# **STAIR-STEP TAPERED WIGGLER FOR HIGH-EFFICIENCY FEL \***

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# Abstract

A concept of a high-efficiency wiggler called the stair-step tapered wiggler is presented. The stair-step tapered wiggler consists of several uniform wiggler segments with decreasing wiggler periods (or decreasing  $a_w$ ). The relatively large bucket in each wiggler segment enables a substantial fraction of the electrons to be captured, resulting in high extraction efficiencies. The stair-step tapered wiggler provides other advantages, such as ease of fabrication and flexibility in the taper rate. Numerical simulations using MEDUSA will be presented to show the high-efficiency performance of a representative FEL with a stair-step tapered wiggler.

# **INTRODUCTION**

The FEL is known for its wavelength tunability and scalability to high power. However, the extraction efficiency, defined as the FEL power divided by the electron beam power, can be low, with typical values around 1%. The low extraction efficiency increases the requirements for the electron beam's average power and puts stringent demands on the high-power RF systems driving the accelerators. One way to increase the extraction efficiency is through the use of tapered wigglers. Most tapered wigglers are continuous, *i.e.* the periods  $(\lambda_w)$  or dimensionless field  $(a_w)$  decreases continuously to maintain the resonance condition,

$$\gamma_R = \sqrt{\frac{\lambda_w}{2\lambda} \left( 1 + a_w^2 \right)} \,, \tag{1}$$

where  $\gamma_{k}$  is the resonance Lorentz factor; and  $\lambda$  is the wavelength.

Continuously tapered wigglers are only optimized for a narrow range of input radiation intensities and wavelengths. For an oscillator FEL, extraction efficiencies as high as 4.6% inside the resonator have been achieved but the efficiency outside the optical cavity is not significantly higher than that of a uniform wiggler [1].

In this paper, we discuss a different concept of tapering to achieve high extraction efficiency using a series of uniform wigglers. Our approach is an extension of the compound wiggler [2], also known as the step-tapered wiggler [3]. In our approach, a long uniform wiggler is used to bunch the electrons longitudinally and a number of short uniform segments with decreasing the wiggler periods (Fig. 1) or increasing the gaps between the magnets (Fig. 2) are used to extract the energy from the \*Work supported by the Office of Naval Research and the High-Energy Laser Joint Technology Office. Author email: dcnguyen@lanl.gov electrons as they execute synchrotron oscillations inside the ponderomotive potentials.



Figure 1: Plot of wiggler period versus z in a stair-step tapered wiggler with taper in wiggler period.



Figure 2: Plot of gaps between magnets versus z in a stair-step tapered wiggler with taper in magnetic field.

# THEORY

In a uniform wiggler, the FEL extraction efficiency is the product of the capture efficiency ( $\eta_c$ ) and the height of the bucket, as given by [4]

$$\eta = 2\eta_C \sqrt{\frac{a_w a_s}{1 + a_w^2}} \quad , \tag{2}$$

where the term in the radical denotes the half-height of the bucket, and  $a_s$  the dimensionless optical field, is proportional to the square root of the optical intensity,  $I_s$ 

$$a_s = \frac{e\lambda\sqrt{2Z_0I_s}}{2\pi nc^2} \tag{3}$$

where e is electronic charge,  $Z_0$  impedance of free space, m electron mass, and c speed of light.

As the electrons interact with the high optical intensity, they undergo synchrotron motion in the first uniform wiggler and, after becoming trapped, rotate to the bottom of the bucket. With a judicious combination of  $\lambda_w$  and  $a_w$ (see Eq. 1), the resonance energy of the next uniform wiggler segment can be lowered such that the electrons are at the top of the bucket of the new wiggler segment (Fig. 3). The electrons that are trapped in the bucket of the new segment now undergo synchrotron motion in this bucket to an even lower energy. The step-tapered wiggler efficiency is the sum of the first and second segments' efficiencies.

$$\eta = 2\eta_{C1} \sqrt{\frac{a_{w1}a_{s1}}{1 + a_{w1}^2}} + 2\eta_{C2} \sqrt{\frac{a_{w2}a_{s2}}{1 + a_{w2}^2}}$$
(4)



Figure 3: Longitudinal phase space of electrons at the transition between the first and second wiggler segment. The red line denotes the separatrix of the second wiggler.

The indices 1 and 2 in Eq. 4 correspond to the first two wigglers, respectively. The typical efficiency for a twosegment step tapered wiggler is twice that of a uniform wiggler, *i.e.* 2%. If there are n segments, then the efficiency can be increased n-fold. The limit in how far we can taper the wiggler is due to a steady reduction in the capture efficiency in each subsequent bucket, and in the induced energy spread. This is due to the non-adiabatic transitions from one segment to the next. However, since the bucket heights are large, it is possible to maintain high capture efficiency, and thus high extraction efficiency.

# SIMULATION

We use the three-dimensional code MEDUSA [6,7] to model a seeded FEL amplifier with a stair-step tapered wiggler and a comparable linearly tapered wiggler. Table I summarizes the FEL and beam parameters used in the MEDUSA simulations. These parameters are chosen for a 1.05-micron wavelength where high-power seed lasers exist. Both wigglers are of a conventional permanentmagnet design with parabolic pole faces to provide equal two-plane, sextupole focusing. The first wiggler segment is the same for both linear and stair-step tapered wigglers. With an input power of  $10^6$  W (1  $\mu$ J pulse energy and 1 ps FWHM, for instance) the first wiggler saturates in 1.83 m. The subsequent wiggler segments are different for the two cases. For the linear taper, it has one 2.44-m long continuously tapered segment with a field taper rate of 0.48 kG per meter of wiggler length. For the stair-step taper, there are four short uniform wiggler segments, each with a smaller  $a_w$ . These wiggler segments can be short because the electrons are already bunched at the entrance of these segments. The pre-bunched electrons radiate power immediately and the electrons execute the synchrotron motion to the bottom of the bucket in each wiggler segment. For these simulations, the electron beam is matched in the wiggler. It is conceivable that a scalloped beam could be used to enhance the FEL interaction and/or to pinch the optical beam [5].

Table 1: MEDUSA simulation parameters and results.

Parameters	Values
Beam energy	80.8 MeV
Peak current	1000 A
Emittance	10 mm-mrad
Energy spread	0.25%
Wiggler period	2.18 cm
Wavelength	1.052 μ
First wiggler segment a <sub>w</sub>	1.187
First wiggler segment length	1.831 m
Second wiggler segment aw	1.159
Second wiggler segment length	0.698 m
Third wiggler segment a <sub>w</sub>	1.116
Third wiggler segment length	0.567 m
Fourth wiggler segment a <sub>w</sub>	1.058
Fourth wiggler segment length	0.523 m
Fifth wiggler segment $a_w$	1.007
Fifth wiggler segment length	0.654 m
Linear taper segment length	2.44 m
Linear taper rate	0.48 kG/m
Input power (peak)	1 MW
Stair-step taper output power (peak)	3.6 GW
Stair-step taper efficiency	4.5%
Linear taper output power (peak)	3.5 GW
Linear taper efficiency	4.4%



Figure 4: Plots of magnetic field versus distance along the wiggler for the linear (red) and stair-step (blue) tapered wigglers.

The MEDUSA simulation results are shown in Fig. 5. The stair-step and linear tapered wigglers achieve peak powers of 3.6 and 3.5 GW, respectively, corresponding to extraction efficiencies of 4.5% and 4.4%. While the FEL power grows continuously for the linear taper wiggler, the FEL power growth curve in the stair-step taper wiggler exhibit plateau regions in between segments. This is not due to a lack of wiggler magnets in the transition regions (there are no gaps in the stair-step wiggler). Rather, these plateaus are needed to rotate the electrons in the bucket so that they enter the next bucket in the correct phase of the synchrotron oscillation period. It is worth noting that the FEL peak power increase is approximately 0.7 GW per wiggler segment, and the extraction efficiency of the five-segment stair-step tapered wiggler is about 5 times that of the uniform wiggler.



Figure 5: Plots of power versus distance for the linear (red) and stair-step (blue) tapered wigglers.

An important consideration in designing any highefficiency wiggler is the energy spread in the electron beam exiting the wiggler. For energy recovery linac, it is necessary to minimize the energy spread of the spent electron beam so that one can transport the electron beam back through the linac for energy recovery. To this end, we plot the energy distributions of the electrons exiting the linear taper (red) and stair-step taper (blue) wigglers in Fig. 6. While the linear taper has a double-humped distribution with most electrons localized near the injected energy and maximum decelerated energy, the stair-step taper has electrons distributed in multiple peaks within the same energy bandwidth. The full energy spread for both tapered wigglers is 13%, three times the extraction efficiency of 4.5%. This energy spread is presently outside the 8% energy acceptance of the energy recovery linac [8]. However, we expect a reduction in electron energy spreads will be realized with further optimization of the stair-step tapered wiggler.



Figure 6: Energy distribution of electrons exiting the stair-step tapered wiggler.

#### **SUMMARY**

We have studied a different approach to high-efficiency wiggler called the stair-step tapered wiggler. The stair-step wiggler consists of uniform wigglers with decreasing magnetic fields (or wiggler periods) to extract power from a decelerated electron beam. MEDUSA simulations show the performance of the stair-step tapered wiggler is the same as that of a comparable linear tapered wiggler. For a representative example of an infrared FEL, the peak power (and extraction efficiency) that can achieved with a five-segment stair-step tapered wiggler is five times that of a uniform wiggler. The full energy spread of the electron beam exiting the stair-step wiggler is three times the extraction efficiency.

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### REFERENCES

- D. Feldman et al., Nucl. Instrum. Meth. A285 (1989) 11-16.
- [2] R.W. Warren and D. Piovella, Nucl. Instrum. Meth. A304 (1991) 696.
- [3] J. Blau et al., Nucl. Instrum. Meth. A483 (2002) 148.
- [4] C.A. Brau, Free Electron Lasers, Academic Press, Oxford (1990).
- [5] D.C. Nguyen, H.P. Freund, and W. Colson, Phys. Rev. ST-AB 9 (2006) 050703.
- [6] H.P. Freund, S.G. Biedron, and S.V. Milton, IEEE J. Quantum Electron. 36 (2000) 275.
- [7] H.P. Freund, Phys. Rev. ST-AB 8 (2005) 110701.
- [8] L. Merminga, D.R. Douglas and G.A. Krafft, Annu. Rev. Nucl. Part. Sci., 53 (2003) 387-429.