SIMULATION OF MIRROR DISTORTION IN FREE-ELECTRON LASER OSCILLATORS*

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Abstract

Thermal distortions in high-average power FEL cavity mirrors can alter mode quality and degrade performance. We address these issues by developing simulation tools, and then benchmarking the simulation against observations on the 10 kW-Upgrade experiment at the Thomas Jefferson National Accelerator Facility in Newport News, VA. The modelling and simulation will rely on the MEDUSA code, which is a three-dimensional FEL simulation code that is capable of treating both amplifiers and oscillators in both the steady-state and time-dependent regimes. MEDUSA employs a Gaussian modal expansion, and treats oscillators by decomposing the modal representation at the exit of the wiggler into the vacuum Gaussian modes of the resonator and then analytically determining the propagation of these vacuum resonator modes through the resonator back to the entrance of the wiggler in synchronism with the next electron bunch. Knowledge of the power loading on the mirrors allows us to model the mode distortions using Zernike polynomials, and this technique will be incorporated into MEDUSA.

INTRODUCTION

Thermal distortions in cavity mirrors in high-average power FELs can alter mode quality and negatively impact performance; hence, it is important to predict the character and magnitude of the distortions and to be able to model their effect on FEL performance. To this end, we address these key issues by developing modelling and simulation tools that can accomplish these goals, and then benchmarking the simulation against observations on the 10 kW-upgrade experiment [1] at the Thomas Jefferson national accelerator facility in Newport News, VA (henceforth referred to as Jefferson Laboratory). The facility is undergoing continual upgrades; in particular, a new permanent magnet wiggler has been installed that will be used in comparing the experimental and simulation results (see Table 1). The modelling and simulation will rely on the MEDUSA code [2,3], which is a three-dimensional FEL simulation code that is capable of treating both amplifiers and oscillators in both the steady-state and time-dependent regimes. MEDUSA employs a Gaussian modal expansion, and treats oscillators by decomposing the modal representation at the exit of the wiggler into the vacuum Gaussian modes of the resonator and then analytically determining the propagation of these vacuum resonator modes through the resonator back to the entrance of the wiggler in synchronism with the next electron bunch. Knowledge of the power loading on the mirrors allows us to model the mode distortions using Zernike polynomials [4], and this technique will be incorporated into MEDUSA.

In this paper, we report on the progress to date in this activity. The first step is to compare MEDUSA predictions with the observed performance of the experiment at low duty factor where mirror distortions are unimportant. We then go on to determine the effects of the so-called first order properties, which include changes in the Rayleigh range and shifts in the position of the mode waist. Higher order distortions such as coma, astigmatism, and spherical aberration, collectively known as third-order aberrations, will be incorporated in the future. The organization of the paper is as follows. In the second Section we discuss the formulation used in MEDUSA. A description of the experiment is given in the third Section, and of the numerical results in fourth Section. A summary and discussion is given in the fifth Section.

THE NUMERICAL FORMULATION

The MEDUSA code [2,3] employs a three-dimensional formulation that includes the slippage of the radiation relative to the electron beam. MEDUSA can model both helical and planar wiggler geometry and treats the electromagnetic field as a superposition of either Gauss-Hermite or Gauss-Laguerre modes in the slowly-varying amplitude approximation, where

$$\delta\mathbf{A}(\boldsymbol{x},t)=\hat{\boldsymbol{e}}_x\sum_{l,n} e_{l,n}(x,y)\left[\delta A_{l,n}^{(1)}\cos\varphi(x,t)+\delta A_{l,n}^{(2)}\sin\varphi(x,t)\right], \quad (1)$$

where "l" and "n" are transverse mode numbers, "h" is the harmonic number, $e_{l,n,h} = \exp(-r^2/w_h^2)H_l(\sqrt{2}x/w_h)H_n(\sqrt{2}y/w_h)$, H_l is the Hermite polynomial of order l, and w_h is the spot size, $\varphi_h = h(k_0 z - \omega_0 t) + \alpha_h r^2/w_h^2$ ($k_0 = \omega_0/c$). We assume that $\delta A_{l,n,h}^{(1,2)}$, w_h, and α_h, vary slowly in z and t. The dynamical equations are

$$\left(\frac{d}{dz}+\frac{w_h'}{w_h}\right)\begin{pmatrix}\delta a_{l,n,h}^{(1)}\\ \delta a_{l,n,h}^{(2)}\end{pmatrix}+K_{l,n,h}\begin{pmatrix}\delta a_{l,n,h}^{(2)}\\ -\delta a_{l,n,h}^{(1)}\end{pmatrix}=\begin{pmatrix}s_{l,n,h}^{(1)}\\ s_{l,n,h}^{(2)}\end{pmatrix}, \quad (2)$$

where $\delta a_{l,n,h}^{(1,2)} = e\delta A_{l,n,h}^{(1,2)}/m_e c^2$, $d/dz = \partial/\partial z + c^{-1}\partial/\partial t$, the "prime" superscript denotes the total z-derivative,

$$K_{l,n,h} = (l+n+1)\left(\alpha_h\frac{w_h'}{w_h}-\frac{\alpha_h'}{2}-\frac{1+\alpha_h^2}{hk_0 w_h^2}\right), \quad (3)$$

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$$\begin{pmatrix} s^{(1)}_{l,n,h} \\ s^{(2)}_{l,n,h} \end{pmatrix} = \frac{2\omega_b^2}{h\omega_0 c}\frac{F_{l,n}}{w_h^2}\left\langle \frac{\upsilon_x}{|\upsilon_z|}\, e_{l,n,h} \begin{pmatrix} \cos\varphi_h \\ -\sin\varphi_h \end{pmatrix} \right\rangle \ , \qquad (4)$$

where $\omega_b(z,t)^2 = 4\pi e^2 n_b(z,t)/m_e$ for a beam density n_b, and $F_{l,n} = [2^{l+n} l!n!]^{-1}$, and <(…)> describes an average over the complete 6-D phase space. The spot size and radius of curvature for each harmonic component are given by

$$w_h' = \frac{2\alpha_h}{hk_0 w_h} - w_h Y_h \ , \qquad (5)$$

$$\frac{\alpha_h'}{2} = \frac{1+\alpha_h^2}{hk_0 w_h^2} - \left(X_h + \alpha_h Y_h\right) \ . \qquad (6)$$

These equations constitute the source-dependent expansion [5], which is a self-consistent adaptive eigenmode representation that tracks the optical guiding of the mode based upon the interaction with the electron beam. The field equations are integrated simultaneously with the complete three-dimensional Lorentz force equations for an ensemble of electrons. No wiggler-average orbit approximation is used so that the spatial step size must be small enough to resolve the wiggler motion.

Table 1: Nominal experimental parameters

ELECTRON BEAM	
Energy	115 MeV
Peak Current	310-370 A
Normalized Emittance	9 mm-mrad/7 mm-mrad
Energy Spread	0.35%
Bunch Length	380-420 fsec
Bunch Charge	79 pC
Initial Beam Size	257 microns/212 microns
Twiss-α Parameter	1.25
WIGGLER	
Amplitude	3.746 kG
Period	5.5 cm
Length	30 periods/1.65 m
OPTICAL MODE	
Wavelength	1.57 microns
Rayleigh Range	1.5 (±0.30) m
Mode Waist Position	1.03 (±0.05) m

THE JEFFERSON LAB EXPERIMENT

The Jefferson Lab IR-Upgrade FEL operates an energy recovery accelerator with a high power FEL wiggler and resonator. The electron beam consists of a core distribution and a halo distribution. The charge, emittance, and peak current are of that core beam. The wiggler is very well characterized and is essentially ideal. The optical resonator is nearly concentric, and consists of a high reflector at the upstream end and a transmissive element at the downstream end that out-couples approximately 10% of the power. The Rayleigh range can be varied in the experiment by changing the radius of curvature of the high reflector. However, the cavity is slightly astigmatic and can lead to a difference between the Rayleigh range in the two axes. The uncertainty in the Rayleigh range is about 20%. The measured gain and efficiency are 70±5% and 1.6±0.1% respectively.

The nominal parameters for the experiment that are used in simulation are given in Table 1. Observe that the normalized emittances and beam dimensions shown in the table refer to the wiggle-plane and the plane transverse to the wiggler-plane respectively. In addition, the Twiss-α parameter shown corresponds to a beam that is focused to a waist near the center of the wiggler. The estimate of the Rayleigh range and the location of the mode waist contain some uncertainty, and the values given are the best estimate at the present time. In particular, we note that the mode waist is located about 15 cm downstream from the wiggler center.

NUMERICAL RESULTS

As we mentioned previously, our first goal is to determine whether MEDUSA is in substantial agreement with the experiment when distortion is absent or minimal. This is the case when the experiment operates at low average powers (*i.e.*, low duty factors), where a single pass gain of the order of 65-75% is measured for a peak current of 310 A and a bunch length of 380 fsec.

The parameters shown in Table 1 are the nominal experimental parameters, and we note that the optical waist is located about 15 cm downstream from the wiggler center. The experiment was optimized by focusing the electron beam to a waist near the center of the wiggler, which provides an optimal match to the resonator mode. This was also found in simulation, and is obtained for a Twiss-α parameter of 1.25 in simulation. The single pass gain found using MEDUSA operating in steady-state mode for these parameters is of the order of 400%. However, the slippage time through the wiggler for this experiment is of the order of 160 fsec, which is a substantial fraction of the bunch length. Hence, slippage is important and can be expected to substantially reduce the single pass gain with respect to steady-state predictions.

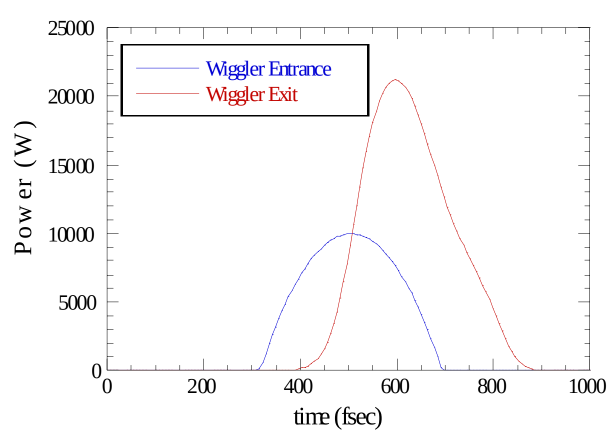

Figure 1: Plot of the power versus time through the pulse at the wiggler entrance and exit.

The slippage of the electromagnetic pulse through the wiggler is illustrated in Fig. 1 where we plot the pulse shapes at the entrance and exit from the wiggler. It is evident from the figure that while the pulse is assumed symmetric at the entrance to the wiggler, it has slipped by at least half the total pulse length over the course of the wiggler. Amplification of the peak power over the pulse has shrunk from the value of 400% found in steady-state simulation to just over 100% when slippage is included. However, in the time-dependent simulation, gain must be calculated based on the overall energy of the pulse, not the peak power. To this end, we plot the amplification of the total pulse energy through the wiggler in Fig. 2. The incident energy is 2.53 nJ and the energy at the output is 4.68 nJ yielding a single pass gain of about 84%. Given the uncertainties in the measured parameters, this represents reasonable agreement with the experiment. For example, there is a 20% uncertainty in the measurement of the Rayleigh range that would result in a reduction of the predicted gain to 73%.

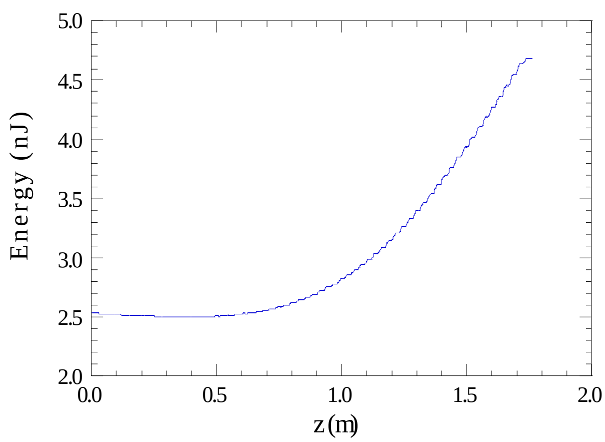

Figure 2: Plot of the amplification of the total pulse energy through the wiggler.

The lowest order mirror distortions involve variations in both the location of the optical waist and the Rayleigh range. In order to study the effects of these distortions, we (1) varied the position of the optical waist while holding the Rayleigh range fixed at 1.5 m, and (2) varied the Rayleigh range for an optical waist that is located at the wiggler center optical and 30 cm downstream from the wiggler center. These results are shown in Figs. 3 and 4 respectively. Figure 3 indicates that, for these parameters, the FEL gain is maximized when the optical waist is located about 20 cm upstream from the wiggler center, in contrast to the actual location that is 15 cm downstream from the wiggler center. Observe that we show the gain variation with Rayleigh range in Fig. 4 for an optical waist that is wiggler-centered and shifted downstream from the wiggler center by 30 cm. However, the optical waist is located 15 cm downstream from the wiggler center, and the actual location was used in the simulations shown in Figs. 1-3. We chose to use the larger displacement in Fig. 4 to better illustrate the performance sensitivity to these parameters, and the actual variation with Rayleigh range in the experiment is between these two lines. It is clear from Fig. 4 that the single pass gain would be larger if the Rayleigh range were smaller, and the optimal Rayleigh range found in simulation varies from about 0.5 m for a wiggler-centered resonator mode to 0.6 m when the mode waist is located 30 cm downstream from the wiggler center. The advantages that accrue from using a short Rayleigh range resonator were first pointed out by W. Colson and his collaborators [6]. The results shown in Figs. 3 and 4 indicate that were mirror distortions to either decrease the Rayleigh range or shift the mode waist upstream, then the single pass gain and FEL performance may actually be enhanced. Whether such an effect can actually be allowed for in the design of a high power FEL oscillator is currently under consideration.

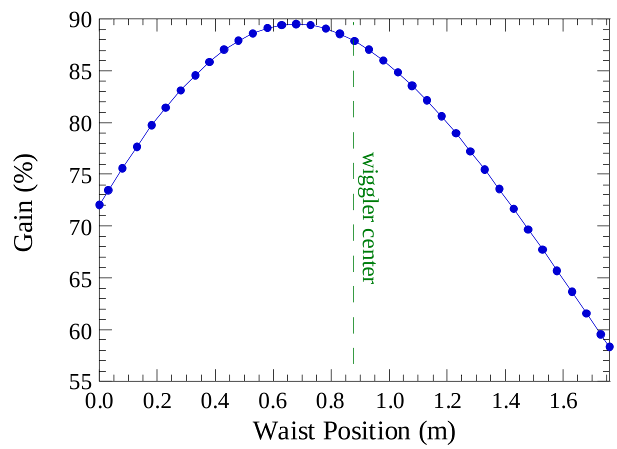

Figure 3: Variation of the single pass gain with the position of the optical waist.

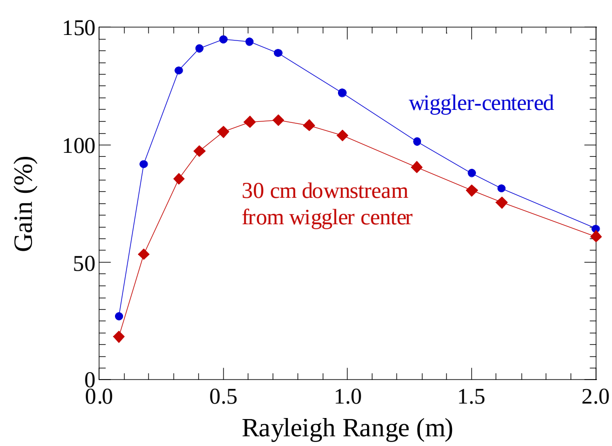

Figure 4: Variation of the single pass gain with the Rayleigh range.

SUMMARY AND DISCUSSION

In this paper we report on the initial work involved in a study of the effect of mirror distortions on the performance of a high power FEL oscillator using the MEDUSA simulation code. To this end, we first undertook to validate MEDUSA for low power (and duty factor) operation where mirror distortion was small. In this case MEDUSA predicted a single pass gain of 84%, which is in reasonable agreement with the measured range of 65-75% given the experimental uncertainties in the Rayleigh range, location of the optical waist, astigmatism in the resonator, and uncertainties related to the electron beam distribution. Experimentally, one derives the mode waist and position from the radii of curvatures (ROC) of the cavity mirrors. Repeated measurements set this uncertainty at ±5 cm. In turn, this creates a 20% uncertainty in the value of the Rayleigh range, but a relatively small (±5 cm) change in the waist position. There are also uncertainties associated with the electron beam parameters, especially those associated with the longitudinal distribution. Simulations indicate that the predicted gain is very sensitive to uncertainties of this magnitude. For example, a 20% uncertainty in the Rayleigh range and a ±5 cm uncertainty in the optical waist position can lead to a variation in the predicted gain of between 71% - 96%. As a consequence, the simulation is in substantial agreement with the experiment. Given the agreement between the simulation and experiment, we then undertook to investigate the variation in performance versus the Rayleigh range and the location of the optical waist. We found that the small signal gain would be substantially larger for much smaller Rayleigh ranges and for an optical waist located upstream from the wiggler center.

Future work will involve the inclusion of higher order mirror perturbations mentioned earlier, as well as validation of the harmonic generation predictions of MEDUSA.

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