PARAMETRIC OPTIMIZATION OF A X-RAY FEL BASED ON A THOMSON SOURCE*

L. Serafini, A. R. Rossi, V. Petrillo, C. Maroli, A. Bacci, INFN-Milano, Milano, Italy
M. Ferrario, INFN-LNF, Frascati, Italy.

Abstract

We present a study based on a parametric optimization of a Thomson Source operated in FEL mode. This deals with the proposed scheme to use a high intensity laser pulse colliding with a high brightness electron beam of low to medium energy (around 10 MeV). Electrons undulating in the incoming laser field may emit radiation in a FEL coherent mode as far as some conditions are satisfied. A set of simple analytical formulas taking into account 3D effects is derived, in order to express these conditions in terms of three free parameters, namely the wavelength of the colliding laser pulse, the amplitude of the ripples in the time profile of the laser field, and the peak current carried by the electron beam. A few examples of possible operating points are compared with results of 3D numerical simulations, showing the FEL coherent emission of X-rays in the 0.1 to 5 nm range with tens of MeV high brightness electron beams colliding with high intensity ps-long laser beams carrying pulse energies of about 10 J.

INTRODUCTION

It has been recognized by several authors in the past [1,5] that the interaction between a high brightness electron beam and a counterpropagating - head-on colliding - high intensity laser pulse could lead to coherent emission of radiation, in the direction of the electron beam motion, according to a FEL-like mechanism driven by a collective instability that induces exponential growth of the radiation intensity. This coherent part of the emitted radiation overlaps with the spontaneous incoherent radiation generated by the Thomson back-scattering effect. Only recently, however, detailed 3D simulations[6] able to model the FEL collective instability showed the potential existence of this effect under particular conditions of electron beam emittance and current as well as laser field amplitude in the focal region, where the interaction between the two beams occurs. In this paper we derive a set of practical analytical formulas describing the existence of operating conditions in the dynamical range of the system where 3D effects can be mitigated so to allow the onset of the FEL instability, hence the generation of coherent radiation in a SASE-FEL emission mode.

Generally speaking, the situation is at all similar to a conventional SASE-FEL based on a magnetostatic undulator through which the electron beam propagates: the magnetostatic field of the undulator is replaced by the e.m. field of the incoming laser pulse, which causes the electrons to wiggle while they propagate through the pulse. Since the laser field is a classical description of a flux of real photons (to be compared with virtual photons for the case of a magnetostatic undulator), the resonance relationship for a Thomson source is at all similar to the FEL resonance apart for a factor 4 in the denominator.

$$\lambda_R = \frac{\lambda}{4\gamma^2}\left(1 + a_0^2 + \gamma^2\vartheta^2\right) \quad (1)$$

where λ is the wavelength of the colliding laser pulse, γ the kinetic energy of the electron beam (expressed in terms of its dimensionless relativistic factor), λ_R the wavelength of the forward emitted radiation (within a small angle ϑ around the electron beam propagation axis) and a_0 is the laser parameter (dimensionless amplitude of the vector potential associated to the laser field), given by

$$a_0 = 8.5 \cdot 10^{-6}\,\frac{\lambda\sqrt{P}}{R_0} \quad (2)$$

where R_0 is the laser focal spot size and P the peak power in the laser pulse (in TW).

The interaction between the electron beam and the laser pulse is assumed to take place in a drift space where no external forces (focusing or deflecting) act on the two beams, which are tightly focused by their individual final focus lens systems, taking them down to micron-size focal spots. Under the assumption that the laser pulse has a uniform transverse intensity profile of hard edge radius R_0 in the focus position, we can neglect ponderomotive transverse effects on the electron trajectories[7].

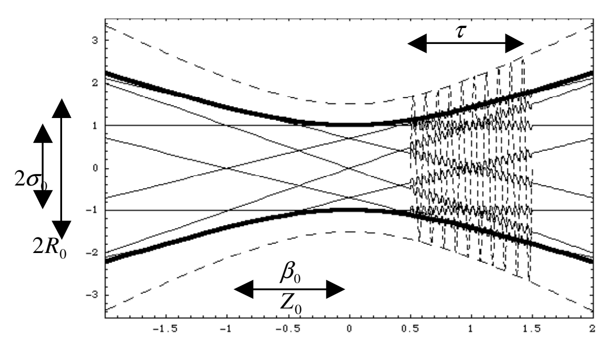

Figure 1: Electron orbits in the focal region (intersecting thin solid lines). Electron beam envelope (bold solid lines) and laser beam envelope (dashed lines) are also shown. Electrons are moving to the right, laser pulse (duration τ) is moving to the left.

As depicted in Fig.1, electrons cross the focus area traveling on rectilinear orbits: when they traverse the laser field their secular trajectories remain rectilinear, with a superimposed slightly wiggling motion.

PARAMETER DEFINITIONS

We break-up the set of parameters describing the whole system into three groups, one for each of the three interacting beams: the electron beam, the colliding laser beam and the emitted FEL radiation beam, respectivley.

The system is described by 11 main free parameters: 4 parameters for the electron beam, summarized in Table 1, 5 parameters for the laser beam, summarized in Table 2, and 2 parameters for the FEL radiation beam, summarized in Table 3. Note that units indicated in the tables are just the ones used for simplicity in the final set of formulas: all intermediate calculations are performed in standard MKS units.

Table 1: Electron Beam parameters

Energy γ	Current I [A]	Focal spot σ_0 [μm]	Emittance ε_n [μm]

There are some other additional parameters which represents ancillary quantities useful for handling the system of conditions relating the 11 main free parameters. These are: the electron beam beta-function in the focus, β_0, which is defined by the usual relation

$$\sigma_0 = \sqrt{\frac{\varepsilon_n \beta_0}{\gamma}} \qquad (3)$$

the electron bunch rms length, σ_z, and the electron beam rms relative energy spread $\frac{\Delta\gamma}{\gamma}$. These last two quantities do not enter in the derivation of the final set of formulas: they are only used for an afterward check on additional ancillary conditions.

Table 2: Laser parameters

Wavelength λ [μm]	Power P [TW]	Pulse length τ [ps]	Focal spot R_0 [μm]	Intensity ripples Δ [%]

Additional parameters for the laser beam are the Rayleigh range Z_0, given by

$$Z_0 = \frac{4\pi R_0^{\,2}}{\lambda} \qquad (4)$$

and the laser pulse energy, defined by $U = P\tau$. Note that the definition of the laser ripple parameter Δ is

$$\Delta \equiv \frac{\Delta a_0}{a_0} \qquad (5)$$

which represents the fluctuations of laser field amplitude along the pulse, which is assumed to have (at $\Delta = 0$) an ideal flat-top time profile.

Table 3: FEL radiation parameters

Wavelength λ_R [Å]	FEL parameter ρ

Additional parameters for the FEL radiation beam are the gain length, $L_g = \frac{\lambda}{4\pi\rho}$, and the quantum parameter, $\overline{\rho} = \rho\frac{\gamma\lambda_R}{\lambda_C}$ (with $\lambda_C = 0.024$ Å). As discussed elsewhere[8], as far as $\overline{\rho} \geq 0.5$ quantum effects are negligible ant the system can be described by means of classical FEL-like equations[6].

CONDITIONS FOR FEL EMISSION

Let us now analyze what are the conditions to be satisfied by the 11 main parameters (γ, I, σ_0, ε_n, λ, P, τ, R_0, Δ, λ_R, ρ) in order to operate the Thomson source as a Free Electron Laser.

The FEL resonance condition

$$\lambda_R = \frac{\lambda}{4\gamma^2}\left(1 + a_0^2\right) \qquad (C.1)$$

The definition of the FEL parameter

$$\rho = \frac{10^{-2}}{\gamma}\sqrt[3]{I\lambda^4 P/\sigma_0^4} \qquad (C.2)$$

Two conditions for optimal geometrical beam overlap of the envelopes of the two colliding beams: the first one is to ensure that the electrons will observe transversally constant undulator field

$$R_0 \geq 2\sigma_0 \qquad (C.3)$$

and the second one is to minimize the hour-glass effect in the collision of the two beams

$$c\tau \leq 2Z_0 \qquad (C.4)$$

Note that we will further check that the condition

$\beta_0 \geq Z_0$ is satisfied (assuring that the electron beam envelope is contained within the laser beam envelope). However, this condition is not explicitely used in order to simplify the following derivation.

We want of course that the interaction between the two beams, *i.e.* the equivalent undulator length, be longer than the FEL saturation length, which is typically set at 10 times the gain length. Hence

$$c\tau \geq 10L_g \qquad \text{(C.5)}$$

Now we must take into account how 3D effects and non-uniformities in the laser filed (which is our undulator field) may affect the FEL instability, avoiding inhomogeneous broadening effects of the gain bandwidth that may damp the onset of the FEL exponential instability. We know that the gain bandwidth of a SASE FEL is set by $\Delta\lambda_R/\lambda_R = 2\rho$. This implies that 3D and non-uniformity effects must produce bandwidth broadening smaller than 2ρ .

For a Fourier trasnform limited laser pulse the spectrum line width is $\frac{\Delta\lambda}{\lambda} = \frac{\lambda}{c\tau}$, therefore $\frac{\Delta\lambda_R}{\lambda_R} = \frac{\Delta\lambda}{\lambda} = \frac{\lambda}{c\tau}$, hence the condition

$$c\tau \geq \frac{\lambda}{2\rho} \qquad \text{(C.6)}$$

The FEL frequency broadening due to fluctuations in the undulator field amplitude, in our case represented by Δ, is given by $\frac{\Delta\lambda_R}{\lambda_R} = \frac{2a_0^2}{1+a_0^2}\Delta$, which in turns implies

$$\Delta \leq \rho\frac{1+a_0^2}{a_0^2} \qquad \text{(C.7)}$$

The transverse motion of the electrons in the focal region, which is mainly determined by the electron beam emittance, produces a random distribution of the angle ϑ in the resonance relationship reported in eq.1, which in turns induces a broadening of the FEL bandwidth. As extensively discussed elsewhere[6,9], the limitation on this random angle can be casted in terms of an upper limit on the emittance. This criterion is generally known as Kim-Pellegrini criterion: it has been generalized in ref.6 to the expression

$$\varepsilon_n \leq \sqrt{\frac{Z_R}{L_G}}\frac{\lambda_R\gamma}{2\pi} \qquad \text{(C.8)}$$

whewre Z_R is the Rayleigh range of the emitted FEL radiation, $Z_R = \frac{4\pi R_0^{\,2}}{\lambda_R}$.

FINAL SET OF FORMULAS

In order to simplify the derivation of a solution for the system of equations (C.1-C.8) we assume equalities for all the conditions instead of inequalities: this will allow to derive the minimum condition that 8 parameters have to fulfill, expressed as functions of three free parameters. We choose as free parameters the laser wavelength λ, the electron beam current I and the laser ripple parameter Δ.

The electron beam parameters must obey:

$$\varepsilon_n = 0.18\lambda \ \text{(F.1)} \ ; \quad \gamma = 0.05\sqrt[3]{\frac{I}{\Delta^2}} \ \text{(F.2)}$$

$$\sigma_0 = 0.21\lambda/\sqrt{\Delta} \ \text{(F.3)} \ ; \quad \beta_0 = 0.009\lambda\sqrt[3]{\frac{I}{\Delta^5}} \ \text{(F.4)}$$

the ancillary condition $\beta_0 \geq Z_0$, as anticipated, is respected if $I > 576\ \Delta^2$, which is easily satisfied, since Δ assumes values definitely lower than 0.1, while the beam curent I has expected values in excess of several hundreds Amps. Note that the additional condition $\Delta\gamma/\gamma \leq \rho$ has to be satisfied, though it was not explicitely considered in the derivation.

The laser parameters must obey:

$$P = 0.0018/\Delta \ \text{(F.5)} \ ; \quad U = 18.6\lambda/\Delta^2 \ \text{(F.6)} \ ;$$

$$a_0 = 1. \ \text{(F.7)} \ ; \quad \tau = 1.1\cdot 10^{-8}\lambda/\Delta \ \text{(F.8)} \ ;$$

$$Z_0 = 1.6\lambda/\Delta \ \text{(F.9)}$$

The FEL radiation is characterized by:

$$\lambda_R = 200\lambda\sqrt[3]{\frac{\Delta^4}{I^2}} \ \text{(F.10)} \ ; \quad \rho = 0.25\Delta \ \text{(F.11)} \ ;$$

$$L_G = 0.32\lambda/\Delta \ \text{(F.12)} \ ; \quad \bar{\rho} = 10^{12}\lambda\sqrt[3]{\frac{\Delta^5}{I}} \ \text{(F.13)}$$

EXAMPLES

A relevant example is for the case of a CPA Ti:Sa laser system, which is nowadays capable of delivering fs to ps long pulses carrying energies in excess of a J, focused down to micron-size spots. In this case, setting $\lambda = 0.8\ \mu$m , the formula set (F.1-F.13) reduces to the following (expressing Δ in units of %):

$$\varepsilon_n = 0.14\ \mu\text{m} \ ; \quad \gamma = 1.08\sqrt[3]{\frac{I}{\Delta^2}} \ ; \quad \sigma_0 = \frac{1.4}{\sqrt{\Delta}}\ \mu\text{m}$$

$$\beta_0 = \frac{0.015}{\Delta}\sqrt[3]{\frac{I}{\Delta^2}}\ [\text{mm}]\ ;\ P = \frac{0.18}{\Delta}\ [\text{TW}]\ ;$$

$$U = \frac{0.15}{\Delta^2}[\text{J}]\ ;\ \tau = \frac{0.85}{\Delta}\ [\text{ps}]\ ;\ Z_0 = \frac{0.13}{\Delta}\ [\text{mm}]\ ;$$

$$\lambda_R = 3436\Delta\sqrt[3]{\frac{\Delta}{I^2}}\ [\text{Å}]\ ;\ \rho = 2.5\cdot 10^{-3}\Delta\ ;$$

$$L_G = \frac{26}{\Delta}\ [\mu\text{m}]\ ;\ \bar{\rho} = 386\Delta\sqrt[3]{\frac{\Delta^2}{I}}\ .$$

We take into account now two specific examples, one in the classical SASE-FEL regime, the other in the quantum regime: all parameters now depend only on the beam current and the laser ripple parameter.

1) we set $\Delta = 0.15\ \%\ ;\ I = 1500\ A$: we find

$$\varepsilon_n = 0.14\ \mu\text{m}\ ;\ \ \gamma = 44\ ;\ \ \sigma_0 = 3.7\ \mu\text{m}\ ;$$
$$\beta_0 = 4.1\ \text{mm}\ ;\ P = 1.2\ \text{TW}\ ; U = 6.6\ \text{J}\ ;$$
$$\tau = 5.6\ \text{ps}\ ;\ Z_0 = 0.85\ \text{mm}\ ;\ \lambda_R = 2.1\ \text{Å}\ ;$$
$$\rho = 4.\cdot 10^{-4}\ ;\ L_G = 159\ \mu\text{m}\ ;\ \bar{\rho} = 1.4\ .$$

2) we set $\Delta = 0.05\ \%\ ;\ I = 2500\ A$: we find

$$\varepsilon_n = 0.14\ \mu\text{m}\ ;\ \ \gamma = 108\ ;\ \ \sigma_0 = 6.4\ \mu\text{m}\ ;$$
$$\beta_0 = 30\ \text{mm}\ ;\ P = 3.5\ \text{TW}\ ; U = 60\ \text{J}\ ;$$
$$\tau = 17\ \text{ps}\ ;\ Z_0 = 2.5\ \text{mm}\ ;\ \lambda_R = 0.35\ \text{Å}\ ;$$
$$\rho = 1.2\cdot 10^{-4}\ ;\ L_G = 531\ \mu\text{m}\ ;\ \bar{\rho} = 0.2\ .$$

The paremeter values predicted for case 1), which is in the calssical regime, are in agreement with the results of 3D simulations reported in ref.6,9, where a 15 MeV electron beam carrying 1.5 kA of current, focused down to 10 µm rms focal spot size, was considered colliding with a Ti:Sa pulse with $a_0 = 0.8$, and a 20 µm spot size, which corresponds to a power of 5.5 TW. The saturation of the FEL instability is reached, for an emittance of 0.44 µm, at 4 ps of laser pulse length, implying the need of 22 J of laser pulse energy and an effective gain length of about 120 µm. The radiation wavelength was 3.64 Å. Since these simulations were performed before deriving the set of formulas reported in this paper, the agreement is only on a general frame. More detailed comparisons between this set of formulas, that are meant to drive the initial choice of parameters for the simulations, and the simulation results, will be the subject of a future work.

As a last remark we report here the results of 1D simulations (performed with a code described elsewhere[10]) evaluating the effects of laser ripples on the growth of the FEL instability, taking same laser and electron beam parameters as for the previously mentioned 3D simulations. The laser amplitude modulation was taken as $\Delta\sin(k_{fluct}z + ct)$. The FEL saturation intensity is plotted as a function of ω_{fluct}/ω_L ($\omega_L \equiv 2\pi c/\lambda$) for different values of the laser ripple parameter Δ.

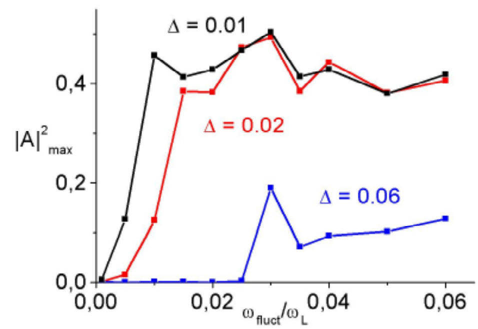

Figure 2: Saturation intensity in presence of laser ripples.

It is clearly visible in Fig.2 how the FEL instability is damped, *i.e.* the exponential growth is no longer attained, when Δ assumes values greater than a few percent, for almost any value of the modulation scale length k_{fluct}. Also, the system seems to tolerate laser amplitude modulations occurring on a scale much shorter than the gain length, as indicated by the fact that the saturation intensity goes to zero for any value of Δ when the scale of the laser modulation gets close to gain length, *i.e.* when $L_G/\lambda = \omega_{fluct}/\omega_L = 1/4\pi\rho = 0.0055$.

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