SPACE-FREQUENCY MODEL OF ULTRA WIDE-BAND INTERACTIONS IN FREE-ELECTRON LASERS

Y. Pinhasi*, Yu. Lurie, and A. Yahalom
The College of Judea and Samaria, Ariel, Israel.

Abstract

The principle of operation of intense radiation devices such as microwave tubes, free-electron lasers (FELs) and masers, is based on a distributed interaction between an electron beam and radiation. We developed a three-dimensional, space-frequency theory for the analysis and simulation of radiation excitation and propagation in electron devices and free-electron lasers operating in an ultra wide range of frequencies. The total electromagnetic field is presented in the frequency domain as an expansion in terms of cavity eigen-modes. The mutual interaction between the electron beam and the field is fully described by coupled equations, expressing the evolution of mode amplitudes and electron beam dynamics. The approach is applied in a numerical particle code WB3D, simulating wide band interactions in free-electron lasers operating in the linear and non-linear regimes. The code is used to study the statistical and spectral characteristics of multimode radiation generation in a free-electron laser, operating in various operational parameters. The theory is demonstrated also in the case of "grazing", resulting in a wide-band interaction between the electron beam and the radiation.

INTRODUCTION

In this paper we continue development of the previously stated space-frequency, 3D model, which describes broadband phenomena occurring in electron devices, masers and FELs [1, 2, 3, 4]. In the model, the total electromagnetic field is presented in the frequency domain in terms of transverse eigen-modes of the cavity, in which the field is excited and propagates. A set of coupled excitation equations, describing the evolution of each transverse mode, is solved self-consistently with beam dynamics equations.

This coupled-mode model is employed in a three-dimensional numerical simulation code WB3D (see [1, 2] for the details). In the present work, the code is used to study the statistical and spectral characteristics of multimode radiation generation in a free-electron laser, operating in various operational parameters. The theory is demonstrated also in the case of "grazing", when the group velocity of the radiation mode is equal to the axial velocity of the electrons, resulting in a wide-band interaction between the electron beam and the generated radiation.

* e-mail: yosip@eng.tau.ac.il

THE MODEL

In the frequency domain the total electromagnetic field can be expanded in terms of transverse eigenmodes of the medium in which it is excited and propagates (see [1, 2] for the details). The perpendicular component of the electric and magnetic fields are given in any cross-section as a linear superposition of a complete set of transverse electric filed $\widetilde{\mathcal{E}}_{q\perp}(x,y)$ and magnetic field $\widetilde{\mathcal{H}}_{q\perp}(x,y)$ eigenmodes:

$$\widetilde{\mathbf{E}}_{\perp}(\mathbf{r},f) = \sum_q C_q(z,f)\, e^{+jk_{z_q}z}\, \widetilde{\mathcal{E}}_{q\perp}(x,y) \quad (1)$$

$$\widetilde{\mathbf{H}}_{\perp}(\mathbf{r},f) = \sum_q C_q(z,f)\, e^{+jk_{z_q}z}\, \widetilde{\mathcal{H}}_{q\perp}(x,y) \quad (2)$$

here $C_q(z,f)$ is amplitude of the mode q, that may be found from the excitation equations:

$$\frac{d}{dz}C_q(z,f) = \frac{1}{\mathcal{N}_q}\sum_i Q_i \left\{ \frac{\mathbf{v}_{\perp i}}{v_{z_i}} \widetilde{\mathcal{E}}^*_{q_\perp}(x_i,y_i) + \widetilde{\mathcal{E}}^*_{q_z}(x_i,y_i) \right\} e^{j[\omega t_{z_i} - k_{z_q} z]} \quad (3)$$

Spectral energy distribution of the electromagnetic field is defined in the model as a sum of energy spectrum of the excited modes:

$$\frac{d\mathcal{W}(z)}{df} = \sum_q \frac{1}{2}|C_q(z,f)|^2\, \Re\{\mathcal{N}_q\} \quad (4)$$

MULTIMODE SUPER-RADIANT EMISSION

To demonstrate the utilization of the model, we present a study of super-radiant emission in a waveguide-based, pulsed beam free-electron maser (FEM), with operational parameters are given in Table 1. Such FEM is expected to operate at millimeter and sub-millimeter (THz) wavelengths. When a FEL utilizes a waveguide, the axial wavenumber of transverse mode q follows the dispersion relation:

$$k_{z_q}(f) = \frac{2\pi}{c}\sqrt{f^2 - f^2_{co_q}} \quad (5)$$

where $f_{co_q} = \frac{c}{2\pi}k_{\perp q}$ is the cut-off frequency of the mode. In synchronism with that mode, the dispersion relation for

Table 1: Operational parameters of millimeter and sub-millimeter wave free-electron maser.

Accelerator	
Electron beam energy:	E_k=1÷6 MeV
Electron beam current:	I_0=1 A
Electron beam pulse duration:	T=0.1 pSec
Wiggler	
Magnetic induction:	B_w=2000 G
Period:	λ_w=5 cm
Number of periods:	N_w=20
Waveguide	
Rectangular waveguide:	15×7.5 mm
mode	Cut-off frequency
TE_{01}	20.0 GHz
TE_{21} , TM_{21}	28.3 GHz
TE_{41} , TM_{41}	44.7 GHz
TE_{03}	60.0 GHz

the electron beam is given by

$$k_{z_q}(f) = \frac{2\pi f}{v_{z0}} + k_w \qquad (6)$$

where v_{z0} is the average velocity of the accelerated electrons and $k_w = \frac{2\pi}{\lambda_w}$ (λ_w is the wiggler's period). The corresponding curves of synchronism frequency vs. beam energy for the FEM are shown in Fig. 1. Only waveguide modes which have in their field profile components that interacts efficiently with the wiggling electrons are shown. Table 2 summarizes several examined cases resulted from Eq. (6) in the multi-transverse mode operational regime. For each transverse mode q, the acceleration energy E_k can be set to excite two frequencies corresponding to the "slow" ($v_{g_q} < v_{z0}$) and "fast" ($v_{g_q} > v_{z0}$) synchronism frequencies or to the special case of "grazing", where $v_{g_q} = v_{z0}$ and a single synchronism frequency is obtained. Here

$$v_{g_q} = 2\pi \frac{d\,f}{d\,k_{z_q}} = \frac{c^2}{2\pi f} k_{z_q}(f) \qquad (7)$$

is the group velocity of the excited mode q.

The effect of super-radiance emerges when the duration of the electron beam pulse is much less then the period of the electromagnetic waves expected to be excited at synchronism frequencies according to Table 2. The waveguide and e-beam dispersion curves when the acceleration energy is $E_k = 2$ MeV are shown in Fig. 2. In this case a single waveguide mode TE_{01} is excited at two separated synchronism frequencies ("slow" and "fast") 46.1 GHz and 149.5 GHz, respectively. The spectral density of energy flux calculated with the code WB3D is shown in Fig. 3a. The spectrum peaks at the two synchronism frequencies with main lobe bandwidth of $\Delta f_{1,2} \approx \frac{1}{\tau_{sp_{1,2}}}$, where $\tau_{sp_{1,2}} \approx N_w \lambda_w \left| \frac{1}{v_{z0}} - \frac{1}{v_{g_{1,2}}} \right|$ is the slippage time. The corresponding temporal wave-packet (shown in Fig. 3b) consist of two "slow" and "fast" pulses with durations equal to the slippage times modulating carriers at their respective synchronism frequencies. Lowering the beam energy to $E_k \approx 1.62$ MeV, results in grazing between the e-beam and the waveguide dispersion curves at a single synchronism frequency 69.6 GHz. The spectrum in the case of grazing, as well as the corresponding temporal wavepacket are shown in Fig. 4.

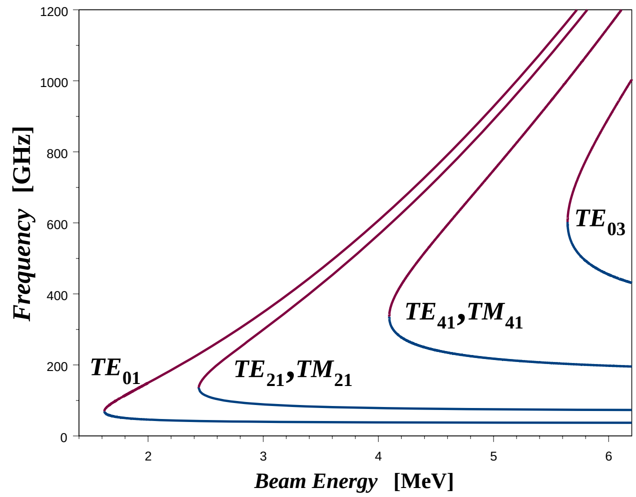

Figure 1: Energy dependence of the dispersion solutions.

As the acceleration energy is increased, transverse modes of higher orders are being excited simultaneously (in addition to the mode TE_{01}) extending the radiation spectrum over a wide rage of frequencies from few tens of GHz to more then THz. Fig. 5 shows the energy spectral densities of the excited waveguide modes as the beam energy is increased.

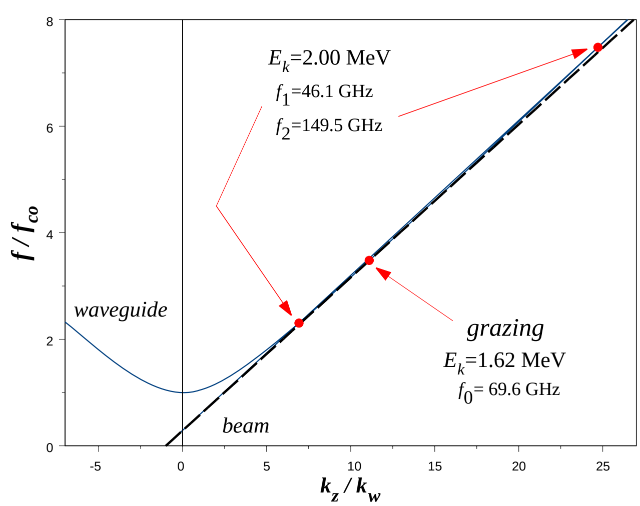

Figure 2: Dispersion solutions for TE_{01} transverse mode for $E_k = 2$ MeV.

Table 2: Synchronism frequencies for several beam energies.

Beam Energy	Synchronism frequencies [GHz]			
[MeV]	TE_{01}	TE_{21},TM_{21}	TE_{41},TM_{41}	TE_{03}
1.62	69.6 (grazing)	—	—	—
2.00	46.1 , 149.5	—	—	—
2.44	42.0 , 230.5	136.2 (grazing)	—	—
3.00	39.8 , 348.4	88.9 , 299.3	—	—
4.09	38.2 , 632.9	78.1 , 592.9	336.1 (grazing)	—
5.00	37.5 , 927.8	75.0 , 890.3	217.5 , 747.8	—
5.60	37.3 , 1151.0	73.9 , 1114.5	203.6 , 984.8	602.6 (grazing)

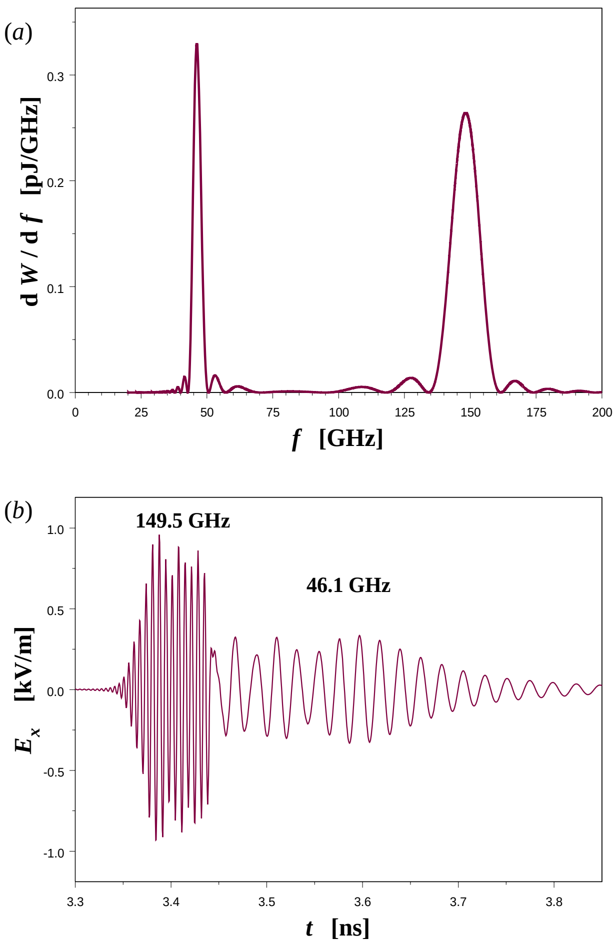

Figure 3: Super-radiant emission from an ultra short bunch when the beam energy is E_k=2 MeV and a single TE_{01} mode is excited: (a) Energy spectrum, and (b) temporal wavepacket.

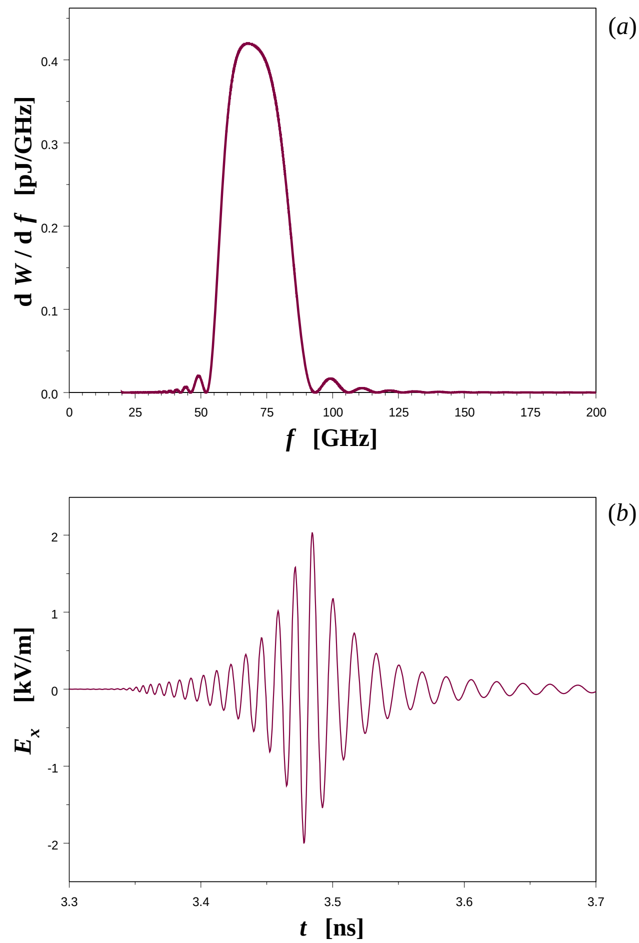

Figure 4: That of Fig. 3, but at grazing condition for mode TE_{01} (the beam energy is $E_k \approx 1.62$ MeV).

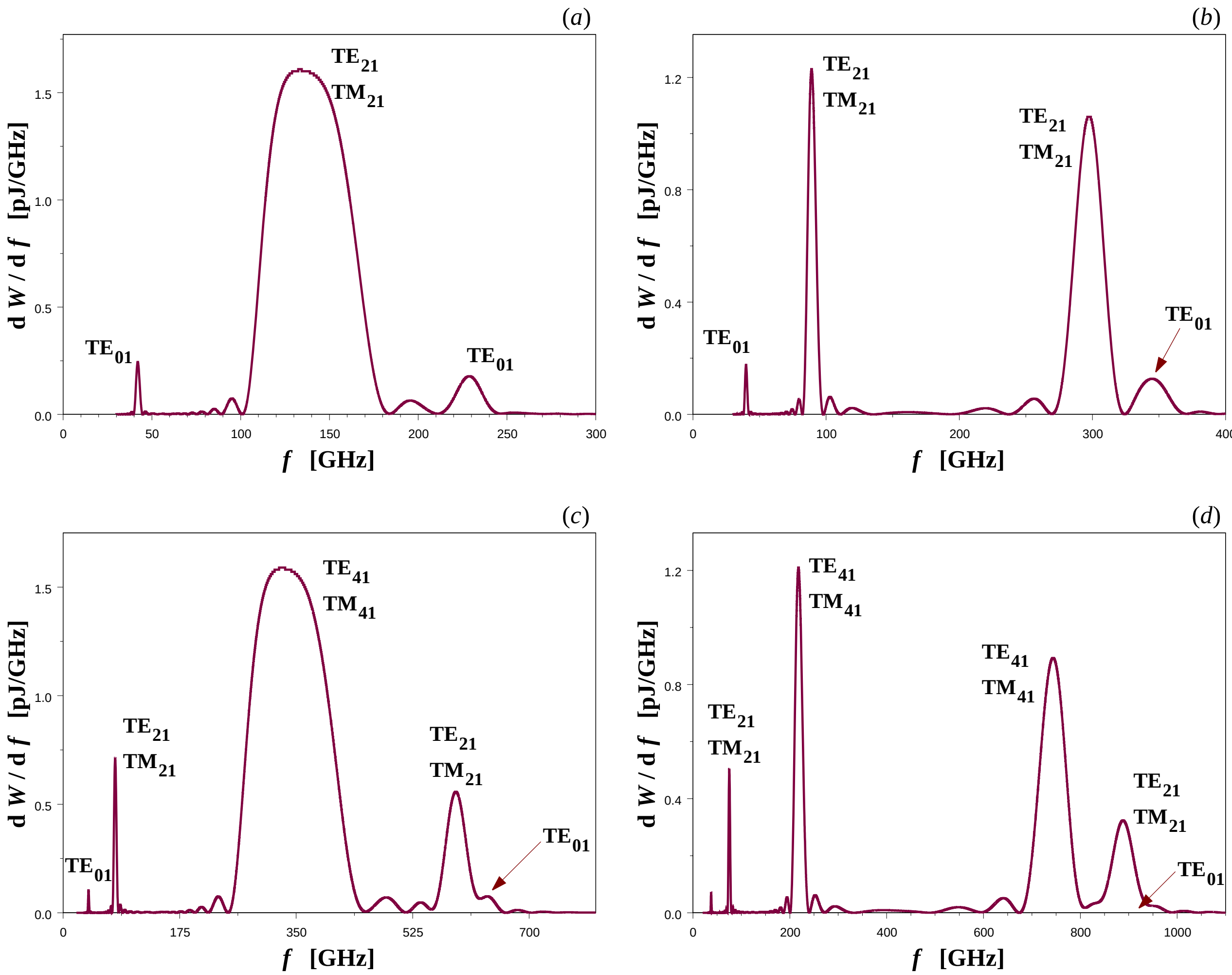

Figure 5: Energy spectra for different acceleration energies: (a) $E_k = 2.44$ MeV (grazing in the TE_{21},TM_{21} modes); (b) $E_k = 3.00$ MeV; (c) $E_k = 4.09$ MeV (grazing in the TE_{41},TM_{41} modes); (d) $E_k = 5.00$ MeV.

CONCLUSIONS

The presented coupled-mode theory, formulated in the frequency domain, enables development of a three-dimensional model, which can accurately describe wide-band interactions between radiation and electron beam in electron devices and free-electron lasers. Space-frequency solution of the electromagnetic equations considers dispersive effects arising from the resonator and gain medium. Such effects play a role also in the special case of grazing, and can not be accurately treated in approximated space-time approaches. We also note that our space-frequency model described here, also facilitates the consideration of statistical features of the electron beam and the excited radiation, enabling simulation of the interaction of a free-electron laser operating in the linear and non-linear regimes.

REFERENCES

[1] Y. Pinhasi, Yu. Lurie and A. Yahalom, Nucl. Instr. and Meth. in Phys. Res. A, **475**, 147 (2001).

[2] Y. Pinhasi, Yu. Lurie, A. Yahalom, and A. Abramovich, Nucl. Instr. Meth. Phys. Res. A, **483**, 510 (2002).

[3] Y. Pinhasi, Yu. Lurie, A. Yahalom, Phys. Rev. E **71**, 036503 (2005).

[4] Y. Pinhasi and Yu. Lurie, Phys. Rev. E **65**, 026501 (2002).