equation for the $n$th space harmonic of axial electric field is given and the transverse components of electric and magnetic fields can be expressed in terms of the axial electric field as follows[14]:

$$[\frac{\partial^2}{\partial x^2} + \varepsilon_{l,n}(\frac{\omega^2}{c^2} - k_n^2)]\delta \hat{E}_{z,n,l}(x) = 0 \quad , \quad l=1,2,3$$

(6)

$$\delta \hat{E}_{x,n,l} = \frac{jk_n}{\kappa_n^2} \frac{\partial}{\partial x} \delta \hat{E}_{z,n,l}$$

(7)

$$\delta \hat{B}_{y,n,l} = \frac{j\omega}{c\kappa_n^2} \frac{\partial}{\partial x} \delta \hat{E}_{z,n,l}$$

(8)

$$\varepsilon_{3,n} = 1 - \sum_{i=1}^{2} \frac{\omega_{pi}^2}{\gamma_i^3 (\omega - k_n v_i)^2} \quad , \quad \varepsilon_{1,n} = \varepsilon_{2,n} = 1$$

(9)

where $\omega_{pi} = (e^2 n_{i0} / m_0 \varepsilon_0)^{1/2}$ is the beam-plasma angular frequency, $\varepsilon_{l,n}$ is the dielectric function, $\kappa_n = [(\omega/c)^2 - k_n^2]^{1/2}$ is the transverse wave number. $l = 1,2,3$ represents the region Ⅰ, Ⅱ, Ⅲ, respectively.

## *Components of electromagnetic fields in each region* [13]

Employed the wave Eq.6 and expressions (7) and (8), the components of electromagnetic fields in the three regions can be treated as follows:

The fields in Region Ⅰ $-a \le x < -b$ and Ⅱ $b < x \le a$ can be expressed in terms of two coefficients, respectively:

$$\delta \hat{E}_{z,n,l_0} = A_n^{l_0} \sin \kappa_n x + B_n^{l_0} \cos \kappa_n x$$

(10)

$$\delta \hat{E}_{x,n,l_0} = \frac{jk_n}{\kappa_n} (A_n^{l_0} \cos \kappa_n x - B_n^{l_0} \sin \kappa_n x)$$

(11)

$$\delta \hat{B}_{y,n,l_0} = \frac{j\omega}{c\kappa_n} (A_n^{l_0} \cos \kappa_n x - B_n^{l_0} \sin \kappa_n x)$$

(12)

where superscript $l_0 = 1,2$ respects the region Ⅰ and region Ⅱ, respectively.

Region Ⅲ ( $-b \le x \le b$ )

$$\delta \hat{E}_{z,n,3} = C_n \sin \beta_n x + D_n \cos \beta_n x$$

(13)

$$\delta \hat{B}_{y,n,3} = \frac{j\omega \beta_n}{c\kappa_n^2} (C_n \cos \beta_n x - D_n \sin \beta_n x)$$

(14)

where $\beta_n = \sqrt{\varepsilon_{3,n}}\kappa_n$ .

Region Ⅳ and Ⅴ is the lower groove $-a - h \le x < -a$ and upper groove $a < x \le a + h$ , respectively. As the grating period is less than the free-space wavelength, we represent the fields inside the $1^{th}$ groove using the form of TEM standing wave modes as follows:

$$\delta \hat{E}_z = S_{\pm a} \sin[\frac{\omega}{c}(x \mp (a+h)] \exp(ik_0 D) \qquad (15)$$

$$\delta \hat{B}_y = jS_{\pm a} \cos[\frac{\omega}{c}(x \mp (a+h)] \exp(ik_0 D) \qquad (16)$$

where the "$\mp$" denotes the upper interface $x=a$ and the lower face $x=-a$, respectively. The $A_n^{l_0}$, $B_n^{l_0}$, $C_n$, $D_n$, $S_{\pm a}$ mean unknown coefficients, which are depended on the boundary conditions.

## *Dispersion equation*

Applying the boundary conditions at the grating surfaces $x = \pm a$ ,the beam surfaces $x = \pm b$ ,solving simultaneously Eqs.10-16 and then eliminating the coefficients of $A_n^{l_0}$, $B_n^{l_0}$, $C_n$, $D_n$, $S_{\pm a}$ ,the odd mode and even modes dispersion equation is obtained as follows, respectively.

$$\frac{D}{d}\cot\left(\frac{\omega h}{c}\right)+\sum_{n=-\infty}^{\infty}\frac{\omega}{c\kappa_n}\mathrm{sinc}^2\left(\frac{k_n d}{2}\right)\frac{Q_n-R_n/\varepsilon_{3,n}^{1/2}}{\varepsilon_{3,n}^{1/2}+T_n}=0$$

(17)

is the dispersion equation for the odd modes, and

$$\frac{D}{d}\cot\left(\frac{\omega h}{c}\right)-\sum_{n=-\infty}^{\infty}\frac{\omega}{c\kappa_n}\mathrm{sinc}^2\left(\frac{k_n d}{2}\right)\frac{1+T_n\varepsilon_{3,n}^{1/2}}{Q_n/\varepsilon_{3,n}^{1/2}-R_n}=0$$

(18)

is the dispersion equation of the even modes, where $Q_n=\varepsilon_{3,n}^{1/2}\cot[\kappa_n(a-b)]$, $R_n=\varepsilon_{3,n}^{1/2}\cdot\tan\beta_n b$, $T_n=\tan\beta_n b\cdot\cot[\kappa_n(a-b)]$ .The Eqs.17and 18 is the similar to Ref.[13] when the voltages of two electron beams are the same. In the absence of beams, the dispersion equations of odd modes and even modes can be reduced to:

$$\frac{D}{d}\cot\left(\frac{\omega h}{c}\right)+\sum_{n=-\infty}^{\infty}\frac{\omega}{c\kappa_n}\mathrm{sinc}^2\left(\frac{k_n d}{2}\right)\cot\kappa_n a=0$$

(19)

is the dispersion equation for the odd modes, and

$$\frac{D}{d}\cot\left(\frac{\omega h}{c}\right)-\sum_{n=-\infty}^{\infty}\frac{\omega}{c\kappa_n}\mathrm{sinc}^2\left(\frac{k_n d}{2}\right)\tan\kappa_n a=0$$

(20)

is the dispersion equation for the even modes.

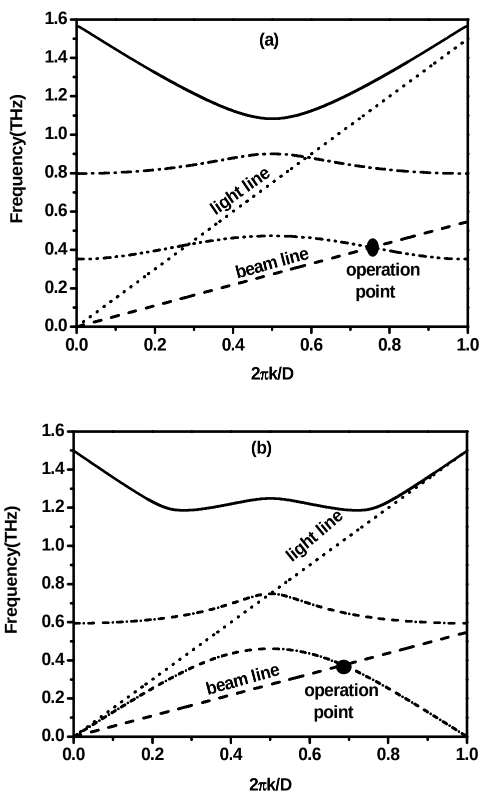


**Fig2.** Dispersion curves of (a) even mode and (b) odd mode. The parameters are period *D=0.2mm*, slot width *d=0.1mm*, slot depth *h=0.1mm* and the distance of two-gratings 2*a=0.3mm*

## NUMERICAL SOLUTIONS

In this section, we focus on the characteristics of dispersion equation through the numerical solutions.

The dispersion curves without beams for the even modes are found by the solutions of Eq.20, which are shown in Fig.2 (a). The beam line with voltage 40kv is plotted for reference. It is observed that the lowest order mode has a cutoff frequency of 0.345THz and has a maximum frequency 0.475THz. There is a stop band between the two lowest order modes extending from 0.475THz to 0.8THz.

Similarly, the dispersion curves without beams for the odd modes are obtained by the solutions of Eq.19, which are shown in Fig.2 (b). The band gap is evident extending from 0.46THz to 0.6THz. Obviously, the band gap of even modes and the frequency of operation point (solid dot) which is the intersection point of beam-wave are greater than that of the lowest odd mode.

The two admixed beams are turned into a single beam when the beam voltages are the same. The dispersion curves of single beam for the even mode found by the solution of Eq.18 are shown in Fig.3, where the left

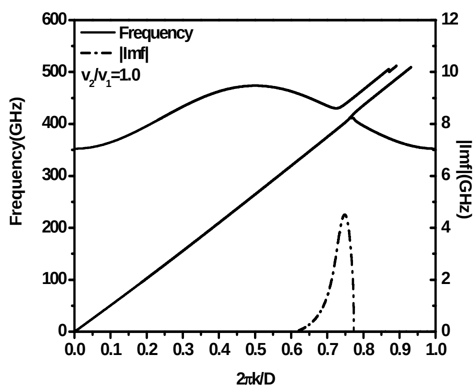


**Fig3**. Dispersion curves showing the real and imaginary components of the frequency for the lowest-order even mode, the parameters of grating are the same as that of in Fig.2. The beam voltage is *40*kv, electron beam density is $n_0=5.97\times10^{18}$ m$^{-3}$

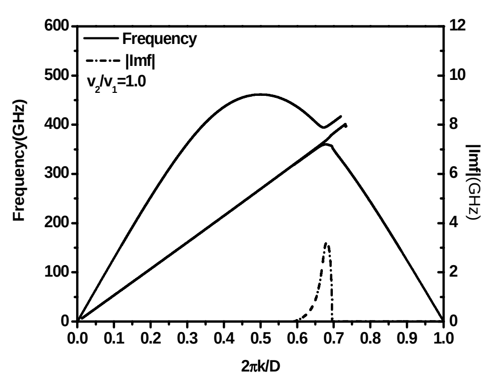


**Fig4**. Dispersion curves showing the real and imaginary components of frequency for the lowest-order odd mode, the parameters are the same as in Fig.3

axis is the real component of frequency and the right axis is the growth rate. Seen from Fig.3, the growth rate is found at the vicinity where the slow-wave and electron beam are synchronous, which occurs at a frequency near 0.41 THz. The peak growth about 4.5GHz is found at a frequency of 0.403THz. Similarly, the dispersion curves of single beam for the odd modes are shown in Fig.4. The peak growth rate about 3.24GHz is found at the normalized wave number of 0.68, which occurs at a frequency of 0.371THz.Obviously, the peak growth rate of the lowest order even mode are greater than that of odd mode.

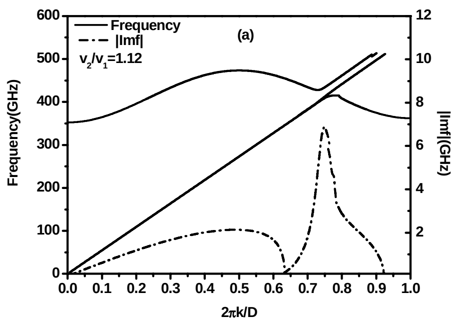


**Fig5**. Dispersion curves showing the real and imaginary components of the frequency for the lowest-order even mode, the parameters of grating are the same as that of in Fig.2. The beam voltage is *40*kv, electron beam density is $n_0=5.97\times10^{18}$ m-3,the voltage ratio is $v_r$=1.12

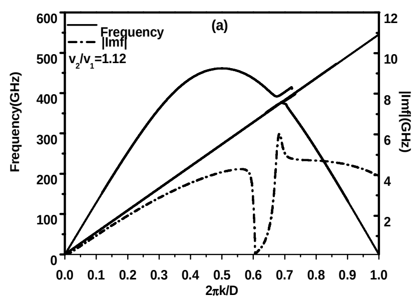


**Fig6**. Dispersion curves showing the real and imaginary components of the frequency for the lowest-order odd mode. The parameters of grating are the same as that of in Fig.2. The beam voltage is *40*kv, electron beam density is $n_0=5.97\times10^{18}$ $m^{-3}$, and the voltage ratio is $v_r$=1.12

The dispersion curves of two-stream for the lowest-order even mode obtained by the solution of Eq.18 are shown in Fig.5. Obviously, the growth rate is remarkably different from that of single beam, the cambered growth rate is produced by the TSI, and the growth rate is remarkably enhanced when the TSI wave is synchronous with the slow-wave. The peak growth rate about 6.91GHz is found at the frequency of 0.412THz, which is enhanced by a factor of 2 than that of single beam. Outside the synchronous range of TSI and electro-magnetic wave, the growth rate is still existed, which is thought to be yielded by the TSI

Similarly, the dispersion curves of two-stream for the lowest-order odd mode solved the Eq.29 are shown in Fig.6. The peak growth rate about 6.04GHz is found at the frequency of 0.375THz, which is enhanced by a factor 2. We may note from Figs.3, 4, 5and 6 that the even mode could be an operation mode for which the growth rate is greater than that of odd mode. To obtain the higher operation frequency, we should make the even mode as an operation mode.

## CONCLUSIONS

In this paper, we have studied the two-stream Smith-Purcell Free-Electron Laser with a dual-grating. The linear dispersion equations for the even modes and odd modes are derived by making use of fluid theory, and those of characteristics are analyzed through the numerical solutions. We find that the peak growth rate is enhanced by a factor of 2 than that of single beam when the TSI wave is synchronous with the electromagnetic wave. The nonlinear effect is under consideration, which will be reported in the future.

## ACKNOWLEDGEMENT

This work is supported by National Natural Science Foundation of P. R. China (60571020, 60178011).

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