THE EFFECT OF AN AC AXIAL MAGNETIC FIELD PERTURBATION ON MODE TRANSITION OF A CIRCULAR FEL

M. Rezvani Jalal[1], F. M. Aghamir[2]
University of Tehran Iran.

Abstract

The effect of a modifying ac axial magnetic field perturbation on single mode operation of a circular FEL is presented. The mode transition and the effect of perturbation field are investigated. The mode transition in the very short interval of each shot can be important in time-averaged mode superposition and also in production of pulse modes. In the present work we consider the subject of mode transition by adding an ac axial magnetic field perturbation to the axial dc guide field and investigate the radiation modes and allowed transitions as a function of perturbation frequency

INTRODUCTION

Many experimental and theoretical studies have been carried out on conventional linear geometry Free Electron Lasers [1.2]. These devices have gain limitations due to the finite length of the interaction region. The circular geometry free electron laser has in principal infinite interaction length, thus the gain of these devices does not saturate. There are, however; some issues in regard to operation of circular FEL, which have not been addressed. First, high quality single mode operation of these lasers has not been represented in literature. Secondly, it is rather difficult to incorporate circular FEL's as compared with conventional linear form with high power accelerators. In a circular FEL magnetic field configuration consists of a uniform axial field $B_0\hat{a}_z$ and an azimuthally periodic wiggler field. In this article an oscillating axial magnetic field $B_1(\rho)Sin(\omega_0 t)\hat{a}_z$ $(B_1(\rho) << B_0)$ is added to the axial guide field. The mode transition of circular FEL is investigated through the effect of the modifying magnetic field and the induced perturbing electric field. Our study shows that the application of the ac magnetic field leads to the operation of the circular FEL in transitional TM mode. The induced electric field affects the cyclotron mode and changes its frequency. This results in mode transition in transverse magnetic (TM) modes.

THEORETICAL MODEL

In our analysis, an annular electron beam of small thickness is considered which rotates within the space between the inner and the outer conductors of a coaxial metallic waveguide (see Fig. 1). The modifying axial ac magnetic field is superimposed on the axial guide field and an azimuthally periodic wiggler field which perturbs the electron beam. The time dependent magnetic field, present inside the waveguide consists of two parts, one being the magnetic field of the electromagnetic wave generated by the laser action $\vec{B}_2(\vec{r},t)$, and the other part [1]is the modifying ac field $B_1(\rho)\sin(\omega_0 t)\hat{a}_z$:

$$\vec{B}(\vec{r},t) = B_1(\rho)\sin(\omega_0 t)\hat{a} + \vec{B}_2(\vec{r},t) \quad (1)$$

Using the Maxwell's equations, the resulting wave equations for the magnetic field and the induced electric field are:

$$\nabla^2 \vec{B}_2(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{B}_2(\vec{r},t) = -\frac{\omega_0^2}{c^2}B_1(\rho)\sin(\omega_0 t)\hat{a}_z \quad (2)$$

$$\nabla^2 \vec{E}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}(\vec{r},t) = 0 \quad (3)$$

The wave equation for the magnetic field, Eq. (2) can be broken up into logitudinal and transverse parts, then

$$\nabla^2 \vec{B}_{2t}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{B}_{2t}(\vec{r},t) = 0 \quad (4)$$

$$\nabla^2 B_{2z}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}B_{2z}(\vec{r},t) = -\frac{\omega_0^2}{c^2}B_1(\rho)\sin(\omega_0 t) \quad (5)$$

The above equations reveal that the purturbing axial magnetic field only couples to TE waveguide modes. Assuming the resonance of a TE mode at frequency ω_0 with the ac purturbing field, the solution of the wave equations can be written as:

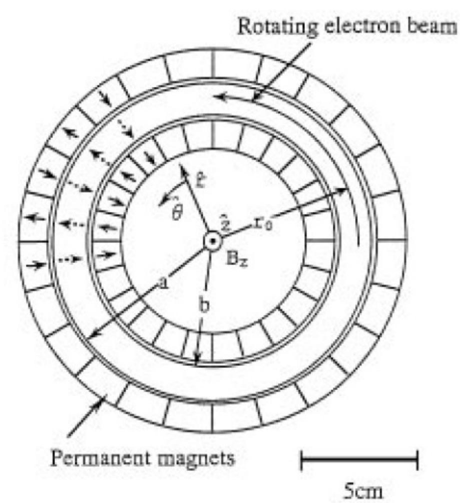


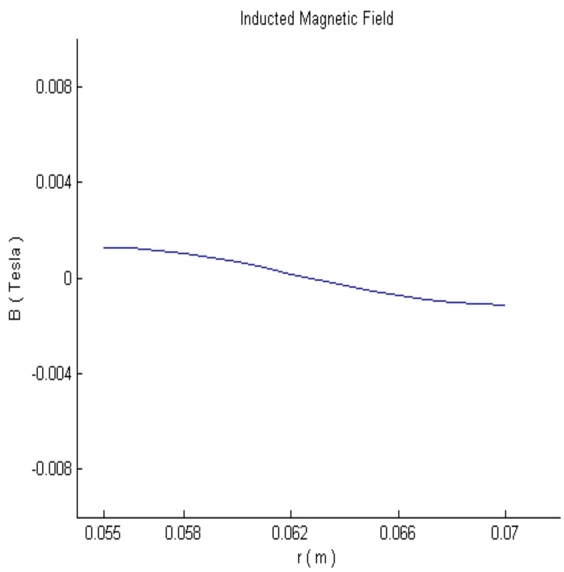
Figure 1: Layout of circular FEL.

[1]Mrj59@phymail.ut.ac.ir
[2]agamir@khayam.ut.ac.ir

$$\vec{B}(\vec{r},t)=[a_0 J_0(\frac{\omega_0}{c}\rho)+b_0 N_0(\frac{\omega_0}{c}\rho)]e^{-i\omega_0 t}\hat{a}_z \quad (6)$$

$$\vec{E}(\vec{r},t)= ic[a_0 J_1(\frac{\omega_0}{c}\rho)+b_0 N_1(\frac{\omega_0}{c}\rho)]e^{-i\omega_0 t}\hat{a}_\varphi \quad (7)$$

Typical radial profiles of the above magnetic and electric fields across the waveguide cross section are shown in Fig. 2.



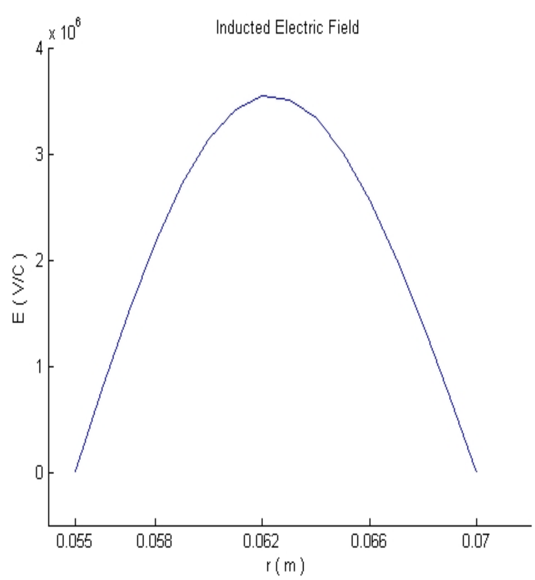


Figure2: The induced magnetic field (top) and the induced electric field (bottom) radial profiles across the waveguide cross section. Here $\omega_0 = 6.297\ GHz$, b= 0.07m, and a=0.055m.

a_0 and b_0 are determined by the application of the boundry conditions.

DISPERSION RELATION

The motion of an electron in the induced electromagnetic fields can be described through the Lorentz force equation:

$$\frac{d}{dt}\gamma\vec{v}=-\frac{e}{m}(\vec{E}+\vec{v}\times\vec{B}) \quad (8)$$

Writing the perturbed quantities $\vec{v}$, γ , $\vec{E}$, and $\vec{B}_t$ in the following form [3]:

$$\vec{v}_t=\vec{v}_{0t}+\delta\vec{v} \qquad \gamma_t=\gamma_{0t}+\delta\gamma$$
$$\vec{E}_t=\vec{E}_{0t}+\delta\vec{E} \qquad \vec{B}_t=\vec{B}_{0t}+\delta\vec{B} \quad (9)$$

Where:

$$\begin{aligned}\vec{v}_{0t}&=-\frac{|\vec{E}(\vec{r}_0)|}{B_0}Sin(\omega_0 t)\hat{a}_\rho\\ &+(r_0\Omega_0-\frac{e|\vec{E}(\vec{r}_0)|}{\gamma_0 m\omega_0}Cos(\omega_0 t))\hat{a}_\varphi\\ &+2^{\frac{1}{2}}ca_w(\frac{1}{\gamma_0}+\frac{e\beta_{\varphi 0}\gamma_0|\vec{E}(\vec{r}_0)|}{\gamma_0 m\omega_0 c}Cos(\omega_0 t))Cos(N\varphi)\hat{a}\end{aligned} \quad (10)$$

$$\vec{E}_{0t}=\vec{E}(\vec{r},t) \qquad (The\ induced\ electric\ field)$$

$$B_{0t}\approx 0$$

$$\gamma_{0t}\approx\gamma_0=\frac{1}{\sqrt{1-\frac{(r_0\Omega_{Cyc})^2}{c^2}}}$$

and considering only TM waves, the force balance equation becomes:

$$\begin{aligned}&\delta v_z(\frac{e\gamma_0^3\beta_{\varphi 0}|\vec{E}(\vec{r}_0)|}{mc}Sin(\omega_0 t))\\ &+(\gamma_0 t-\frac{e\gamma_0^3\beta_{\varphi 0}|\vec{E}(\vec{r}_0)|}{mc\omega_0}Cos(\omega_0 t))\frac{d}{dt}\delta v_z=\\ &\quad -\frac{e}{m}(\delta E_z-\delta B_\rho\frac{e|\vec{E}(\vec{r}_0)|}{m\omega_0}Cos(\omega_0 t)\\ &\quad -\delta B_\varphi\frac{|\vec{E}(\vec{r}_0)|}{B_0}Sin(\omega_0 t))\end{aligned} \quad (11)$$

It should be noted that we have considered only the TM modes where $\delta B_z=0$, $\delta\vec{E}_{\rho,\varphi}=0$. In order to obtain the dispersion properties, the spatial parts of the perturbed quantitie are separated as follows [4].

$$\delta f(\vec{r},t)=\delta f(\rho)e^{-i\omega(t)t} \quad (12)$$

Substituting in Eq.(11) yields:

$$\delta v_z\left(\frac{e\gamma_0^3\beta_{\varphi 0}\left|\vec{E}(\vec{r}_0)\right|}{mc}Sin(\omega_0 t)\right)+$$
$$\left(\gamma_0 t-\frac{e\gamma_0^3\beta_{\varphi 0}\left|\vec{E}(\vec{r}_0)\right|}{mc\omega_0}Cos(\omega_0 t)\right)\Omega(t)\delta v_z=$$
$$-\frac{e}{m}\left(\delta E_z-\delta B_\rho\frac{e\left|\vec{E}(\vec{r}_0)\right|}{m\omega_0}Cos(\omega_0 t)\right.$$
$$\left.-\delta B_\varphi\frac{\left|\vec{E}(\vec{r}_0)\right|}{B_0}Sin(\omega_0 t)\right) \tag{13}$$

Where $\Omega(t)=i[\omega(t)+\frac{d\omega}{dt}]$.

For TM_{pq} modes with axial symmetry, through the use of the Maxwell's equations we would then have

$$\frac{p}{\rho}\delta E_{z,p}=[\omega(t)+t\frac{d\omega}{dt}]\delta B_{\rho,p}$$
$$\frac{\partial}{\partial\rho}\delta E_{z,\rho}=i[\omega(t)+t\frac{d\omega}{dt}]\delta B_{\varphi,p}$$
$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\delta B_{\varphi,p}+\frac{ip}{\rho}\delta B_{\rho.p}= \tag{14}$$
$$\mu_0\delta J_{z,p}+\frac{i}{c^2}[\omega(t)+t\frac{d\omega}{dt}]\delta E_{z,p}$$

Setting p=0, the equations reduce to

$$\delta B_\rho=0$$
$$\frac{\partial}{\partial\rho}\delta E_z=i[\omega(t)+t\frac{d\omega}{dt}]\delta B_\varphi \tag{15}$$
$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\delta B_\varphi=\mu_0\delta J_{z,p}+\frac{i}{c^2}[\omega(t)+t\frac{d\omega}{dt}]\delta E_z$$

Now integrating over the raduis of the waveguide the last equation becomes:

$$\int_{r1}^{r2}\frac{\partial}{\partial\rho}\rho\delta B_\varphi d\rho=\mu_0\int_{r1}^{r2}\rho\delta J_z d\rho$$
$$+\frac{i}{c^2}[\omega(t)+t\frac{d\omega}{dt}]\int_{r1}^{r2}\rho\delta E_z d\rho \tag{16}$$

Where r_1 and r_2 are the inner and outer radius of electron ring, respectively. The first term on the right hand side can be neglected if the electron beam is considered tenuous:

$$[\omega(t)+t\frac{d\omega}{dt}]\delta E_z(r_0)=-\delta B_\varphi(r_0)c^2 \tag{17}$$

Where r_0 is the mean radius of electron ring. One more equation is needed to set up the dispersion relation. The time variation of the energy balance equation can be expressed as:

$$\frac{d}{dt}\left(\frac{1}{2}\gamma_0 mv^2\right)=\frac{d}{dt}\left(\frac{1}{2}\varepsilon_0 E^2+\frac{1}{2\mu_0}B^2\right) \tag{18}$$

Substituting for $\vec{v},\vec{E},$ and $\vec{B}$ the axial component of $\delta\vec{v}$ as a function of $\delta\vec{E}$ can be obtained as

$$\delta v_z=\frac{\varepsilon_0\left|\vec{E}(\vec{r}_0)\right|Sin(\omega_0 t)}{\gamma_0 m\omega(t)v_{0tz}}\delta E_z \tag{19}$$

From equations 13, 17, and 19 the time dependent differential dispersion equation is obtained (see Eq. 20). This determinan will give quadratic equation in terms of $\omega(t)+t\frac{d\omega}{dt}$ so it will have tow solutions. Every solution is a differential equation that in turn should be solved for $\omega(t)$. Real and imaginary parts of the obtaind frequency can give the oscillation frequency and lifetime of each exited TM mode. Both of these are oscillatory by frequency of applied perturbation field. By plotting imaginary part of abtained frequency it can be found that evry oscillation that have a positive imaginary part couldn' t been amplified and would disappear.

$$\begin{vmatrix} \left(\frac{e\gamma_0^3\beta_{\varphi 0}\left|\vec{E}(\vec{r}_0)\right|}{mc}Sin(\omega_0 t)+(\gamma_0 t-\frac{e\gamma_0^3\beta_{\varphi 0}\left|\vec{E}(\vec{r}_0)\right|}{mc\,\omega_0}Cos(\omega_0 t)[-i(\omega(t)+t\frac{d\omega}{dt})]\right) & \frac{e}{m} & -\frac{e\left|\vec{E}(\vec{r}_0)\right|}{\delta_0 mB_0}Sin(\omega_0 t) \\ 0 & [\omega(t)+t\frac{d\omega}{dt}] & c^2 \\ 1 & -\frac{\varepsilon_0\left|\vec{E}(\vec{r}_0)\right|Sin(\omega_0 t)}{m\gamma_0[\omega(t)+t\frac{d\omega}{dt}]v_{0tz}} & 0 \end{vmatrix}=0 \tag{20}$$

REFERENCES

[1]T. Mizuno, T. Otsuki, Phys. Rev. Lett. 77,2686, (1996).
[2] Ronald C. Davidson, Wayane A. Mcmullin, , Phys. Fluids, 27 (1), 233, (1984).
[3] H. Saito, J. S. Wurtele, Phys. Fluids, 30 (7), 2209, (1987).
[4]Yasushi Kawai, Hirobumi Saito, Phys. Fluids, 3 (6), 1485, (1991).