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TIME-DOMAIN ANALYSIS OF ATTOSECOND PULSE GENERATION IN AN X-RAY FREE-ELECTRON LASER

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Abstract

The method of enhanced self-amplified spontaneous emission (eSASE) is one of the strongest candidates for the generation of sub-femtosecond X-ray pulses in a free-electron laser. The optimization of an eSASE experiment involves many independent parameters, which makes the exploration of the parameter space with 3-D simulations computationally intensive. Therefore, a robust theoretical analysis of this problem is extremely desirable. We provide a self-consistent, analytical treatment of such a configuration using a one-dimensional, time-dependent FEL model that includes the key effects of linear e-beam chirp and linear undulator taper. Verified via comparison with numerical simulation, our formalism is also utilized in parameter studies that seek to determine the optimum setup of the FEL.

INTRODUCTION

Because of their attractiveness to users, the generation of ultrashort X-ray pulses is one of the main objectives of research into advanced operation modes in a modern FEL facility. One of the more prominent schemes for generating sub-fs X-rays in an FEL is eSASE [1] (enhanced Self-Amplified Spontaneous Emission). This technique involves the interaction of an electron beam with an optical laser pulse in the presence of a short wiggler, prior to the beam being sent into a conventional undulator. This process relies on some intense manipulation of the longitudinal phase space of the e-beam, after which the strongly chirped beam typically has to travel through a tapered undulator in order to achieve lasing with the required properties. Apart from a significant improvement in the performance of the FEL, this method provides an attractive scheme for generating X-ray pulses in the attosecond range.

In this paper, we provide an self-consistent, analytical treatment of such a configuration using a simple, one-dimensional (1D) FEL model that includes the effects of startup from noise (SASE), slippage, electron beam chirp (linear and nonlinear) and undulator taper. 3D effects such as radiation diffraction, emittance and focusing are excluded. This allows us to calculate various key properties of the FEL radiation in the latter stage of the exponential gain regime. After verifying its validity through comparison with the output of a 1D FEL simulation code, our analysis is also utilized in parameter studies that seek to determine the optimum setup of the FEL. This enables us to obtain a more thorough understanding of the physics behind the experimental method.

1D FEL ANALYSIS

In this section, we outline the main results of our theoretical analysis, leaving the details of the derivation for another publication. In the context of our model, the main properties of the radiation can be extracted from a slowly-varying complex amplitude $a(\theta, z)$, which can be related to the actual electric field through the relation $E_{\rm rad} = a(\theta, z)e^{ik_r(z-ct)}/2 + \text{c.c.}$ Here, $k_r = 2\pi/\lambda_r$ is the radiation wave number and $\theta = k_u z + k_r(z - ct)$ is the ponderomotive phase ($k_u = 2\pi/\lambda_u$, where λ_u is the undulator period). The θ variable also satisfies the relation $\theta = k_r s$, where s is an internal bunch coordinate. The main FEL parameters satisfy the resonance condition $\lambda_r = \lambda_u(1 + K_0^2/2)/(2\gamma_0^2)$, where λ_r is the radiation wavelength, K_0 is the (initial) undulator parameter and γ_0 is the average relativistic factor of the beam. The longitudinal phase space coordinates are (θ, η), where $\eta = \gamma/\gamma_0 - 1$ is the energy deviation variable.

As far as the key properties of the e-beam are concerned, we assume that the current is given by $I(\theta) = I_0\chi(\theta)$, where $0 \le \chi(\theta) \le 1$ is a scaled profile and I_0 is the peak current, while the correlated energy (chirp) profile is $\eta = -\mu(\theta - \theta_m) - \Upsilon(\theta)$. Here, $\theta_m = \theta_b/2$ is the phase corresponding to the middle of the bunch (we have $\chi(0) = \chi(\theta_b) = 0$), μ is a constant linear chirp coefficient and the Υ function represents a nonlinear chirp component (we assume zero uncorrelated energy spread). For our purposes, we select a parabolic current profile of the form $\chi(\theta) = 1 - (\theta - \theta_m)^2/\theta_m^2$ and a nonlinear chirp profile given by $\Upsilon(\theta) = \mu_3(\theta - \theta_m)^3$, though the formalism can also accommodate the general case. The logic of this particular selection will be justified later on. Finally, we also assume a linear taper profile of the form $K = K_0(1 + \epsilon z)$.

We follow the self-consistent analysis of Ref. [2]. In the linear regime of the interaction, we can show that the complex radiation amplitude can be expressed as

$$a(\theta, z) \propto \sum_j e^{-i\theta_j} G(\theta, \theta_j, z)\,, \tag{1}$$

where θ_j are the random initial electron phases (at $z = 0$) and $G(\theta, \theta_j, z)$ is a Green's function. The latter is non-zero only when $0 < \theta - \theta_j < k_u z$, in which case it is given in contour integral form (up to a phase term) by

$$G = -\frac{1}{2\pi i}\int_{-\infty+iy}^{+\infty+iy} \frac{d\hat{\lambda}}{\hat{\lambda}} \exp(-i\hat{\lambda}[\bar{z} - (\hat{\theta} - \hat{\theta}_j)] - i\int_{\hat{\theta}_j}^{\tau(\hat{\theta})} dt$$
$$\times\, \hat{\chi}(t)[\hat{\lambda} + \hat{\Delta}_0(t - \hat{\theta}_j) + \hat{\mu}_3\{(t - \hat{\theta}_m)^3 - (\hat{\theta}_j - \hat{\theta}_m)^3\}]^{-2})\,. \tag{2}$$

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In the above equation, we have introduced the following scaled variables: $\bar{z} = 2\rho k_u z$, $\hat{\theta} = 2\rho\theta$, $\hat{\theta}_j = 2\rho\theta_j$, $\hat{\theta}_m = 2\rho\theta_m$ (where ρ is the dimensionless FEL (or Pierce) parameter), $\hat{\mu}_3 = \mu_3/(8\rho^4)$ and $\hat{\Delta}_0 = (\mu - \bar{a}_1)/(2\rho^2)$, where $\bar{a}_1 = a_1/(2k_u^2)$ and $a_1 = -2\epsilon k_u K_0^2/(2+K_0^2)$. Moreover, y is a real constant that is larger than the imaginary part of all the integrand singularities.

Apart from the current profile itself, finite pulse effects are reflected in the function $\tau(\hat{\theta})$, which is equal to $\hat{\theta}$ for $0 \le \hat{\theta} \le \hat{\theta}_b = 2\hat{\theta}_m$ and $\hat{\theta}_b$ for $\hat{\theta} > \hat{\theta}_b$. For the case of zero taper and zero nonlinear chirp ($a_1 = 0$, $\mu_3 = 0$), the above expressions reduce to the Green's function for the case of linear chirp [3]. On the other hand, we can easily show that taking $\hat{\Delta}_0 = 0$ reproduces the well-known compensation condition between linear chirp and linear taper [4]. In any case, calculation of the Green's function via the contour integral is facilitated by the stationary phase approximation, which is accurate enough in the latter stage of the linear regime. It is worth noting that, for the case of parabolic current/cubic chirp, the t-integral in Eq. (2) can be determined analytically.

Given the Green's function, we can determine various properties of the radiation, the most important of which is the radiation power. This quantity, averaged over a large number of shots (i.e. ensembles of random phases), is given by (see [5] and [1])

$$P_{\rm rad}(\hat{\theta}, \bar{z}) = 2\gamma_0 mc^3 k_r \rho^2 \int d\hat{\theta}_j \, \hat{\chi}(\hat{\theta}_j) \left| G(\hat{\theta}, \hat{\theta}_j, \bar{z}) \right|^2 . \quad (3)$$

Moreover, we can also quantify the state of the e-beam by calculating the bunching factor $b = \left|\left\langle e^{-i\theta_j}\right\rangle_\Delta\right|$, where the Δ-index refers to average within a radiation wavelength. The shot-averaged version of this quantity is, in turn, given by

$$\left\langle b^2(\hat{\theta}, \bar{z})\right\rangle_{\rm shot} = \frac{4\pi\rho}{n_0\lambda_r} \int d\hat{\theta}_j \, \hat{\chi}(\hat{\theta}_j) \left| G_b(\hat{\theta}, \hat{\theta}_j, \bar{z}) \right|^2 / \hat{\chi}^2(\hat{\theta}), \quad (4)$$

where $n_0 = I_0/(ec)$ is the peak number density and $G_b = (\partial/\partial\bar{z} + \partial/\partial\hat{\theta})G$ is a derivative Green's function (unlike G, G_b is non-zero only within the electron bunch).

Finally, we note the relationship between the nonlinear chirp coefficient μ_3 and the linear chirp μ. Though these two parameters are - in principle - independent, we correlate them in the following way: since the chirp profile of the beam is shaped by space charge effects before the amplification process, it can be modeled by the θ-derivative of the actual current profile, which is closer to a Gaussian. Thus, we have

$$\eta = -\mu(\theta - \theta_m) - \mu_3(\theta - \theta_m)^3 \propto \frac{d}{d\theta} \exp\left(-\frac{(\theta - \theta_m)^2}{2\sigma_\theta^2}\right), \quad (5)$$

up to third order terms in $\theta - \theta_m$. This yields the relation $\mu_3 = -\mu/(2\sigma_\theta^2)$. As far as σ_θ is concerned, we can either choose it in an ad-hoc way or derive it by matching the parabolic and Gaussian current profiles up to second order in $\theta - \theta_m$. The latter manipulation yields $\sigma_\theta = \theta_m/\sqrt{2}$. Though not entirely self-consistent, this strategy allows us to adequately model the space-charge induced chirp while preserving some degree of analyticity as far as the Green's function is concerned.

NUMERICAL RESULTS

In what follows, we present a brief numerical illustration of the theory outlined in the previous section. We select a parameter set that roughly approximates a plausible configuration of the X-LEAP eSASE experiment at SLAC. This involves the generation of 800 eV photons ($\lambda_r = 1.55$ nm) with a standard LCLS undulator ($\lambda_u = 3$ cm, $K_0 = 3.5$) and a 4.24 GeV beam with a peak current of 4.5 kA. The average beta function is about 10 m, which corresponds to an rms beam size of approximately 25 μm (for a transverse normalized emittance of 0.5 μm). The ρ-parameter is about 2×10^{-3}, while we also assume zero uncorrelated energy spread. In the first case we consider, μ is given by $\mu = -(\Delta\gamma/\gamma_0)/\theta_b$, where $\Delta\gamma = \Delta E/mc^2$ and $\Delta E = 30$ MeV is the total energy variation due to the linear chirp. This leads to a negative linear chirp, which is compensated by the appropriate (reverse) taper, leaving only the cubic component contribution.

In Figs. 1 and 2, we plot the (shot-averaged) radiation power and bunching factor (the latter defined as $\sqrt{\langle b^2\rangle_{\rm shot}}$) as functions of the position s along the bunch. The theoretical values are calculated by Eqs. (3) and (4) while the simulation values are obtained from a 1D FEL simulation code (some details in Ref. [2]). Reasonable agreement is observed between the two approaches, which helps us build up confidence in the formalism.

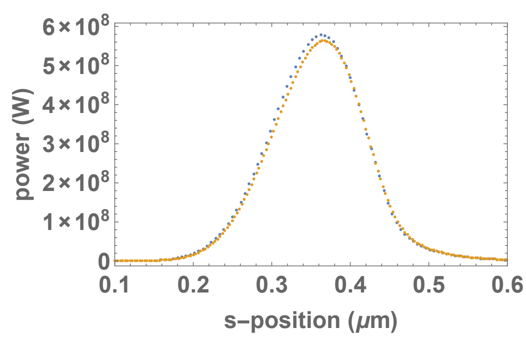

Figure 1: Shot-averaged radiation power along the bunch ($z = 11.45$ m, 0.5 μm bunch length, $\sigma = \sigma_\theta/k_r = 160$ nm, $\Delta E = 30$ MeV, 400 shot average, matched linear chirp). The blue/brown curve represents theory/simulation data.

Moving on, we scan the electron pulse duration t_e while keeping constant both the peak current I_0 and the product $\mu \times t_e$. According to our previous discussion, the latter is proportional to the total energy variation ΔE, which is fixed at 35 MeV. Using the Green's function, we obtain the power profiles for the pure, matched linear chirp case (red data in Figs. 3-4) and for the case with the added nonlinear chirp component (blue data). In Fig. 3, we plot these two profiles for a bunch length of 1.5 fs. A power suppression due to the cubic chirp is immediately evident. Moreover, we find that the radiation full-width-at-half-maximum (FWHM) is *also reduced*. This is made explicit in Fig. 4, where the FWHM

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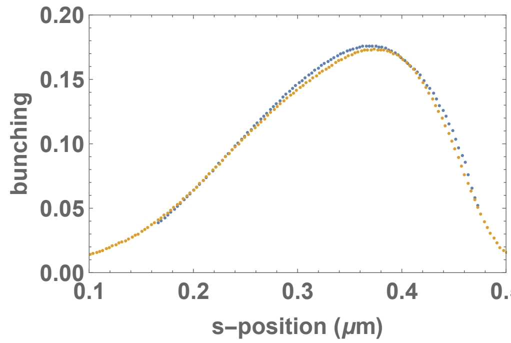

Figure 2: Shot-averaged bunching factor along the bunch (same parameters/color convention as in Fig. 1).

is plotted as a function of t_e. A marked *linear dependence* is observed, along with what appears to be a constant vertical shift (reduction) due to the cubic chirp.

The former feature can actually be obtained in a heuristic way by the following simple argument: since the ρ-parameter scales according to $L_g^{-1} \sim \rho \sim I^{1/3}$, we can define a θ-dependent ρ by plugging in the parabolic profile $\chi(\theta) = 1 - (\theta - \theta_m)^2/\theta_m^2$ (recall that $I \sim \chi(\theta)$). Combining this with the power growth relation $P = P_0 \exp(z/L_g)$, we find a power profile of the form $\exp(-(z/L_0)(\theta - \theta_m)^2/(3\theta_m^2))$, where $L_0 = \lambda_u/(4\pi\sqrt{3}\rho)_{I\to I_0}$ is the basic (minimum) value for the power gain length L_g (about 0.70 m for our parameters). This predicts maximization of the radiation power in the middle of the electron bunch, which is not very accurate (in fact, it happens closer to the head of the beam, see Fig. 3). On the other hand, the FWHM is simply $t_e \times \sqrt{3 \log 2/(z/L_0)}$, a result which also exhibits linear dependence with a slope of about 0.35. This is very close to the value calculated from the Green's function data (0.32), even though the simple model does not take into account detuning effects etc. This scaling may be of some use when doing back-of-the-envelope calculations involving attosecond-style pulses.

In conclusion, we also point out that the Green's function formalism allows us to get a sense of what the optimum setup of the FEL configuration is. Apart from determining the proper matching strategy involving the linear chirp and taper, we can study the interplay of power suppression and

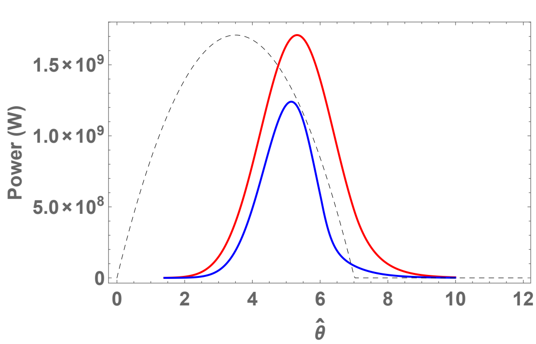

Figure 3: Power profiles for a matched linear chirp with or without the cubic chirp component (blue and red curves, respectively). The dashed line denotes the current profile.

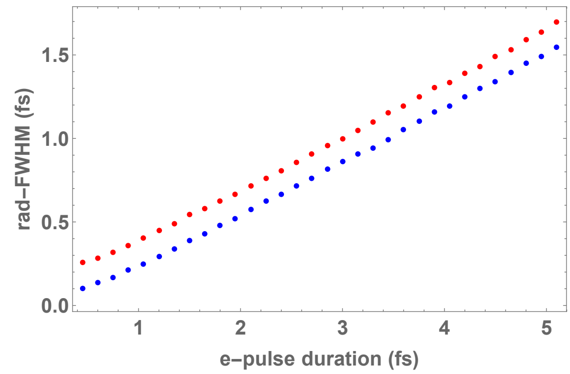

Figure 4: Radiation FWHM vs the electron bunch length (same color convention as in Fig. 3). All of the data shown in Figs. 3-4 have been calculated assuming that $z = 12.3$ m.

FWHM reduction due to the nonlinear chirp in order to find a suitable working point. Though our results are only one-dimensional, the essential conclusions remain valid in a more complicated (3D) setup.

CONCLUSIONS

We have developed a one-dimensional, time-dependent theory which can adequately model an eSASE-based FEL. Our formalism includes startup from noise, radiation slippage, e-beam chirp (linear and nonlinear) and undulator linear taper. Using a Green's function approach, we are able to determine the basic properties of the radiation pulse prior to the onset of saturation. As part of our derivation, we can provide a rigorous proof of the well-known compensation condition between linear chirp and taper. Reasonable agreement is observed between our semi-analytical treatment and the output of a 1D FEL code. Moreover, our technique is robust enough to allow us to perform simple parameter studies, which show some interesting features like the pulse-shortening effect due to nonlinear chirp.

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