EMITTANCE COMPENSATION OF SUPERCONDUCTING GUN AND LINAC SYSTEM FOR BEAMS WITH LARGE CHROMATIC VARIANCE

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Abstract

Among the methods of emittance compensation for a superconducting rf gun and linac system include utilizing a solenoid and drift space after the gun to achieve a specific beam envelope with zero beam divergence before entrance into the linac. Studies on this method have assumed minimal energy spread in the beam. However, in cases where chromatic effects cannot be ignored this one solenoid emittance compensation technique is inadequate. Proposed is a new method of emittance compensation utilizing two solenoids in order to minimize emittance in beams with large energy spread. We present a theoretical basis for the new technique along with a computer optimized configuration. The results are compared with previous methods of emittance compensation.

INTRODUCTION

First introduced by Carlsen [1] and expanded upon theoretically by Serafini and Rosenzweig [2] is a scheme for emittance compensation for an rf gun and linac system that uses a solenoid magnetic field to compensate for space charge defocusing. Chang et al. [3] extended the method for systems with a superconducting gun for which the solenoid must be placed outside the gun. Such studies and further experiments have achieved good emittance compensation results for beams with little energy spread [4].

Touched upon by Chang et al. are the effects on emittance caused by varying energy along the bunch. Energy dependence in solenoid focusing, space charge defocusing and linac ponderomotive focusing leads to emittance increases when large energy spread in the bunch is present. Previous studies have laid the foundation for cancelling out chromatic effects at the linac entrance [3]. However, there are other important chromatic effects that should be looked into.

In this paper we study the chromatic effects of the solenoid focusing and space charge defocusing. We introduce a method for compensating for emittance increases caused by these effects by adding a second solenoid. The mechanism of this emittance compensation scheme is studied with analysis of the beam through the system. The relationship between the placement and strengths of the two solenoids is studied theoretically and compared to a computer optimized setup. Finally, this two solenoid method is compared to the one solenoid method by running Parmela [5] simulations with both setups and comparing final normalized emittance values. We find that the two solenoid method is superior to the one solenoid method at minimizing emittance at the exit of the linac.

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EMITTANCE COMPENSATION

The essence of the emittance compensation method described in detail in Ref [2] involves focusing the diverging beam immediately after the gun with a solenoid and reaching a point of zero convergence after a drift space at which point the beam enters the linac. The solenoid field must be of a certain strength in order to have the beam enter the linac at the invariant envelope of the linac. This envelope is given by:

$$\hat{\sigma} = \left(\frac{2}{\gamma'}\right) \cdot \sqrt{\frac{I}{3I_A\gamma}} \qquad (1)$$

where I_A= 17,054 A is the Alfven current, I is the current in the beam, and γ' is the average accelerating gradient of the linac. Entering at this envelope with zero divergence, the beam will exit the linac with zero divergence and with an envelope satisfying Eq. (1).

Chromatic Effects

Chromaticity comes into play in the energy dependence of the solenoid, space charge and linac focusing. As previously stated, we will not focus on the latter in this paper.

The envelope equation for a cylindrical space charge dominated beam through a focusing element, in this case a solenoid, is given by [2]:

$$\sigma'' = -\alpha_S\sigma + \frac{Q\beta c}{2I_A\sigma_z(\gamma\beta)^3\sigma} + \frac{\varepsilon_n^2}{(\gamma\beta)^2\sigma^3} \qquad (2)$$

where ε_n is the normalized emittance and α_S is the solenoid focusing strength

$$\alpha_S = \left(\frac{eB}{2\gamma\beta mc}\right)^2 \qquad (3)$$

where e is the charge of electron and B is the magnetic field of the solenoid. From the dependence on γ in each of these terms one can see that slices along the bunch with different energies will transform differently through the solenoid-drift space element.

ANALYSIS OF PHASE SPACE ANGLE

To illustrate these chromatic effects on the emittance we take a look at the transformation of the phase space angle

$$\theta = \frac{\sigma'}{\sigma} \tag{4}$$

through the solenoid and drift space elements.

Through Solenoid

In this analysis we will assume a small solenoid length in which no change in σ occurs, focusing only on the first term on the right hand side of Eq. (2). Taking the integral and dividing by σ we achieve

$$\Delta\theta = -\alpha_s L \tag{5}$$

with L the length of the solenoid.

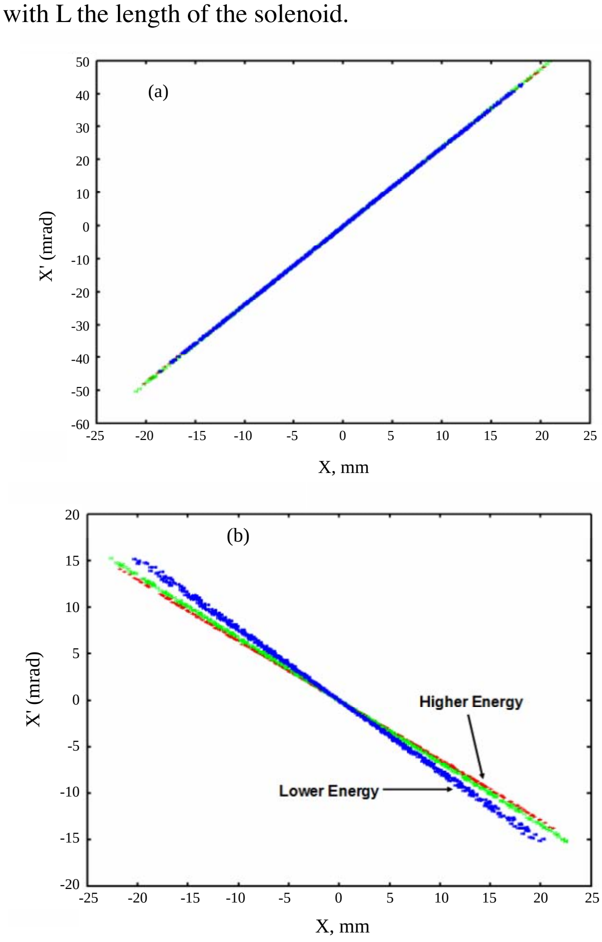


Figure 1: Phase space plots of three different slices of varying energy. Average energy of 2.5 MeV, energy spread of 3% and a total charge of 5nC, one of the possibilities for the RHIC e-cooling project at Brookhaven National Laboratory [6]. (a) Slices aligned before solenoid, immediately after gun. (b) Slices misaligned after solenoid focusing.

From Eq. (3) the dependence on energy is clear. Figures 1(a) and 1(b) illustrate the effect on the phase space distributions of three slices of varying energy. After the solenoid the phase space slices are no longer aligned leading to a rise in projected emittance.

Through Drift Space

For beam parameters typical for BNL's ERL injector (Q ~ 1 nC, γ ~ 5, $\sigma_z \sim \sigma$ ~ 1cm, and a normalized slice emittance of ε_n ~1 mm·mrad) the ratio between third and second terms on the right of Eq. 2 is very low: $I_A\sigma_z\gamma\varepsilon_n^2/Qc\sigma^2 \sim 5\cdot 10^{-5}$. Hence, only the second term on the right side of the envelope equation is necessary:

$$\sigma'' = \frac{P}{\sigma}; \tag{6}$$

$$P = \frac{Q\beta c}{2I_A\sigma_z(\gamma\beta)^3} \tag{7}$$

with P being the perveance, a function of energy. Near the waist, the envelope can be approximated by a parabola [7]:

$$\sigma \cong \sigma_w + \frac{P(z-z_w)^2}{2\sigma_w} \tag{8}$$

with divergence of

$$\sigma' \cong \frac{P(z-z_w)}{\sigma_w} \quad . \tag{9}$$

Dividing the two we obtain a phase space angle of

$$\theta \cong \frac{2P(z-z_w)}{2\sigma_w^{\ 2} + P(z-z_w)^2} \quad . \tag{10}$$

The envelope at the waist is given by integrating the differential equation of Eq. 6 to obtain the equivalent first order differential equation and rearranging to achieve

$$\sigma_w = \sigma_i e^{-\frac{\sigma_i'^2}{2P}} \quad . \tag{11}$$

The position at the waist is found by taking the equivalent integral equation of Eq. 6 and using a polynomial approximate solution [3]:

$$z_w \cong \frac{\sigma_i}{\sqrt{P}}\left(.009165 + .95449\frac{\sigma'_i}{\sqrt{P}} + .21824\frac{\sigma'^2_i}{P}\right) \ . \quad (12)$$

From these equations we are able to analyze how the spread in the phase space angle varies through the solenoid and drift space. In Fig. 2 we plot the difference in angle between the head and tail slices of the same bunch from Fig. 1, the head having a higher energy than the tail.

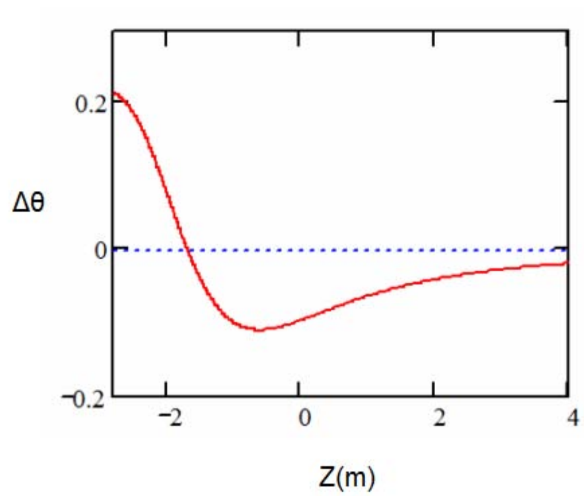


Figure 2: Difference between higher energy and lower energy slices in bunch immediately after solenoid. The beam waist occurs at z = 0 meters. The calculations and plots were done with Mathcad [8].

The plot begins after the solenoid at the maximum spread in angle and continues through drift space. We see that the slices align before z=0, the waist position, and are thus misaligned at the waist. This leads to a non-optimized emittance before entrance into the linac.

What one notices is that since the lower energy slices rotate past the higher energy slices it would be possible to use a second solenoid to realign the slices in phase space. The above equations allow one to calculate the spread in θ after an arbitrary distance and thus the relationship between the solenoid strengths of the first and second solenoids and the distance between them, which would result in realignment of the phase space slices.

OPTIMIZATION

A two solenoid, rf gun and linac system was optimized for lowest normalized emittance by optimizing position and field strengths of the two solenoids and position of the linac. Plots of normalized emittance and envelope can be found in Fig. 3 where they are compared to the same plots for a non-optimized one solenoid setup. The one solenoid configuration is set up following the invariant envelope method [2]. Since it is not optimized, looking at final emittances does not make for a definitive comparison. Instead, the plots serve to illustrate the overall advantage the two solenoid method has over the invariant envelope one solenoid method when dealing with large energy variation.

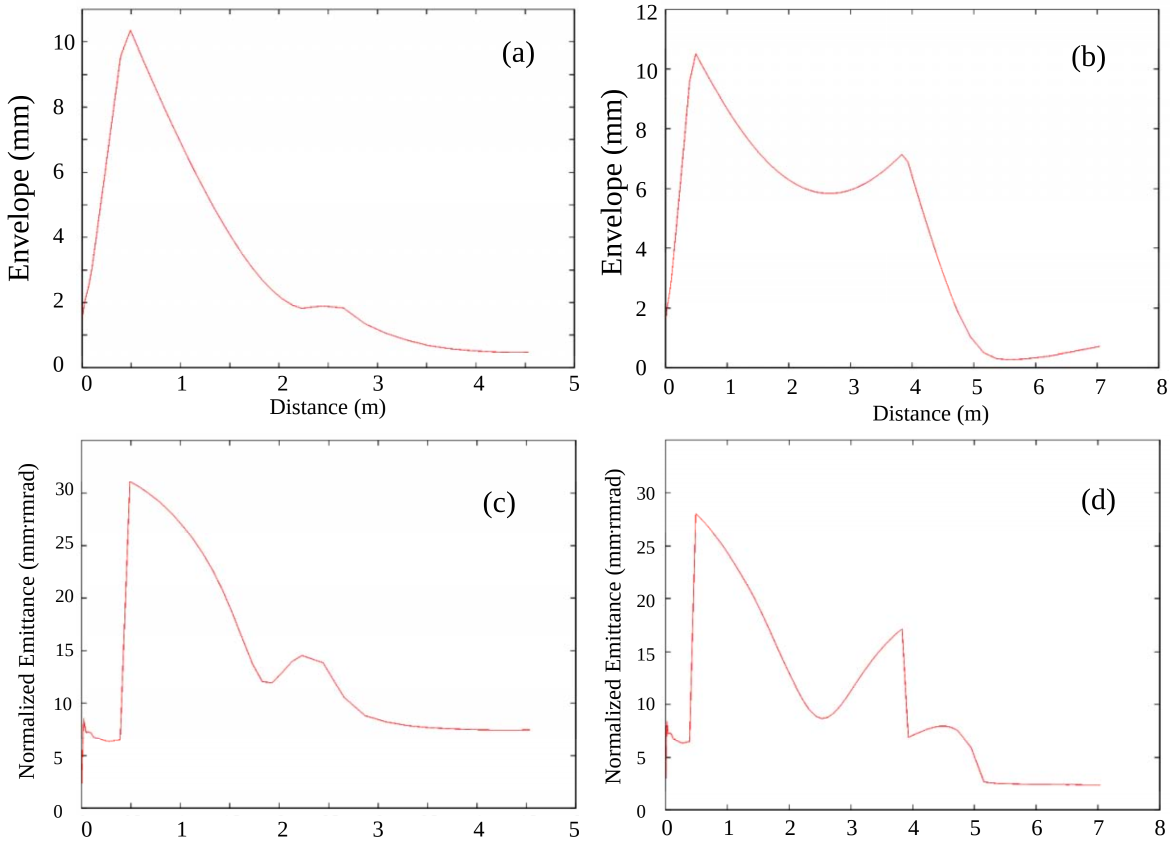
The optimized magnetic fields of the first and second solenoids and drift length between them are respectively 854 Gauss, 607 Gauss and 333.9 cm. To test the accuracy of the above equations describing the alignment of phase space slices we use them to calculate the magnetic field of the second solenoid, which would realign the phase space slices with all other optimized parameters unchanged. We obtain a theoretical magnetic field strength of roughly 520 Gauss for the second solenoid. This is significantly lower than the optimized value above. However, when comparing standard deviations in θ with 500 slices we find that the theoretical value has a lower spread in θ, .0846 inverse meters, than does the optimized value, .0897 inverse meters. This should not come as a surprise as the theoretical calculation minimized spread in θ after the second solenoid while the optimized value minimized normalized emittance after the linac, not necessarily minimizing spread in θ. What the comparison of spread in θ does is validate the accuracy of our theoretical approach, even if it should not be used as the exact value when attempting to minimize emittance.

DISCUSSION AND CONCLUSION

We have introduced a new method of emittance compensation that has advantages compared with previous methods using one solenoid in minimizing emittance of beams with significant energy spread. Our theoretical analysis of the transformations of phase space angle due to solenoid focusing and space charge defocusing has proven to be accurate and may serve as a basis for future analysis into this new emittance compensation scheme.

Further studies however must be undertaken in order to gain a better understanding of this new method. Firstly, alignment of sliced phase space angles is not the only parameter affecting total normalized emittance. Individual sliced emittances must also be minimized along with spread in slices and the two might not be simultaneously minimized. Studies into optimizing spread in θ after the linac have resulted in increases in normalized emittance.

Another aspect that should be looked into further are the chromatic effects of the linac focusing. As seen in Fig. 3, for the optimized two solenoid set up the beam does not enter the linac at the waist as is deemed necessary in previous one solenoid methods. Instead the beam enters at a significant convergence. Non-linearities in the linac focusing due to this convergent entrance perhaps might be playing a large role in minimizing individual sliced emittances. Efforts at moving the linac further from the second solenoid have not resulted in lower values of normalized emittance.



Final Normalized Emittance: 7.5 μm-rad

Final Normalized Emittance: 2.8 μm-rad

Figure 3: Comparison of one solenoid and two solenoid methods: Electron beam envelope with one solenoid (a) and two solenoids (b). (c) Normalized projected emittance with one solenoid (c) and two solenoids (d).

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