# DESIGN OF THE CAVITY BPM FOR FERMI@ELETTRA

P. Craievich<sup>\*</sup>, D. Castronovo, M. Ferianis, ELETTRA, Trieste, Italy M. Poggi, Laboratori Nazionali di Legnaro, INFN, Legnaro, Italy

### Abstract

High resolution Beam Position Monitors (BPM) are fundamental diagnostics for a seeded FEL, like FERMI@ELETTRA, as they allow to measure the electron beam trajectory non destructively and on a shot-by shot basis. Cavity BPMs provide the required sub-micrometer resolution relying on the excitation of the  $TM_{110}$  dipole mode when the beam passes through the cavity off axis. For the seeded FEL FERMI, we adopted a pair of cavity BPMs located upstream the modulating undulator to measure the electron beam trajectory at the sub-micrometer level. In this paper we first discuss the requirements for such a cavity BPM including those for the beam based alignment. The scaling from an X-band design to the final C-band design is presented. The resolution to stay below one micrometer has been cross-checked, both analytically and numerically. The losses of the common mode  $TM_{010}$  have been also checked, leading to the final dimensions which keep the losses at the level of the X-band cavity BPM.

### **INTRODUCTION**

An accurate measurement of the transverse beam position is an important issue throughout all the machine; in the undulator sections, the position and angle of the electron beam at the entrance is a fundamental parameter to be measured in order to match it to the seed laser trajectory to maximise the interaction, laser to beam. In order to provide the required single shot resolution for the position measurement two cavity BPMs have been adopted in front of the modulator. According to the FERMI layout based on two undulator sections (FEL-1 and FEL-2), two pairs of cavity BPMs will be located in front of each FEL. The design of this cavity BPM is based on a previous development [1, 2]. It is a resonant pill-box cavity where the information on beam position is encoded in the amplitude and phase variations of the dipole mode  $(TM_{110})$  with beam transverse position, as measured on the two ports located on opposite sides of the cavity. A scaling from X-band to C-band of cavity BPM has been performed. We evaluated by means of analytical and numerical models that the resolution at C-band remains below 1 micron. In the scaling process we paid attention at the losses in the common mode: the new dimensions have been set to get same losses as for the X-band common mode. Using previous analysis we have fixed geometrical dimensions for a 3-D model of the cavity BPM and coupling waveguide used to extract the signals excited by electron bunch. Here we present results of the simulations obtained by means of Microwave Studio 3-D code [3].

### BACKGROUND

The cavity BPM is essentially a pill-box cavity; the analytical formulas for a cylindrical cavity with circular section without beam pipe are an upper limit to estimate beam losses and RF parameters. From a theoretical point of view the energy losses experienced by a charge q when passing through a cavity are:  $\Delta U_{110} = q^2 k_{110} x^2$  and  $\Delta U_{010} = q^2 k_{010}$  for  $TM_{110}$  and  $TM_{010}$  modes respectively. The loss factors are defined as  $k_{110} = (\omega_{110}/2)(R/Q)_{110}$  and  $k_{010} = (\omega_{010}/2)(R/Q)_{010}$ .  $\omega_{110}$  and  $\omega_{010}$  are the resonant frequencies of dipole and monopole modes. The ratio R over Q is defined for the two modes as:

$$\left(\frac{R}{Q}\right)_{110} = \frac{|V_z(x)|^2_{110}}{x^2 2\omega_{110} U_{110}} \tag{1}$$

$$\left(\frac{R}{Q}\right)_{010} = \frac{|V_z(x)|^2_{010}}{2\omega_{010}U_{010}} \tag{2}$$

where  $|V_z(x)|_{110}^2$  and  $|V_z(x)|_{010}^2$  are the squared voltages for each mode seen by a charge particle when it flies along the longitudinal axis at some offset x (x=0 for monopole).  $U_{110}$  and  $U_{010}$  are stored energy in cavity for the  $TM_{110}$ and  $TM_{010}$  modes respectively. The ratio R over Q of a given mode can be also theoretically computed for a cavity without beam pipe.

### Dipole mode $TM_{110}$

$$\left(\frac{R}{Q}\right)_{110} = \frac{2LJ_1(\frac{a_{11}}{b}x)^2 T^2}{\pi\epsilon_0\omega_{110}b^2J_0(a_{11})^2x^2}$$
(3)

where L is cavity gap, b is the cavity radius,  $\epsilon_0$  is the permeability of the vacuum,  $J_0$  is the  $0^{th}$  order Bessel function of the  $1^{st}$  kind,  $J_1$  is the  $1^{st}$  order Bessel function of the  $1^{st}$ kind, with its first zero  $a_{11} = 3.832$  and T is the transit time factor defined by:

$$T = \frac{\sin\left(\frac{\omega_{110}L}{2c_0}\right)}{\frac{\omega_{110}L}{2c_0}} \tag{4}$$

being  $c_0$  the speed of light. It is worthwhile to mention that for small x,  $J_1(\frac{a_{11}}{b}x) \approx \frac{a_{11}}{2b}x$ ; furthermore R over Q and also  $k_{110}$  become independent from x. The loss factor  $k_{110}$ , as a function of the cavity gap L and of the resonant frequency of  $TM_{110}$ , is plotted in figure 1. In general for a small cavity gap, the transit time factor T is almost equal to 1, but from figure 1 we can see that in the X-band range

<sup>\*</sup> paolo.craievich@elettra.trieste.it

and with a cavity gap longer than 15mm, approximately, there is a reduction of the loss factor due to influence of T.



Figure 1: Loss factor of the dipole mode  $TM_{110}$  versus cavity gap L and resonant frequency.

Monopole mode  $TM_{010}$ 

$$\left(\frac{R}{Q}\right)_{010} = \frac{LJ_0(\frac{a_{01}}{b}x)^2 T^2}{\pi\epsilon_0 \omega_{010} b^2 J_1(a_{01})^2}$$
(5)

where  $a_{01} = 2.405$  is the first zero of the Bessel function  $J_0$  and T defined as above. For the monopole mode the R over Q is calculated on longitudinal axis (x=0) where  $J_0(\frac{a_{01}}{b}x) = 1$ . The loss factor  $k_{010}$  as a function of cavity gap L and resonant frequency of  $TM_{010}$  is shown in figure 2. The energy lost in the  $TM_{010}$  mode produces undesirable wake fields and has to be minimized in the design. We can see from figure 2 that, using a lower resonant frequency, we can increase cavity gap L without increasing the value of the loss factor.



Figure 2: Loss factor of the monopole mode  $TM_{010}$  versus cavity gap L and resonant frequency.

### **BPM RESOLUTION**

As presented in [1, 2] it is possible to design the coupling waveguide to selectively couple out to the  $TM_{110}$  mode but not the  $TM_{010}$  mode. Without the contamination of the monopole mode, the intrinsic BPM resolution is limited by the signal to thermal noise ratio of the system. For a given coupling coefficient, the external signal voltage on matched impedance  $Z_0$  for a charge q is:

$$V_{ext} = q \sqrt{Z_0 \frac{\omega_{110}}{Q_{ext}} k_{110}} x \tag{6}$$

The thermal noise voltage of the BPM is given by  $N = \sqrt{Z_0 k T_k \Delta f}$  where k is the Boltzmann's constant,  $T_k$  the temperature in Kelvin and  $\Delta f = f_{110}/Q_{ext}$  is the bandwidth. The upper limit of resolution can be achieved when the external signal  $V_{ext}$  is equal to the thermal noise N for:

$$x_{min} = \frac{1}{q} \sqrt{\frac{kT_k}{2\pi k_{110}}} \tag{7}$$

Assuming a charge q = 0.8nC and  $T_k = 300K$ , in figure 3 we plot the BPM resolution with respect to the resolution at X-band as a function of resonant frequency of dipole mode  $TM_{110}$ . As a reference case for an X-band design we have assumed a pill-box cavity without beam pipe, with gap length L = 3mm and cavity radius b =14.7mm. In table 1 we report the parameters for the Xband cavity used as reference.



Figure 3: Resolution of the cavity BPM with respect to the resolution of an X-band cavity BPM versus its resonant frequency.

Table 1: Loss factors and resonant frequencies for X-band cavity BPM without beam pipe used as reference.

$f_{110}$	$k_{110}$	$x_{min}$	$f_{010}$	$k_{010}$
[GHz]	$[V/nC/mm^2]$	[nm]	[GHz]	[V/nC]
12.4	49.7	0.14	7.8	907.6

# SCALING OF THE CAVITY BPM

At X-band frequencies the mechanical tolerances are more stringent than at lower frequencies. Considering also the requirement on the minimum beam pipe radius, being equal to about 10mm in the undulator region, we have explored the achievable performance adopting a scaling at frequencies lower than X-band. In fact, dipole mode  $TM_{110}$  will be coupled away into the beam pipe if its resonant frequency is higher than the lowest cut-off frequency of the beam pipe. Assuming a beam pipe radius of 10mm, the beam pipe cut-off frequencies are: 8.8GHzand 11.5GHz, respectively for the  $TE_{11}$  and  $TM_{01}$  circular waveguide modes. Considering the available C-band hardware together with the above motivations, a resonant frequency of 6GHz (C-band) for the cavity BPM (radius cavity b = 30.4mm) has been selected. From figure 3, we can see that the resolution at C-band is still within an acceptable range. Looking at figures 2 we can increase the cavity gap up to about L = 10mm without increasing losses in  $TM_{010}$  mode if compared to the losses in the X-band cavity. Assuming a charge q = 0.8nC, at room temperature, in table 2 are reported the parameters for Cband BPM cavity.

Table 2: Loss factors and resonant frequencies for C-band BPM cavity without beam pipe (cavity radius b = 30.4mm and gap length L = 10mm).

$\begin{array}{c} f_{110} \\ [GHz] \end{array}$	$\frac{k_{110}}{[V/nC/mm^2]}$	$x_{min}$ [nm]	$\begin{array}{c} f_{010} \\ [GHz] \end{array}$	$\frac{k_{010}}{[V/nC]}$
6.0	8.3	0.35	3.8	684.9

### **C-BAND CAVITY BPM**

Preliminary numerical simulations by means of CST MWS[3] were done on a 3-D model with beam pipe to have a comparison with previous analytical treatments. For 3-D model we have adopted the following dimensions: b =30.4mm, L = 10mm and beam pipe radius a = 10mm. Table 3 shows results from analytical and numerical calculations and we can observe that the analytical one without beam pipe is a good estimation of cavity BPM with beam pipe for both dipole and monopole modes. We have adopted as waveguide the commercially available standard WR137 with internal dimensions of  $34.85 \times 15.80 mm^2$ . To couple this waveguide to the cavity we have left unchanged the width while we have used a transition to 6mm for the height. This standard works in the frequency range of 5.85-8.20 GHz thus we have fixed the resonant frequency at 6.5GHz decreasing the radius of the cavity to 26.4mm. Figure 4 shows the 3-D model of the C-band cavity BPM where we can observe the waveguide transition to the standard WR137.

The external voltage in eq. 6 depends on  $Q_{ext} = Q_0/\beta$ where  $\beta$  is the coupling coefficient and  $Q_0 = 8530$  is the



Figure 4: C-band cavity BPM with its four coupling waveguides and reference cavity.

Table 3: Comparison between numerical obtained by means of CST MWS and analytical results for C-band BPM.

	analytical	numerical	unit
$f_{110}$	6.0	5.7	[GHz]
$(R/Q)_{110}$	0.44	0.39	$[\Omega/mm^2]$
$k_{110}$	8.3	7.0	$[V/nC/mm^2]$
$Q_0$		8400	
$f_{010}$	3.8	4.0	[GHz]
$k_{010}$	685	523	[V/nC]

unloaded Q factor of the cavity with new dimensions. Figure 5 shows the dipole  $Q_{ext}$  as a function of the distance between beam axis and bottom of the waveguide.



Figure 5:  $Q_{ext}$  of the dipole mode as a function of distance of the bottom of waveguide to beam axis.

The waveguide is always under coupled to the cavity and for distance below 13 mm the  $Q_{ext}$  remains about constant. We have fixed the distance between waveguide and beam axis at 12 mm thus  $\beta = 0.1$  and  $Q_{ext} = 10500$ . In this situation we only have a loss of 0.9 dB in the external voltage with respect to a critical coupling  $\beta = 1$ . In figure 4 the reference cavity is also indicated. The cavity will be used to provide the reference signal in the RF electronics and is designed to resonate at 6.5 GHz approximately with  $TM_{010}$  mode. The external coupling is foreseen by means of an antenna placed on the cavity lateral side.

A prototype C-band cavity BPM is being designed and cold test are foreseen. Table 4 shows dimensional and RF parameters of the C-band and reference cavity prototypes. From the mechanical point of view, we are faced with some challenges. First of all the required machining tolerance was fixed to  $\pm 10 \mu m$  for cavities, waveguides and their relative positioning. An important item is to decide how to build the BPM and reference cavities from bulk copper. One idea is to divide in two parts the whole geometry along the longitudinal symmetrical plane. The first part consists of half cavity together with waveguides, as shown in figure 6 [4], and second part consists of the other half cavity without waveguides. Both parts will be milled with the profile of the cavity and waveguides and subsequently the beam pipes, cavity and waveguides are brazed together. So the challenge concerns the brazing process and in particular to find the best brazing material. During the process it is important not to pollute the copper nor to deform the construct. This last item must be investigated through additional numerical simulations.

Table 4: Collections of the dimensional and RF parameters of the C-band cavity BPM and reference cavity.

C-band cavity BPM			
Cavity gap	10mm		
Cavity radius	26.4mm		
Beam pipe radius	10mm		
Coupling WG	$34.85\times 6mm^2$		
Distance WG to beam axis	12mm		
WG standard	WR137		
Resonant frequency	6.5GHz		
Unloaded Q factor	8530		
External Q	10500		
Coupling coefficient	0.1		
Reference cavity			
Cavity gap	10mm		
Cavity radius	17.6mm		

# CONCLUSION

In this paper we have explored a possible scaling of the cavity BPM presented in [1, 2]. For our considerations, we have used a model of a pill-box without beam pipe and have shown that a scaling from X-band to C-band frequencies is possible without loss of resolution. In addition, we have done simulations on a 3-D model to validate the analytical treatment and we have analyzed the position of the coupling waveguide to the cavity. These studies have allowed



Figure 6: Preliminary technical drawing of the cavity BPM [4].

fixing dimensional parameters for a C-band prototype of the cavity BPM.

## REFERENCES

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