THEORY OF RADIATION OF ELECTRONS IN THE FIELD OF A LINEAR POLARIZED STATIONARY ELECTROMAGNETIC WAVE

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Abstract

In the paper the results of exact integration of Lorentz equation for a free electron in the field of a linear polarized standing electro-magnetic wave are presented. Standing wave is considered as a sum of two running in opposite directions linear polarized waves. Projections of equations on coordinate axes can be integrated once. It allows us to reduce the task to solution of nonlinear equation of the second order for electron coordinate. The axis of projection coincides with a wave line. For approximate integration of the second order equation the expansion on two small parameters are used. Velocity and coordinate of electron in parametric form are presented in the paper. It is shown that under interaction of a relativistic electron with stationary wave there is a motion, which has of beating character. The amplitude and period of the beating were calculated.

INTRODUCTION

The theory of the electron interaction with a standing light wave originates in description of the Kapiza-Dirac effect [1]. The physical sense of the effect is stipulated radiation of electrons in the field of a stationary wave. After that sufficiently large amount of papers were devoted to the theoretical investigations of electron radiation in the field of a light wave.

The interest to the subject has been revived lately due to huge progress in intense laser technique. The latest works use both quantum and classical electrodynamics approach.

The main difficulty in using of classical electrodynamics approach is determination of solution of equations of electron motions in the form which will be convenient for analytical calculations of the radiation spectrum and for estimations of the electron velocity and coordinate evolutions. For example, in basic works [2,3] the solving of motion equations is reduced to the solving of the equation system of the two first order equations. But in this case one can find only approximate solution and the solution can be formulate as a function of intrinsic time of the electron. Such approach makes calculations of the radiation spectrum and other characteristics quite difficult. In the paper [4] an electron trajectory in the field of a standing linear polarized electromagnetic wave was reduced to the solving of nonlinear differential equation system with time dependent coefficients. After linearization of the system the Hill equations has to be solved.

In the presented paper, the approximate solutions of the Lorenz equation for an electron in the field of linear polarized standing wave are presented. The standing wave is considered as a sum of two running in opposite directions waves with the same polarizations.

Two projections of the Lorenz equation can be integrated once [4]. It allows to reduce the task to the solving of a second order nonlinear equation. For the approximate solution the expanding on two small parameters was used.

In the work the expressions for velocity and coordinates of an electron were derived in a form of parametrical functions of time. Using [5] and with the help of derived formulas one can calculate radiation spectrum of an electron in the field of standing linear polarized electromagnetic wave.

As one can see from the derived solutions, the electron motion in the standing wave has beating character and can lead to electron grouping in the propagation direction. The period and amplitude of the beatings were calculated.

MOTION EQUATIONS

An electron motion in the field of standing wave can be described with Lorenz equation:

$$\frac{d}{dt}\frac{m_0\vec{v}}{\left(1-\beta^2\right)^{1/2}} = c\vec{E} + \frac{e}{c}\left[\vec{v}\vec{H}\right], \qquad (1)$$

where, $\beta = v/c$, m_0 the rest mass of the electron, c is the velocity of light, e is the electron charge, $\vec{v} = \frac{\partial \vec{r}}{\partial t}$ is the vector of an electron velocity, $\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$, t is time, $\vec{E},\vec{H}$ are vectors electrical and magnetic field.

We will consider the standing wave as a sum of two linear polarized running in opposite direction waves:.

$$\begin{aligned} &\vec{E}=\vec{E}_1+\vec{E}_2;\ \vec{E}_1=\vec{k}E_{1z};\ \vec{E}_2=\vec{k}E_{2z};\\ &E_{1x}=0;\ E_{1y}=0;\ E_{2x}=0;\ E_{2y}=0; \end{aligned} \qquad (2)$$

$$\begin{aligned} &\vec{H}=\vec{H}_1+\vec{H}_2;\quad \vec{H}_1=[\vec{i};\vec{E}_1]=-\vec{j}E_{1z};\\ &\vec{H}_2=[-\vec{i};\vec{E}_1]=\vec{j}E_{1z};\\ &H_{1z}=0;\ H_{1x}=0;\ H_{2z}=0;\ H_{2x}=0; \end{aligned} \qquad (3)$$

$$\begin{aligned} &\vec{E}_1=\vec{k}E_{01}\cos\left[2\pi\nu_1\left(t-\frac{x}{c}\right)+\delta_1\right];\\ &\vec{E}_2=\vec{k}E_{02}\cos\left[2\pi\nu_2\left(t+\frac{x}{c}\right)+\delta_2\right]; \end{aligned} \qquad (4)$$

Substituting (4) in (2) and (3) one can get:

$$\vec{E}=\vec{k}\left(E_{01}\cos\left[2\pi\nu_1\left(t-\frac{x}{c}\right)+\delta_1\right]+\right.$$
$$\left.+E_{02}\cos\left[2\pi\nu_2\left(t+\frac{x}{c}\right)+\delta_2\right]\right); \tag{5}$$

$$\vec{H}=-\vec{j}\left(E_{01}\cos\left[2\pi\nu_1\left(t-\frac{x}{c}\right)+\delta_1\right]-\right.$$
$$\left.-E_{02}\cos\left[2\pi\nu_2\left(t+\frac{x}{c}\right)+\delta_2\right]\right); \tag{6}$$

Using expression: $mc\Big/\left(1-\beta^2\right)^{1/2}=\xi$ (where ξ - is electron energy), and expressions (5),(6) we project equation (1) on coordinate axes:

$$\frac{1}{c}\frac{d}{dt}(\xi\beta_z)=e(1-\beta_x)E_{01}\cos\left[2\pi\nu_1\left(t-\frac{x}{c}\right)+\delta_1\right]+$$
$$+e(1+\beta_x)E_{02}\cos\left[2\pi\nu_2\left(t+\frac{x}{c}\right)+\delta_2\right] \tag{7}$$

$$\frac{1}{c}\frac{d}{dt}(\xi\beta_y)=0 \tag{8}$$

$$\frac{1}{c}\frac{d}{dt}(\xi\beta_x)=e\beta_z\left(E_{01}\cos\left[2\pi\nu_1\left(t-\frac{x}{c}\right)+\delta_1\right]-\right.$$
$$\left.-E_{02}\cos\left[2\pi\nu_2\left(t+\frac{x}{c}\right)+\delta_2\right]\right) \tag{9}$$

where $\beta_x=\frac{1}{c}\frac{dx}{dt}$, $\beta_z=\frac{1}{c}\frac{dz}{dt}$, $\beta_y=\frac{1}{c}\frac{dy}{dt}$.

Equations (7,8) can be integrated once. As a results the following expressions can be derived:

$$\beta_z/\sqrt{1-\beta^2}=\psi \tag{10}$$

$$\beta_y/\sqrt{1-\beta^2}=A \tag{11}$$

where $\psi=PF+\xi_z$ (12)

$$F=\left(Sin\left[2\pi\nu_1\left(t-\frac{x}{c}\right)+\delta_1\right]+Sin\left[2\pi\nu_2\left(t+\frac{x}{c}\right)+\delta_2\right]\right)-$$
$$\left(Sin\left[2\pi\nu_1\left(t_0-\frac{x(t_0)}{c}\right)+\delta_1\right]+Sin\left[2\pi\nu_2\left(t_0+\frac{x(t_0)}{c}\right)+\delta_2\right]\right); \tag{13}$$

$$P=\frac{eE_0}{mc(2\pi\nu)};\quad \xi_z=\gamma(t_0)\beta_z(t_0);\quad A=\gamma(t_0)\beta_y(t_0) \tag{14}$$

t_0 is initial time; $\vec{E}_0=\vec{E}_{01}=\vec{E}_{02}$.

With use of two integrals (10) and (11) one can express, β_z,β_y,ξ through β_x,ψ,A

$$\beta_z^2=\psi^2(1-\beta_x^2)/(1+\psi^2+A^2) \tag{14}$$

$$\beta_y^2=A^2(1-\beta_x^2)/(1+\psi^2+A^2) \tag{15}$$

$$\xi=m_0c^2(1+\psi^2+A^2)^{1/2}/(1-\beta_x^2)^{1/2} \tag{16}$$

Substituting (14-16) to (9) and using expression:

$$\frac{d\xi}{dt}=e(\vec{v}\vec{E}), \tag{17}$$

one can derive the second order equation for electron coordinate x.

$$\frac{\dot{\beta}_x}{1-\beta_x^2}=\frac{eE_0}{m_0c}\frac{\psi}{(1+\psi^2+A^2)}\Big((1-\beta_x)$$
$$Cos\left[2\pi\nu_1\left(t-\frac{x}{c}\right)+\delta_1\right]-$$
$$-(1-\beta_x)Cos\left[2\pi\nu_1\left(t-\frac{x}{c}\right)+\delta_1\right]-$$
$$-(1+\beta_x)Cos\left[2\pi\nu_2\left(t+\frac{x}{c}\right)+\delta_2\right]\Big) \tag{18}$$

where $\dot{\beta}_x=\frac{d}{dt}\beta_x=\frac{1}{c}\frac{d^2x}{dt^2}$.

Integrating (18), we derive:

$$\ln\frac{1+\beta_x}{1-\beta_x}=\frac{2eE_0}{m_0c}\int_{t_0}^{t}\frac{\psi}{(1+\psi^2+A^2)}\{(1-\beta_x)Cos[\varphi_-]-$$
$$-(1+\beta_x)Cos[\varphi_+]\}dt+C_2 \tag{19}$$

$$C_2=\ln\frac{1+\beta_x(t_0)}{1-\beta_x(t_0)}.$$

For standing wave we can set:

$\vec{E}_0=\vec{E}_{01}=\vec{E}_{02}$; $\delta_1=\delta_2=0$; $\nu_1=\nu_2=\nu$.

Than $\varphi_-=2\pi\nu\left(t-\frac{x}{c}\right)$; $\varphi_+=2\pi\nu\left(t+\frac{x}{c}\right)$.

As it follows from (19):

$$\frac{1+\beta_x}{1-\beta_x}=\frac{1+\beta_x(t_0)}{1-\beta_x(t_0)}e^{\Phi(t)} \tag{20}$$

where

$$\Phi(t)=\frac{2eE_0}{m_0c}\int_{t_0}^{t}\frac{\psi}{(1+\psi^2+A^2)}\{(1-\beta_x)Cos[\varphi_-]-(1+\beta_x)Cos[\varphi_+]\}dt$$

APPROXIMATE SOLUTION

Assuming $P<<1$, $\xi_z<<1$, $\Phi(t)<<1$ and $e^{-\Phi(t)}\approx 1-\Phi(t)+O\frac{\Phi(t)^2}{2}$

one can get from (20):

$$\beta_x(t)\approx\beta_x(t_0)+\frac{1}{2}(1-\beta_x^2(t_0))\Phi(t) \tag{21}$$

$$\Phi(t)\approx P^2\Phi_1+P\xi_z\Phi_2+P^2\xi_z\Phi_2 \tag{22}$$

where

$$\Phi_1\approx\Gamma^{-2}\{(-\frac{1}{2})(Cos[2\varphi_-]-Cos[2\varphi_+])-2(Sin[\varphi_-(t_0)]+$$
$$+Sin[\varphi_+(t_0)])(Sin[\varphi_-]-Sin[\varphi_+])$$
$$-\frac{1}{2}(Cos[2\varphi_-(t_0)]-Cos[2\varphi_+(t_0)])\} \tag{23}$$

$$\Phi_2 \approx 2\Gamma^{-2}\{(Sin[\varphi_-] - Sin[\varphi_+]) - \\ -(Sin[\varphi_-(t_0)] + Sin[\varphi_+(t_0)])\} \quad (24)$$

$$\Phi_3 = 2(2\pi\nu)\Gamma^{-2}\left\{\int_{t_0}^{t} Sin\left[4\pi\nu\frac{x}{c}\right]dt - \\ -\int_{t_0}^{t}\beta_x(t)Sin[4\pi\nu t]dt\right\} \quad (25)$$

and $\Gamma^{-2} = 1 + \gamma^2(t_0)(\beta^2(t_0) - \beta_x^2(t_0))$.

The approximate solution of (21) we will find in the form:

$$\frac{x}{c} = \beta_x(t_0)(t - t_0) + \alpha_1^m(\alpha_1 x_1 + \alpha_2 x_2) \quad (26)$$

$$\beta_x = \beta_x(t_0) + \alpha_1^m(\alpha_1\dot{x}_1 + \alpha_2\dot{x}_2) \quad (26)$$

$$\dot{x}_1 = \frac{dx_1}{dt}; \quad \dot{x}_2 = \frac{dx_2}{dt}; \quad \alpha_1 << 1; \quad \alpha_2 << 1;$$

Substituting (26,27) to (21) one can get $m = 1$; $\alpha_1 = P$; $\alpha_2 = \xi_z$;

$$\frac{\dot{x}_1}{c} \approx \frac{1}{2}(1 - \beta_x^2(t_0))(\tilde{\Phi}_1 - \tilde{\Phi}_3) \quad (28)$$

$$\frac{\dot{x}_2}{c} \approx \frac{1}{2}(1 - \beta_x^2(t_0))\tilde{\Phi}_2 \quad (29)$$

$$\beta_x = \beta_x(t_0) + P^2\dot{x}_1 + P\xi_z\dot{x}_2;$$

$$\frac{x}{c} = \beta_x(t_0)(t - t_0) + P^2 x_1 + P\xi_z x_2 \quad (30)$$

Expressions for $\tilde{\Phi}_1$, $\tilde{\Phi}_2$ one can produce from the corresponding expressions for Φ_1, Φ_2 by substitution of φ_-, φ_+, to $x = c\beta(t_0)(t - t_0)$, and change $\varphi_- \to \tilde{\varphi}_-, \varphi_+ \to \tilde{\varphi}_+$.

$$\tilde{\varphi}_- = 2\pi\nu((1 - \beta_x(t_0)) + \eta_0)$$

$$\tilde{\varphi}_+ = 2\pi\nu((1 + \beta_x(t_0)) - \eta_0)$$

where $\eta_0 = 2\pi\nu\beta_x(t_0)t_0$

$\tilde{\Phi}_3$ And after approximate integration (25) is equal to:

$$\tilde{\Phi}_3 = \Gamma^{-2}\left\{\left(\frac{1}{\beta_x(t_0)}\right)(1 - Cos[2\pi\nu\beta_x(t)t - 2\eta_0]) + \\ +\beta_x(t_0)[Cos[2\pi\nu t] - Cos[2\pi\nu t_0]]\right\} \quad (31)$$

Integrating (28) and (29) one can get for x_1, x_2

$$\frac{x_1}{c} = \frac{1}{2}(1 - \beta_x^2(t_0))\Gamma^{-2}\frac{1}{2\pi\nu}\left\{\left(-\frac{1}{4}\right)\left[\frac{Sin2\tilde{\varphi}_-}{(1 - \beta_x(t_0))} - \frac{Sin2\tilde{\varphi}_+}{(1 + \beta_x(t_0))}\right] \\ + 4Sin[2\pi\nu t_0]\left[\frac{Cos2\tilde{\varphi}_-}{(1 - \beta_x(t_0))} - \frac{Cos2\tilde{\varphi}_+}{(1 + \beta_x(t_0))}\right] - \frac{7}{2}Sin[4\pi\nu t_0] \\ + \left[\frac{1}{\beta_x(t_0)} - \beta_x(t_0)Cos[4\pi\nu t_0]\right]2\pi\nu(t - t_0) - \frac{\beta_x(t_0)}{2}Sin[4\pi\nu t_0] \\ -\frac{1}{2}\frac{1}{\beta_x^2(t_0)}Sin[2\pi\nu\beta_x(t)t - 2\eta_0] + \frac{1}{2}\beta_x(t_0)Sin[2\pi\nu t]\right\} \quad (32)$$

$$\frac{x_2}{c} = (1 - \beta_x^2(t_0))\Gamma^{-2}\frac{1}{2\pi\nu}\left\{\left[\frac{Sin\tilde{\varphi}_-}{(1 - \beta_x(t_0))} - \frac{Sin\tilde{\varphi}_+}{(1 + \beta_x(t_0))}\right] + \\ +\frac{\beta_x(t_0)}{1 - \beta_x^2(t_0)}Cos[2\pi\nu t_0]\right\} \quad (33)$$

As it is followed from expressions (2-3), at $\beta_x \approx 1$ running in the initial propagation direction wave exits electron oscillations with larger amplitude than the wave running in opposite direction.

Two last items in (32):

$$\frac{1}{2}(1 - \beta_x^2(t_0))\Gamma^{-2}\frac{1}{2\pi\nu}\left\{-\frac{1}{2}\frac{1}{\beta_x^2(t_0)}Sin[2\pi\nu\beta_x t - 2\eta_0] + \\ +\frac{1}{2}\beta_x(t_0)Sin[2\pi\nu t]\right\}$$

At $\beta_x \approx 1$ an electron motion, which has beating character [7] with period T:

$$T = \frac{1}{2\pi\nu(1 - \beta_x(t_0))} \quad (34)$$

The A_{max} maximum and A_{min} minimum beating amplitude can be described with following expressions

$$A_{min} = \frac{p^2c(1 - \beta_x^2(t_0))}{8\pi\nu\Gamma^2}\frac{(1 - |\beta_x(t_0)|^3)}{\beta_x^2(t_0)} \quad (35)$$

$$A_{max} = \frac{p^2c(1 - \beta_x^2(t_0))}{8\pi\nu\Gamma^2}\frac{(1 + |\beta_x(t_0)|^3)}{\beta_x^2(t_0)} \quad (36)$$

Substituting expressions for β_x and x from (30) to (14) and keeping items of the second order on p and ε_z one can derive approximate expression for β_z

$$\beta_z = \Gamma^{-1}(1 - \beta_x^2(t_0))^{1/2}(PF_0 + \xi_z) + 0P^3$$

where

$$F_0 = Sin[2\pi\nu(1 - \beta_x(t))t + \eta_0] + \\ + Cos[2\pi\nu(1 - \beta_x(t_0))t - \eta_0] + 2Sin[2\pi\nu t_0]$$

In the same way, substituting expressions for β_x and x from (30) to (15) and keeping items of the second order on p and ε_z we can produce approximate expression for β_y:

$$\beta_y = \frac{A}{\Gamma^2}(1 - \beta_x^2(t_0))^{1/2}\left[1 - P^2\frac{\beta_x(t_0)}{1 - \beta_x^2(t_0)}\dot{x}_1 - \\ -P\xi_z\frac{\beta_x(t_0)}{1 - \beta_x^2(t_0)}\dot{x}_2 - \frac{P^2}{2\Gamma^2}F_0^2 - \frac{P\xi_z}{\Gamma^2}F_0^2\right] + 0P^3 \quad (38)$$

Integrating (37) and (38) we derive dependence coordinates z and y on time t

$$z=\sqrt{c}\Gamma^{-1}(1-\beta_x^2(t_0))^{1/2}\left\{\frac{P(-1)}{2\pi\nu}*\right.$$
$$\left[\frac{Cos[2\pi\nu(1-\beta_x(t_0))t+\eta_0]}{(1-\beta_x(t_0))}+\frac{Cos[2\pi\nu(1-\beta_x(t_0))t-\eta_0]}{(1+\beta_x(t_0))}\right]+ \tag{39}$$
$$\left.+\frac{P}{2\pi\nu}\frac{2Cos[2\pi\nu t_0]}{(1-\beta_x^2(t_0))}+(\xi_Z+2Sin[2\pi\nu t_0])(t-t_0)\right\}+z_0(t_0)$$

$$y=\frac{A}{\Gamma^2}(1-\beta_x^2(t_0))^{1/2}\left[(t-t_0)-P^2\frac{\beta_x(t_0)}{1-\beta_x^2(t_0)}x_1-\right.$$
$$\left.-P\xi_z\frac{\beta_x(t_0)}{1-\beta_x^2(t_0)}x_2-\frac{P^2}{2\Gamma^2}\int_{t_0}^{t}F_0^2dt-\frac{P\xi_z}{\Gamma^2}\int_{t_0}^{t}F_0^2dt\right]+y_0(t_0) \tag{40}$$

CONCLUSION

The expressions derived above, describe the electron motion in the field of standing linear polarized light wave and allows to calculate spectrum of electron radiation with use of methods described in [5,6].

Today, it is supposed to use interaction of relativistic electron with intense laser beams accumulated in an optical cavity for generation of shirt wave radiation. The transversal sizes of the beams in the interaction point are equal of about several tens of micrometers. For this reason, the evaluation of transversal sizes of the electron beam during interaction is very actual task. It necessary to note, that the largest increasing of the electron beam transversal size is produced with the wave running in the direction of electron beam propagation and not the wave which generates the shirt wave radiation.

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