DISPERSION EFFECTS IN SHORT PULSE WAVEGUIDE FEL

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Abstract

The influence of waveguide dispersion on the FEL operation driven by short electron bunches is studied. Under the assumption of a high quality resonator, a parabolic equation for the evolution of the profile of electromagnetic pulse is derived. The condition of self-excitation are found by means of an analytical theory describing a structure of supermodes as the sum of resonator eigenmodes with locked phases. It is demonstrated that due to waveguide dispersion FEL is able to generate not only for positive but also for negative cavity detuning. The transient and nonlinear stages of the free-electron laser operation are analyzed by the computer simulation, and the optimal mismatches of group and cavity synchronism conditions are found.

INTRODUCTION

The mode-locking regime is typical for free-electron lasers (FEL) driven by a train of short electron bunches. In this regime, the electromagnetic radiation consists of micropulses with a duration nearly equal to that of the electron bunches. Both pulses (electron and electromagnetic) travel together through the resonator, but shift slightly away from each other due the difference between the wave group velocity and the electron velocity. Once they reach the right-hand mirror, the electron pulse escapes from the resonator, while the electromagnetic pulse reflects and comes back to the left-hand mirror at the time when the next current pulse arrives.

In short wavelength (optical, infrared) FEL experiments [1-4], the group velocity of electromagnetic pulses exceeds the velocity of electron bunches. To provide generation under such conditions, a specific mismatch between a period of electromagnetic pulse round trip over a resonator and a period of bunch injection was used. However, in some experimental investigations of long wavelength FELs [5-8], a waveguide may be used, so that the specific waveguide dispersion allows one to realize "zero-slippage" condition, for which the group velocity of electromagnetic pulse is equal to the longitudinal velocity of electrons. Under such conditions, mutual synchronization of radiation from different parts of electron bunches occurs due to dispersive spreading of the electromagnetic field, whereas localization of radiation near electron bunches is caused by its guiding properties [9,10].

In the present paper, a theoretical model of waveguide FEL is developed which takes into account the waveguide dispersion. Under the assumption of a high quality resonator, a parabolic equation for the evolution of the profile of electromagnetic pulse is derived. The linear, transient and nonlinear stages of the FEL operation are investigated, both analytically and numerically, and the optimal conditions are found.

THE MODEL AND BASIC EQUATIONS

Let us suggest that the radiation pulse propagating through a waveguide circulates between two mirrors (with reflection coefficients $R_{1,2}$), which are placed at some distance, L_0. Let FEL be fed by short electron pulses of duration τ_p, which is essentially less than both the round-trip of the radiation in the cavity $T_R = 2L_0 / v_{gr}$, and the repetition period of an electron bunch injection, T_i. During n-the pass through a resonator, the field can be represented as

$$A = \mathrm{Re}\left[E_s(r_\perp)\left\{ \begin{array}{l} A_n^+(z,t)\exp(i(\omega_0 t - h_0 z)) + \\ A_n^-(z,t)\exp(i(\omega_0 t + h_0 z)) \end{array} \right\} \right],$$

where E_S is a function, describing the profile of a given transverse waveguide eigenmode, ω_0 is the reference frequency, and $h_0 = h(\omega_0)$ the longitudinal wave number. Resonant electron-wave interaction takes place under the synchronism condition, $\omega_0 - h_0 v_\| \simeq \Omega_\perp$, where $\Omega_\perp = h_u v_\|$ is the bounce frequency, $h_u = 2\pi / \lambda_u$, λ_u the undulator period. We will consider the excitation of a resonator by a train of short electron bunches under the following conditions:

- the reflection coefficients are close to unity, $R_{1,2} \simeq 1$ and the changes of the wave amplitude during one pass are very small;
- dispersion spreading of the electromagnetic pulse during one pass over the resonator is small as well.

Under these approximations, we can replace the discrete variable n (pass number through the resonator) by a slow time τ with the period of one round trip, T_R taken as a time unit. Evolution of the pulse profile along the resonator can be described by the parabolic equation (for details see [9]):

$$\frac{\partial\alpha}{\partial\hat\tau} + \frac{\omega_0}{2Q}\alpha - \hat\varepsilon\frac{\partial\alpha}{\partial y} + i\frac{\hat\nu V_{gr}}{2}\frac{\partial^2\alpha}{\partial y^2} = \frac{e\pi I_0 V_{gr}\kappa}{2m_e c^3\gamma N_S h_0 l}\int_0^l f(y - z/V_{gr}) I_\omega dz, \quad (1)$$

where $I_\omega = \frac{1}{\pi}\int_0^{2\pi} e^{-i\Theta} d\Theta_0$ is the synchronous harmonic of the beam current, $V_{gr} = \partial\omega / \partial h$ the wave group velocity, $\hat\nu = |\partial^2 h / \partial\omega^2|$ the wave dispersion parameter,

$Q=\omega_0 l/V_{gr}(1-R_1R_2)$ the resonator Q-factor, I_0 the peak current of electron bunch, $\gamma=1/\sqrt{1-\beta^2}$ the relativistic Lorentz factor, n_s the norm of the operating mode, κ the electron-wave coupling coefficient, $f(y)$ a function, describing the electron pulse profile, $\alpha=eA_n/(m_e c\gamma)$ the dimensionless wave amplitude. Taking into account the cavity detuning $\hat{\varepsilon}=(T_i-T_R)/T_R$, we use an independent time variable, $y=t-z/V_{gr}-\hat{\varepsilon}\hat{\tau}$.

Under assumptions described above, we can relay upon the periodical boundary conditions:

$$\alpha(\hat{\tau},y)=\alpha(\hat{\tau},y+T_R) \qquad (2)$$

and expand the field and the beam current into Fourier series:

$$\begin{aligned}\alpha(\hat{\tau},y)&=\sum_{m=-\infty}^{\infty}a_m(\hat{\tau})\exp(i2\pi my/T_R),\\ I_\omega&=\sum_{m=-\infty}^{\infty}J_m(\hat{\tau})\exp(i2\pi my/T_R).\end{aligned} \qquad (3)$$

An amplitude of each harmonic a_m, can be treated as an amplitude of cavity eigenmode with the longitudinal index m.

Assuming a small variation of electron energy $E=m_e c^2\gamma$ and neglecting the Coulomb interaction, the electron motion equation can be presented in the form

$$\left(\frac{\partial}{\partial z}+\left(\frac{1}{V_{\|}}-\frac{1}{V_{gr}}\right)\frac{\partial}{\partial y}\right)^2\Theta=\left(\frac{\omega_0}{c}\right)^2\mu\kappa\,\mathrm{Re}\left(\alpha e^{i\Theta}\right) \quad (4)$$

with the boundary conditions:

$$\left(\frac{\partial}{\partial z}+\left(\frac{1}{V_{\|}}-\frac{1}{V_{gr}}\right)\frac{\partial}{\partial y}\right)\Theta\Big|_{z=0}=\Delta,\ \ \Theta\Big|_{z=0}=\Theta_0\in[0,2\pi],$$

where $\Delta=(\omega_0-h_0V_{\|}-\Omega_\perp)/V_{\|}$ is the initial mismatch of synchronism at the reference frequency, $\mu\simeq\gamma^{-2}$ the inertial bunching parameter. With the use of normalized variables,

$$\xi=\frac{\omega_0 y}{\chi}P,\ \tau=\frac{\omega_0\hat{\tau}}{2Q},\ \zeta=z\frac{\omega_0}{c}P,$$

$$P=\left(\frac{4\pi eI_0\mu\kappa^2\beta_{gr}Q}{m_e c\gamma dbN_s h_0 l\omega_0^2}\right)^{1/3},$$

equations (1), (4) can be transformed to the following form:

$$\begin{aligned}&\frac{\partial a}{\partial\tau}+a-\varepsilon\frac{\partial a}{\partial\xi}+i\nu\frac{\partial^2 a}{\partial\xi^2}=\int_0^L F(\xi-\zeta)I_\omega d\zeta, &(a)\\ &\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\xi}\right)^2\Theta=\mathrm{Re}\left(ae^{i\Theta}\right), &(b)\end{aligned} \qquad (5)$$

with boundary conditions:

$$a(\tau,\xi)=a\left(\tau,\xi+T\right),$$

$$\Theta\big|_{\zeta=0}=\Theta_0\in[0,2\pi],\ \left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\xi}\right)\Theta\big|_{\zeta=0}=\delta,$$

where

$$a=\frac{\alpha\kappa\mu}{P^2},\quad T=\frac{T_R\omega_0 P}{\chi},$$

$$\nu=\left|\frac{\partial^2 h}{\partial\omega^2}\right|\frac{V_{gr}\omega_0 QP^2}{\chi^2},\ \ \varepsilon=\frac{(T_i-T_R)}{T_R}\frac{2QP}{\chi},\ \ \delta=\Delta\frac{c}{\omega_0 P}.$$

$L=l\omega_0P/c$ is the normalized interaction length, $F(\xi)$ the function describing electron-pulse profile, $\chi=\beta_{\|}^{-1}-\beta_{gr}^{-1}$ is the relative value of the detuning of "zero-slippage" condition. The normalized energy stored in electromagnetic pulse is give by relation

$$W=\frac{1}{T_c}\int_0^T|a|^2 d\xi.$$

THE LINEAR THEORY

Linearizing the equation of electron motion (see Eq. 5b), we obtain an equation for the electron current:

$$\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\xi}\right)^2 I_\omega=-ia \qquad (6)$$

Using the expressions (3) we find all harmonics of current

$$\begin{aligned}J_m&=\left[\frac{i}{k_m^2}+\frac{\zeta e^{-ik_m\zeta}}{k_m}-\frac{ie^{-ik_m\zeta}}{k_m^2}\right]a_m,\\ k_m&=2\pi m/T+\delta.\end{aligned} \qquad (7)$$

As a result for the amplitude of each mode a_m from (5a) taken into account (7) we obtain

$$\left[\frac{d}{d\tau}+1-i\varepsilon\frac{2\pi m}{T}-i\nu\left(\frac{2\pi m}{T}\right)^2\right]a_m=\sum_{n=-\infty}^{\infty}C_n^m a_n, \qquad (8)$$

where

$$C_n^m=\int_0^T\int_0^L\left[\frac{i\left(1-e^{-ik_n\zeta}\right)}{k_n^2}+\frac{\zeta e^{-ik_n\zeta}}{k_n}\right]\frac{F(\xi-\zeta)e^{i\xi(k_n-k_m)}}{T}d\zeta d\xi.$$

Obviously the diagonal elements of the matrix C_n^m coincide with the expression for a complex electron permittivity found in [9, 10].

Representing the solution of Eq. 8 in the form $a_m=e^{i\Omega\tau}\hat{a}_m$ where Ω is a complex frequency, we get the algebraic equations for the supermodes of the resonator excited by a train of electron bunches:

$$i\Omega\hat{a}_m=\sum_{n=-\infty}^{\infty}D_n^m\hat{a}_n, \qquad (9)$$

where $D_n^m=C_n^m-1+i\varepsilon(2\pi m/T)+i\nu(2\pi m/T)^2.$

Assumed for simplicity that the electron pulse has the rectangular form with normalized duration $T_c = \tau_c \omega_0 P / \chi$, we obtain the following expression for the elements of the matrix D_n^m:

$$D_n^m = \frac{\left(e^{i(k_n - k_m)T_c/2} - e^{-i(k_n - k_m)T_c/2}\right)}{T} \frac{e^{-ik_m T_c/2}}{(k_n - k_m)^2} \times$$

$$\left[\frac{ie^{ik_m L}}{k_m^{\ 2}} - \frac{ie^{ik_n L}}{k_n^{\ 2}} - \frac{i}{k_m^{\ 2}} + \frac{i}{k_n^{\ 2}} + \frac{L}{k_m} - \frac{L}{k_n}\right] +$$

$$i\varepsilon \frac{2\pi m}{T} + i\nu \left(\frac{2\pi m}{T}\right)^2 - 1.$$

The starting condition of generation corresponds to the equality

$$\mathrm{Im}[\Omega] = 0. \qquad (10)$$

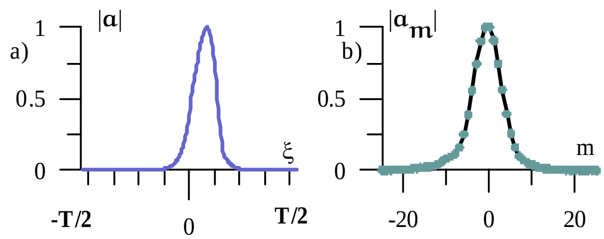

Figure 1: Normalized profile (a) and spectrum (b) of the supermode for the dimensionless parameters: $T_c = 4$, $\varepsilon = 1$, $L = 2.3$, $\nu = .3$. Resonator eigenmodes amplitudes are shown by blue dotes.

A spatial structure of supermodes calculated via eigen vectors of the matrix D_n^m is shown in Fig. 1a; it is a superposition of longitudinal resonator eigenmodes.

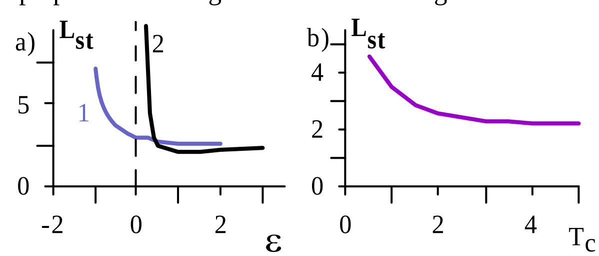

Figure 2: Dependence of the starting length, L_{st} a) on the cavity detuning ε: 1 for dispersive parameter $\nu = .3$, and 2 for the absence of dispersion and b) on the dimensionless electron bunch duration T_c.

Note that wave dispersion, playing a part of feedback, allows FEL to generate even in the case of negative values of the cavity detuning ($\varepsilon < 0$, see Fig. 2a, curve 1), for which the resonator can not be excited in the absence of dispersion (see Fig. 2a, curve 2). The increasing of the electron bunch duration leads to the decrease of the generation threshold (see Fig. 2b). For long electron bunch duration T_c this value as well as the cavity detuning does not practically influence the generation threshold.

COMPUTER SIMULATION OF THE NONLINEAR STAGE

The nonlinear stage of the electron-wave interaction was analyzed on the basis of computer simulation of Eqs. 5. The electron bunch profile has a rectangular form with normalized duration T_c. Three basic regimes of the FEL generation have been observed when the value of current exceeds the generation threshold: a) stationary regime (see Fig. 3), b) periodic self-modulation (see Fig. 4a), c) chaotic self-modulation (see Fig. 4b).

At a small excess over the generation threshold, a profile of the field and its spatial spectrum are closed to those found from the linear analysis (see Fig.1). The dynamics of electromagnetic pulse profile become more complicated with increasing the dimensionless resonator length, L (see Fig.3).

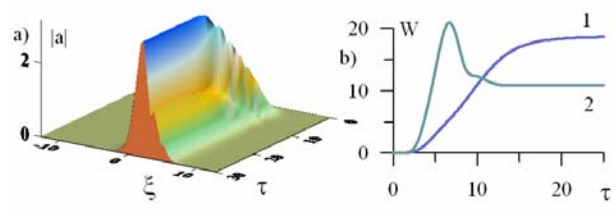

Figure 3: Regime of the stationary generation: $L = 4.4$, T=25.6, $\varepsilon = 0.5$, $T_c = 4$, $\nu = 0.3$,: a) time–space evolution of an envelope of the electromagnetic pulse; b) time dependence of the electromagnetic pulse energy W: for comparison the case of the absence of dispersion is shown by curve 2.

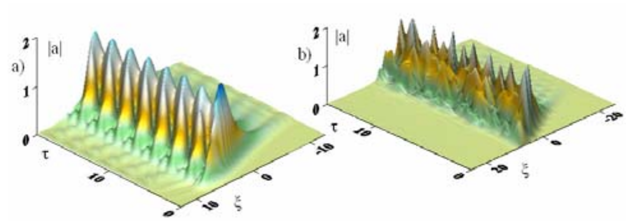

Figure 4:Time–space evolutions of an envelope of electromagnetic pulse in the regime of periodical (a) ($L = 4.4$, T=25.6, $\varepsilon = 4$, $T_c = 4$, $\nu = 1$) and chaotic self-modulation (b) ($L = 10$, T=51.2, $\varepsilon = 1$, $T_c = 6$, $\nu = 1$).

The self-modulation regime may be reached by two ways: via increasing the dimensionless length L and/or via enlarging the cavity detuning ε. Possible quasi-periodical behavior is demonstrated in Fig. 4a.

At large excess over the generation threshold, the chaotic regime of generation takes place (see Fig. 4b). At extremely large excess over the generation threshold, the pulse envelope evolves in a complicated stochastic manner, so that the generated radiation is distributed quasi-homogeneously over very wide spectral range of the resonator eigenmodes. According to estimates all these regimes are reasonable for the waveguide FEL to provide variety of applications (see Fig.5).

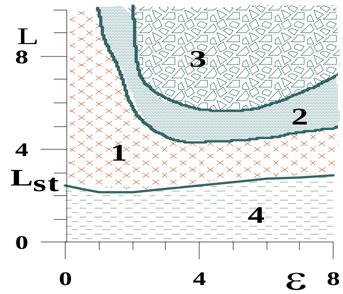

Figure 5: Zones of stationary (1), periodical (2) and chaotic (3) generation on the plane of dimensionless length L and cavity detuning ε for $\nu = 1$, $T_c = 4$, $T = 51.2$. Zone (4) is absence of generation.

The stationary regime, when one supermode is generated, has been investigated in detail for various parameters of dispersion, the detuning of “zero-slippage” conditions and cavity detuning. At the limit $\chi \to 0$ and $\varepsilon \to 0$ the equations (5) transform into the equations for the case of group synchronism [5, 6, 10]. Numerical simulation of the equations (5) at small values of the parameter ε, allows us to determine an optimal relation between all FEL parameters, χ, L, T_C and ν, which gives the maximum field amplitude, i.e., provides the most effective interaction between the electromagnetic pulse and the electron bunches.

Note also that a superradiant (nonstationary) type of operation regime [11] can be realized for small negative cavity detuning ε (see Fig. 6).

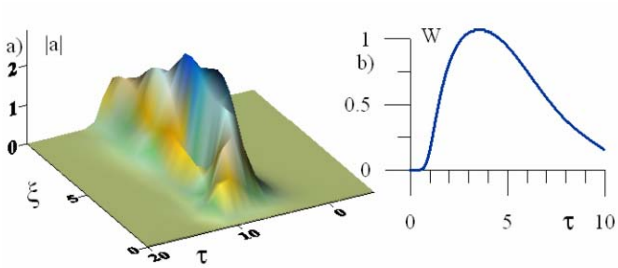

Figure 6: Superradiant operation regime: $L = 4.4$, T=51.2, $\varepsilon = -0.5$, $T_c = 16$, $\nu = 0.3$,: a) time–space evolution of an envelope of the electromagnetic pulse; b) time dependence of the field energy.

CONCLUSION

In conclusion, we develop both linear and nonlinear theory to describe regimes of operation of the waveguide FELs with finite detuning of “zero-slippage” condition and cavity detuning.

On the base carried out theoretical analyze it were estimated parameters of generated radiation for KAERI THz FEL ($\lambda = 100\mu m$) [7, 8]. The experiments were done for an electron bunche of duration $\tau_p \simeq 30$ ps, an electron current ~ 0.5 A, particle energy ~ 6.5 MeV, the undulator period $\lambda_u = 25$ mm, the undulator length –2 m, amplitude of undulator field ~ 6 kG, transverse sizes of the plane waveguide $d = 30$ mm, $b = 2$ mm, resonator losses $\sim 10\%$ Above physical parameters corresponds to dimensionless ones of length of interaction $L = 7$. From simulations the duration of electromagnetic micropulse is ~ 20 ps (relative spectrum width $\sim 1.5\%$) and peak power 30kW. The value of spectrum width corresponds to experimental date, but experimental value of peak power is much less the theoretical limit that in particular can be explained by the parameter spread in the real electron bunches.

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