A FEL AMPLIFIER BASED ON PLANAR BRAGG WAVEGUIDES*

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Abstract

We study a new version of FEL amplifier on the base of wide sheet electron beams. We suggest using a transverse open Bragg structures which can provide radiation waveguiding with simultaneous effective mode selection (filtration). Theory of transverse current FEL amplifier based on planar Bragg waveguides is considered.

INTRODUCTION

A planar periodic Bragg structure can be used not only as resonant elements of FEL oscillators [1-3] but as selective waveguides in different new schemes of FEL amplifiers. Open Bragg waveguide where wave propagates in the direction transverse to the lattice vector (Fig.1) can provide high selectivity over the transverse coordinate when its size essentially exceeds wavelength. In a transverse current amplifier scheme it is beneficial to use a grating with a step of corrugation, which results in the existence of a single low dissipative mode located near defect. The sheet electron beam moves across the waveguide to be resonant to one of partial waves forming the operating mode. Another way is a traditional travelling wave amplifier scheme where electron beam moves along waveguide axis. To increase effective size of operating mode one ca use a structure with regular longitudinal corrugation that couples two partial waves propagating at some angle to the axis to the wave propagating directly along the axis. This wave, which in moving reference frame is transformed into a cut off mode, is excited by the electrons. Analysis shows rather high gain and efficiency of the novel schemes with simultaneous discrimination of parasitic modes.

This paper is organized as follows: in the Sect.1 we study wave propagation in the Bragg waveguide with a step of corrugation to find the mode spectrum of this structure. In the Sect.2 we investigate the model of the transverse current FEL amplifier. In Conclusion we briefly overview another scheme of Bragg amplifier based on coupling of some higher mode of the planar waveguide and two *TEM* modes.

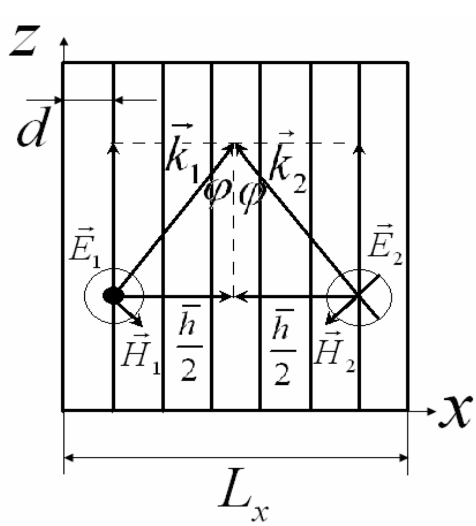

Figure 1: The diagram showing the coupling of waves on the Bragg lattice.

EIGENMODES OF THE OPEN PLANAR BRAGG WAVEGUIDE WITH A STEP OF CORRUGATION

We consider a planar waveguide (Fig.4) with weakly corrugated walls

$$l = l_0 \cos(\bar{h}x) \quad (1)$$

where $2\,l_0$ is the depth of corrugation. Assuming that lattice vector $\bar{h} = \frac{2\pi}{d}$, (*d* is the period of corrugation) is directed in *x* direction. The coupling exists for *TEM* waves with *x*- and *z*- wavenumbers satisfying Bragg resonance condition (see Fig.1)

$$h_{z1} = h_{z2} = h;\, h_{x1} = -h_{x2} = \bar{h}/2$$

We assume that the deflection of the waveguide surface $l(x)$ is much less than the wavelength λ and the corrugation period d: $l_0 << \lambda, d$. In this case perturbation of waveguide plates can be treated by using the equivalent surface magnetic current [4]. We seek the electric and magnetic fields as the linear combination of two TEM modes of a corresponding regular waveguide (Fig.5):

$$\vec{E} = \mathrm{Re}\left[\left(A_1(x,z)\vec{E}_1^{\,0} e^{i\frac{\bar{h}}{2}x} + A_2(x,z)\vec{E}_2^{\,0} e^{-i\frac{\bar{h}}{2}x}\right)\exp(i\omega t - ihz)\right],$$

$$\vec{H} = \mathrm{Re}\left[\left(A_1(x,z)\vec{H}_1^{0} e^{i\frac{\bar{h}}{2}x} + A_2(x,z)\vec{H}_2^{0} e^{-i\frac{\bar{h}}{2}x}\right)\exp(i\omega t - ihz)\right]$$

Slowly varying (in the wavelength scale) partial waves amplitudes, $A_{1,2}(x,z)$ satisfy the coupling equations:

$$\frac{h}{k}\frac{\partial A_{1,2}}{\partial z} \mp \frac{\bar{h}}{2k}\frac{\partial A_{1,2}}{\partial x} + i\alpha A_{2,1} = 0 \quad (2)$$

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where

$$\alpha = \frac{l_0}{4a_0}\frac{\bar{h}^2}{2k}$$

is the coupling parameter (a_0 is the distance between plates), $h = \sqrt{k^2 - (h/2)^2}, k = 2\pi/\lambda$. If we introduce an angle φ between the partial wave propagation directions ($\sin\varphi = \frac{\bar{h}}{2k}$), the coupling parameter can be represented as

$$\alpha = \frac{l}{4a_0}\bar{h}\sin\varphi\,.$$

The maximum value of the coupling parameter takes place when the frequency tends to the cutoff frequency and the wave vectors of the two partial waves are counter directional $\left(\varphi = \frac{\pi}{2}\right)$. The coupling parameter and the angle φ tend to zero while the frequency shift from the cutoff increases. By seeking seek the solution of the eqs.(2) as $A_{1,2}\exp(igx + i\Gamma z)$, we have the dispersion equation for the unbounded grating

$$\left(\tfrac{\bar{h}}{2}g\right)^2 = (k\Gamma)^2 - (k\alpha)^2 \qquad (3)$$

where g and Γ are small amendment to the transverse and longitudinal wavenumbers $g << \bar{h}/2,\ \ \Gamma << h_0$.

The defect of corrugation in the middle of the waveguide can be entered as a *π/2* phase step of the waveguide corrugation (1) which leads to the change of the sign of *α* in (2) at the point $x = L_x/2$:

$$\alpha = \alpha(x) = \begin{cases} +|\alpha|,\ x < L_x/2 \\ -|\alpha|,\ x > L_x/2 \end{cases}$$

Using the coupled waves equations (2) one can find the eigenmodes of an open in *x* direction planar Bragg waveguide with width L_x. In this case the boundary conditions for partial waves correspond to the absence of reflections and can be presented as follows:

$$A_1(0) = A_2(L_x) = 0\,.$$

Taking (2) into account and using the fields continuity condition on the step of corrugation, the characteristic equation for the bounded grating can be presented in a form similar to the theory of Bragg resonators [2]:

$$\begin{aligned} 2\alpha^2 &= \left[\Gamma^2 - \Gamma\left(g\tfrac{\bar{h}}{2k}\right)\right]\exp(igL_x) + \\ &+ \left[\Gamma^2 + \Gamma\left(g\tfrac{\bar{h}}{2k}\right)\right]\exp(-igL_x) \end{aligned} \qquad (4)$$

Equations (3),(4) can be solved approximately in the assumption of strong wave coupling ($\alpha L_x >> 1$). The spectrum of longitudinal wavenumbers consists of the single mode at the exact Bragg resonance ($\mathrm{Re}\,\Gamma = 0$) (which would be treated as “0”th mode):

$$\begin{cases} g_0 = \pm i\alpha\dfrac{2k}{\bar{h}} \\ \mathrm{Re}\,\Gamma_0 = 0 \\ \mathrm{Im}\,\Gamma_0 = -2\alpha\dfrac{k}{h}\exp\left(-\alpha\dfrac{2k}{\bar{h}}L_x\right) \end{cases} \qquad (5)$$

and a set of higher modes:

$$\begin{cases} g_N = 2\dfrac{\pi N}{L_x} - i2\dfrac{\pi N}{\alpha L_x^2}\dfrac{\bar{h}}{2k} \\ \mathrm{Re}\,\Gamma_N = \alpha\dfrac{k}{h}\dfrac{N}{|N|}\left(1 + 2\left(\dfrac{\pi N}{\alpha L_x}\dfrac{\bar{h}}{2k}\right)^2\right) \\ \mathrm{Im}\,\Gamma_N = -\alpha\dfrac{k}{h}4\dfrac{(\pi N)^2}{(\alpha L_x)^3}\left(\dfrac{\bar{h}}{2k}\right)^3 \end{cases} \qquad (6)$$

where $N = \pm1, \pm2, \pm3...$ are the mode numbers.

Fig. 2 demonstrates the dispersion diagrams for the different mode numbers and the diffraction losses curves via the frequency shift from the cutoff $\Omega = \dfrac{2k}{\bar{h}} - 1$.

It should be noted that expressions (5,6) diverge at $h \to 0$, while the frequency tends to the cutoff. These formally infinite diffraction losses appear when the applicability conditions of perturbation theory fail to fulfill. Nevertheless for practical use of Bragg waveguides it is interesting situation of rather large wave group velocities corresponding to large detuning of radiation frequency from the cutoff (Bragg frequency) where expressions (5),6) are non-divergent.

For *N>0* the transverse structure of the N^{th} waveguide mode represents the distribution close to a standing wave with sine wave envelope having $|N|$ half-periods on the length L_x (Fig.3a), while the lowest mode distribution is localized near the defect and has the exponential transverse structure (Fig.3b).

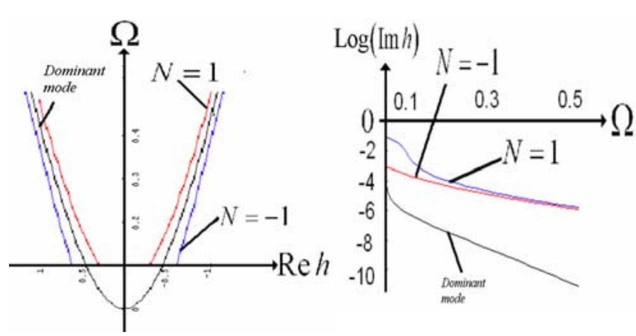

Figure 2: Dispersion diagrams for different mode numbers (left) and diffraction losses curves via the frequency shift (right).

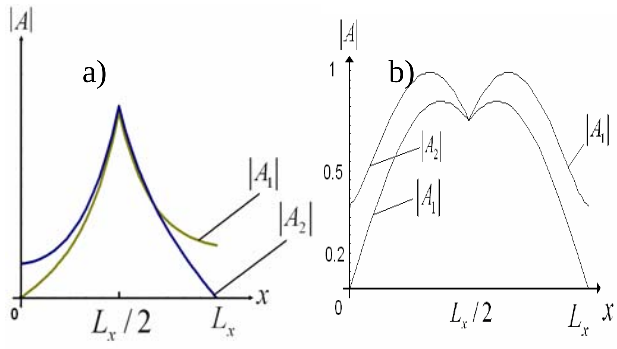

Figure 3: Transverse structure of modes with *N=0* (a) and *N=1* (b).

Diffraction losses of this mode ($\operatorname{Im}\Gamma$) are proportional to $\exp(-\left|2k\alpha L_x/\bar{h}\right|)$ and are very small at large L_x, while the high order modes have significantly greater decrements (they decrease as $1/L_x^3$) than the dominant mode decrement. Thus, such wide structures can be used for highly selective waveguiding.

TRANSVERSE CURRENT FEL AMPLIFIER

Transverse current amplifier schemes are well known in microwave electronics. In case of a rectilinear electron beams such type of interaction can be realized by means of oblique corrugation of slow wave structure [5]. In case of curves beams the synchronous interaction can be realized in smooth electrodynamical systems. One of the problems arising in such systems is distortion of the transverse field structure by the electron beam [6, 7]. This problem can be solved by means of using the open waveguides based on Bragg structures which provides high selectivity over the transverse mode index.

We consider a thin over the *y* coordinate sheet electron beam moving in the undulator field and in the uniform guiding magnet field $\vec{H}_0 = H_0\vec{\xi}_0$ (the directions of the coordinates and ξ are shown in the Fig.4). The beam is injected into the waveguide in the direction corresponding to the direction of propagation of the one of the coupled *TEM* waves in the waveguide described in the previous section and it is synchronous to this mode.

We consider the electron-wave interaction in the case of the undulator synchronism assuming the bounce frequency Ω is far from cyclotron synchronism with electrons:

$$\left|\omega - hv_{II} - \omega_H\right| T >> 2\pi, \quad \left|\Omega - \omega_H\right| T >> 2\pi,$$

where $\omega_{H0} = eH_0/m_0c\gamma_0$ is a cyclotron frequency, v_{II} is the longitudinal velocity of electrons, $\gamma_0 = \left(1 - v^2/c^2\right)^{-1/2}$ is relativistic factor, T is characteristic time of interaction. We also consider the ultra relativistic case $\gamma_0 >> 1$.

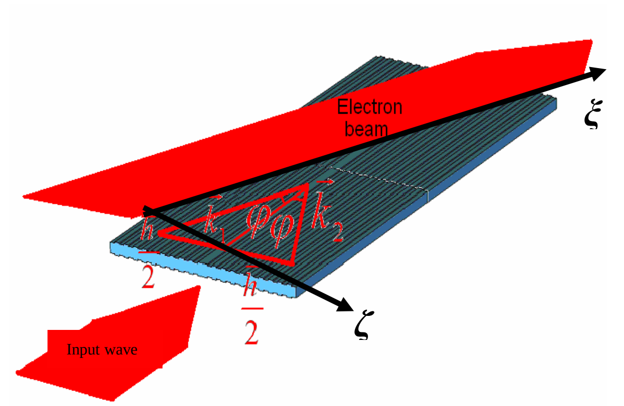

Figure 4: The scheme of a FEL amplifier with transverse current.

The process of monochromatic signal amplification in the transverse current FEL scheme can be described by the equations similar to the nonstationary equations of traditional FEL with 1D Bragg resonator [2] with time variable replaced by spatial one. These equations describe the formation of transverse structure of the fields together with their longitudinal structure

$$\begin{cases} \dfrac{h}{k}\dfrac{\partial a_1}{\partial \hat{z}} + \dfrac{\bar{h}}{2k}\dfrac{\partial a_1}{\partial \hat{x}} - i\delta a_1 + i\alpha a_2 = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} e^{-i\theta} d\theta_0 \\ \dfrac{h}{k}\dfrac{\partial a_2}{\partial \hat{z}} - \dfrac{\bar{h}}{2k}\dfrac{\partial a_2}{\partial \hat{x}} - i\delta a_2 + i\alpha a_1 = 0 \\ \left(\dfrac{h}{k}\dfrac{\partial}{\partial \hat{z}} + \dfrac{\bar{h}}{2k}\dfrac{\partial}{\partial \hat{x}}\right)^2 \theta = \operatorname{Re}\left(a_1 e^{i\theta}\right) \end{cases} \tag{7}$$

Here the normalizations, variables and parameters are defined as follows: $\hat{x}, \hat{z} = Ckx, Ckz$, $\hat{\alpha} = \alpha/(Ck)$, $a_{1,2} = eK\mu A_{1,2}/C^2$, θ is electron phase, C is Pierce parameter, μ is bunching parameter, K is electron-wave coupling parameter, δ is normalized synchronism detuning.

If the input signal is a *TEM* wave packet entering the system under the angle corresponding to the direction of propagation of the partial wave A_1, then the boundary conditions for (7) take the form

$$\begin{gathered} \frac{d\theta}{dX}\Big|_{X=0} = 0,\ \theta\big|_{X=0} = \theta_0 \in [0, 2\pi), \\ a_1\big|_{Z=0} = a_0,\ \ a_1\big|_{X=0} = 0,\ a_2\big|_{X=L_x} = 0 \end{gathered} \tag{8}$$

Results of direct numerical simulation of (7-8) are presented in Fig.5. In the Fig 5a the dependence of the amplification coefficient

$$\Gamma = \frac{\displaystyle\int_0^{L_x}\left(\left|A_1(X)\right|^2 + \left|A_2(X)\right|^2\right)dX}{\displaystyle\int_0^{L_x}\left|A_{inx}(X)\right|^2 dX}$$

on the longitudinal coordinate is depicted.

We can estimate the parameters of such system for an 8mm FEL basing that can be realized on the base of U2 high current accelerator at BINP RAS, Novosibirsk (width of the beam is ~150 cm, energy of electrons 1 MeV, $\gamma_0 = 3$, current density $j_0 = 1kA/cm$). We

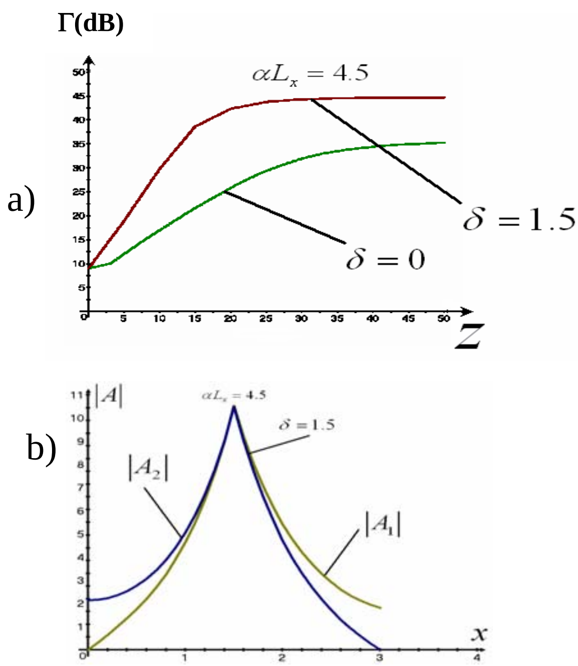

Figure 5: a) Dependence of the gain on the longitudinal coordinate, b) transverse structure of the fields.

assume the undulator period $d_u = 8cm$, undulator field amplitude $H_u = 2kOe$, guiding magnetic field $H_0 = 10kOe$, $a_0 = 1cm$ (the Pierce parameter is $C = 0.01$). At the following parameters of Bragg waveguide $l_0 = 0.4mm$, $d = 1mm$, $L_x = 50cm$, $L_z = 160cm$, $\varphi = 18^0$ one can get the gain up to 40 dB.

CONCLUSION

Another variant of using of an open Bragg structure for provision of higher selectivity in a FEL amplifier can be based on coupling between two TEM waves $A_\pm$ propagating at some angle to the axis and the wave *B* with a higher *y*-index propagating directly along the system axis. The latter wave is synchronous to the beam propagating directly along the axis, while the former waves provide the synchronization of radiation over the *x* coordinate. The coupling is provided by the regular corrugation with the following Bragg resonance conditions $\bar{h} = 2h_x$, where h_x is the transverse wavenumber of *TEM* modes. This interaction is described by the following equations.

$$\frac{\partial A_\pm}{\partial Z} \pm \frac{\partial A_\pm}{\partial X} = i\alpha B$$

$$\frac{\partial B}{\partial Z} + \frac{i}{2}\frac{\partial^2 B}{\partial Z^2} = -i\alpha(A_+ + A_-) + J$$

These equations are similar to those describing the excitation of the FEL oscillator based on coupling of the propagating and the trapped waves (see [10]) with excitation factor removed into the equations for the higher mode. Preliminary estimations show that in this case multiplication coefficient also should be rather high.

It should be noted in conclusion that the considered in section 1 system with defect of periodicity can be treated as a simple realization of photonic bandgap structures [8, 9]. Advantage of these Bragg waveguides is their compatibility with powerful sheet electron beam transportation system.

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