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DYNAMICS OF SUPERRADIANT EMISSION BY A PREBUNCHED E-BEAM AND ITS SPONTANEOUS EMISSION SELF-INTERACTION*

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Abstract

In the context of radiation emission from an electron beam, Dicke's superradiance (SR) is the enhanced "coherent" spontaneous radiation emission from a pre-bunched beam, and Stimulated-Superradiance (ST-SR) is the further enhanced emission of the bunched beam in the presence of a phase-matched radiation wave. These processes are analyzed for Undulator radiation in the framework of radiation field mode-excitation theory. In the nonlinear saturation regime the synchronism of the bunched beam and an injected radiation wave may be sustained by wiggler tapering: Tapering-Enhanced Superradiance (TES) and Tapering-Enhanced Stimulated Superradiance Amplification (TESSA). Identifying these processes is useful for understanding the enhancement of radiative emission in the tapered wiggler section of seeded FELs. The nonlinear formulation of the energy transfer dynamics between the radiation wave and the bunched beam fully conserves energy. This includes conservation of energy without radiation reaction terms in the interesting case of spontaneous self-interaction (no input radiation).

INTRODUCTION

In the context of radiation emission from an electron beam, Dicke's superradiance (SR) [1] is the enhanced "coherent" spontaneous radiation emission from a pre-bunched beam, and Stimulated-Superradiance (ST-SR) is the further enhanced emission of the bunched beam in the presence of a phase-matched radiation wave [2]. These processes are analyzed for Undulator radiation in the framework of radiation field mode-excitation theory. In the nonlinear saturation regime the synchronism of the bunched beam and an injected radiation wave may be sustained by wiggler tapering: Tapering-Enhanced Superradiance (TES) and Tapering-Enhanced Stimulated Superradiance Amplification (TESSA) [3]. In section II we present the radiation modes expansion formulation (in the spectral Fourier frequency formulation) [2] and explain the radiation cases. In section III we derive the radiation from a single bunch and from a finite train of bunches in the spectral Fourier frequency formulations. In section IV we present the single frequency formulation of the radiation field mode-excitation, and calculate the power radiated by an infinite train of bunches, and in section V we derive an energy-conserving non linear model which results in a couple of differential equations and present numerical results of those equations for some cases of interest.

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SUPERRADIANCE AND STIMULATED SUPERRADIANCE IN SPECTRAL FORMULATIONS

As a starting point we review the theory of superradiant (SR) and stimulated superradiant (ST-SR) emission from free electrons in a general radiative emission process. In this section we use a spectral formulation, namely, all fields are given in the frequency domain as Fourier transforms of the real time-dependent fields. We use the radiation modes expansion formulation of [2], where the radiation field is expanded in terms of an orthogonal set of eigenmodes in a waveguide structure or in free space (eg. Hermite-Gaussian modes):

$$\{\tilde{\boldsymbol{E}}_q(\mathbf{r}), \tilde{\boldsymbol{H}}_q(\mathbf{r})\} = \{\tilde{\boldsymbol{\mathcal{E}}}_q(\mathbf{r}_\perp), \tilde{\boldsymbol{\mathcal{H}}}_q(\mathbf{r}_\perp)\}e^{ik_{qz}z}$$

$$\check{\mathbf{E}}(\mathbf{r},\omega) = \sum_{\pm q} \check{C}_q(z,\omega)\tilde{\boldsymbol{E}}_q(\mathbf{r})$$

$$\check{\mathbf{H}}(\mathbf{r},\omega) = \sum_{\pm q} \check{C}_q(z,\omega)\tilde{\boldsymbol{H}}_q(\mathbf{r})$$

The amplitude coefficients $\check{C}_q$ have dimensions of time, are in units of sec V/m and sec A/m.

The excitation equations of the mode amplitudes is:

$$\frac{d\check{C}_q(z,\omega)}{dz} = \frac{-1}{4\mathcal{P}_q}\int \check{\mathbf{J}}(\mathbf{r},\omega)\cdot\tilde{\boldsymbol{E}}_q^*(\mathbf{r})d^2\mathbf{r}_\perp. \quad (1)$$

where the current density $\check{\mathbf{J}}(\mathbf{r},\omega)$ is the Fourier transform of $\mathbf{J}(\mathbf{r},t)$.

The above is formally integrated and given in terms of the initial mode excitation amplitude and the currents:

$$\check{C}_q(z,\omega) - \check{C}_q(0,\omega) = -\frac{1}{4\mathcal{P}_q}\int \check{\mathbf{J}}(\mathbf{r},\omega)\cdot\tilde{\boldsymbol{E}}_q^*(\mathbf{r})dV,$$

where

$$\mathcal{P}_q = \frac{1}{2}Re\iint(\tilde{\boldsymbol{\mathcal{E}}}_q\times\tilde{\boldsymbol{\mathcal{H}}}_q)\cdot\hat{e}_z d^2\mathbf{r}_\perp = \frac{|\tilde{\boldsymbol{\mathcal{E}}}_q(\mathbf{r}_\perp=0)|^2}{2Z_q}A_{em\,q} \quad (2)$$

and Z_q is the mode impedance ($\sqrt{\mu_0/\epsilon_0}$ in free space). In the case of a narrow beam passing on axis near $\mathbf{r}_\perp = 0$, Eq. (2) defines the mode effective area $A_{em\,q}$ in terms of the field of the mode on axis $\tilde{\boldsymbol{\mathcal{E}}}_q(\mathbf{r}_\perp = 0)$.

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For the Fourier transformed fields we define the total spectral energy (per unit of angular frequency) based on Parseval theorem as

$$\frac{dW}{d\omega} = \frac{2}{\pi}\sum_q \mathcal{P}_q |\check{C}_q(\omega)|^2 .$$

This definition corresponds to positive frequencies only: $0 < \omega < \infty$. Considering now one single mode q,

$$\frac{dW_q}{d\omega} = \frac{2}{\pi}\mathcal{P}_q |\check{C}_q(\omega)|^2 .$$

For a particulate current (an electron beam):

$$J(\mathbf{r},t) = \sum_{j=1}^{N} -e\mathbf{v}_j(t)\delta(\mathbf{r}-\mathbf{r}_j(t)) .$$

The field amplitude increment appears as a coherent sum of contributions (energy wavepackets) from all the electrons in the beam:

$$\check{C}_q^{out}(\omega) - \check{C}_q^{in}(\omega) \equiv \sum_{j=1}^{N}\Delta\check{C}_{qj}(\omega) = -\frac{1}{4\mathcal{P}_q}\sum_{j=1}^{N}\Delta\check{\mathcal{W}}_{qj} \text{, or}$$

$$\Delta\check{\mathcal{W}}_{qj} = -e\int_{-\infty}^{\infty}\mathbf{v}_j(t)\cdot\tilde{\boldsymbol{E}}_q^*(\mathbf{r}_j(t))e^{i\omega t}dt . \quad (3)$$

The contributions can be split into a spontaneous part (independent of the presence of radiation field) and stimulated (field dependent) part:

$$\Delta\check{\mathcal{W}}_{qj} = \Delta\check{\mathcal{W}}_{qj}^{0} + \Delta\check{\mathcal{W}}_{qj}^{st} .$$

We do not deal in this section with stimulated emission; however, we note that, in general, the second term $\Delta\check{\mathcal{W}}_{qj}^{st}$ is a function of $\check{C}_q(z)$ through $\mathbf{r}_j(t)$ and $\mathbf{v}_j(t)$ and therefore $\Delta\check{\mathcal{W}}_{qj}^{st}$ cannot be calculated explicitly from the integral in Eq. 3. Its calculation requires solving the electron force equations together with the wave excitation equation in Eq. 1.

Assuming a narrow cold beam where all particles follow the same trajectories, we may write $\mathbf{r}_j(t) = \mathbf{r}_j^0(t-t_{0j})$ and $\mathbf{v}_j(t) = \mathbf{v}_j^0(t-t_{0j})$, change variable $t' = t - t_{0j}$ in Eq. (3) [5], so that the spontaneous emission wavepacket contributions are identical, except for a phase factor corresponding to their injection time t_{0j}:

$$\Delta\check{\mathcal{W}}_{qj}^{0} = \Delta\check{\mathcal{W}}_{qe}^{0}e^{i\omega t_{0j}} ,$$

where

$$\Delta\check{\mathcal{W}}_{qe}^{0} = -e\int_{-\infty}^{\infty} v_e^0(t)\cdot\tilde{\boldsymbol{E}}_q^*(r_e^0(t))e^{i\omega t}dt. \quad (4)$$

The radiation mode amplitude at the output is composed of a sum of wavepacket contributions including the input field contribution (if any):

$$\check{C}_q^{out}(\omega) = \check{C}_q^{in}(\omega) + \Delta\check{C}_{qe}^{0}(\omega)\sum_{j=1}^{N}e^{i\omega t_{0j}} \quad (5)$$

so that the total spectral radiative energy from the electron pulse is

$$\begin{aligned}\frac{dW_q}{d\omega} &= \frac{2}{\pi}\mathcal{P}_q\left|\check{C}_q^{out}(\omega)\right|^2 \\ &= \frac{2}{\pi}\mathcal{P}_q\Bigg\{\left|\check{C}_q^{in}(\omega)\right|^2 \\ &\quad + \left|\Delta C_{qe}^{(0)}(\omega)\right|^2\left|\sum_{j=1}^{N}e^{i\omega t_{oj}}\right|^2 \\ &\quad + \left[\check{C}_q^{in*}(\omega)\Delta C_{qe}^{(0)}(\omega)\sum_{j=1}^{N}e^{i\omega t_{oj}} + c.c.\right]\Bigg\} \\ &= \left(\frac{dW_q}{d\omega}\right)_{in} + \left(\frac{dW_q}{d\omega}\right)_{sp/SR} + \left(\frac{dW_q}{d\omega}\right)_{ST-SR} .\end{aligned}$$

The first term in the {} parentheses ("in") represents the input wave spectral energy. The second term ("sp/SR") is the spontaneous emission, which may also be superradiant in case that all contributions add in phase. The third term has a very small value (averages to 0) if the contributions add randomly, so it is relevant only if the electrons of the beam enter in phase with the radiated mode. It is thus dependent on the coherent mode complex amplitude $\check{C}_q^{in}$, and therefore it is marked by the subscript "ST-SR", i.e. "zero-order"stimulated superradiance.

SINGLE BUNCH AND FINITE TRAIN OF BUNCHES

Using Eq. (4) for a single tight bunch one obtains the spectral energy per unit of angular frequency at the exit of the undulator for SR

$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{N^2e^2Z_q}{16\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)^2\frac{L_w^2}{A_{em}}\,\mathrm{sinc}^2(\theta L_w/2),$$

and for ST-SR

$$\left(\frac{dW_q}{d\omega}\right)_{ST-SR} = |\check{C}_q^{in}(\omega)|\frac{Ne}{2\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)\sqrt{\frac{2Z_q\mathcal{P}_q}{A_{em\,q}}}L_w \times \mathrm{sinc}(\theta L_w/2)\cos(\varphi_{qb0}-\theta L_w/2),$$

where L_w is the undulator length.

Similarly, for a train of N_M tight bunches, one obtains

$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{N^2e^2Z_q}{16\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)^2 \times\frac{L_w^2}{A_{em}}\,|M_M(\omega)|^2\,\mathrm{sinc}^2(\theta L_w/2),$$

$$\left(\frac{dW_q}{d\omega}\right)_{ST-SR} = |\check{C}_q^{in}(\omega)|\frac{Ne}{2\pi}\left(\frac{\bar{a}_w}{\beta_z\gamma}\right)\sqrt{\frac{2Z_q\mathcal{P}_q}{A_{em\,q}}}L_w|M_M(\omega)| \times\mathrm{sinc}(\theta L_w/2)\cos(\varphi_{qb0}-\theta L_w/2),$$

where

$$M_M(\omega) = \frac{\sin(N_M\pi\omega/\omega_b)}{N_M\sin(\pi\omega/\omega_b)} .$$

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INFINITE TRAIN OF BUNCHES IN SINGLE FREQUENCY ANALYSIS

In the single frequency analysis, the radiation modes expansion formulation is expressed by the following equations:

$$\tilde{\mathbf{E}}(\mathbf{r}) = \sum_q \tilde{C}_q(z)\tilde{\mathcal{E}}_q(\mathbf{r})$$

$$\tilde{\mathbf{H}}(\mathbf{r}) = \sum_q \tilde{C}_q(z)\tilde{\mathcal{H}}_q(\mathbf{r})$$

$$\mathcal{P}_q = \frac{1}{2}Re\iint(\tilde{\mathcal{E}}_q \times \tilde{\mathcal{H}}_q)\cdot \hat{e}_z d^2\mathbf{r}_\perp = \frac{|\tilde{\mathcal{E}}_q(\mathbf{r}_\perp = 0)|^2}{2Z_q}A_{em\,q}$$

$$\frac{d\tilde{C}_q(z)}{dz} = \frac{-1}{4\mathcal{P}_q}\int \tilde{\mathbf{J}}(\mathbf{r})\cdot \tilde{\boldsymbol{E}}^*_q(\mathbf{r})d^2\mathbf{r}_\perp$$

$$P = \sum_q \mathcal{P}_q|\tilde{C}_q(\omega)|^2$$

representing the radiated power. We can again use the excitation as described in Eq. (5):

$$\tilde{C}_q^{\text{out}}(\omega) - \tilde{C}_q^{\text{in}}(0) = -\frac{1}{4\mathcal{P}_q}\int \tilde{\mathbf{J}}(\mathbf{r})\cdot \tilde{\boldsymbol{E}}^*_q(\mathbf{r})dV\,.$$

Applying this to an infinite train of tight bunches results in the SR radiated power

$$P_{SR} = \frac{1}{32}Z_q\frac{N^2e^2\omega_0^2|\tilde{\boldsymbol{\beta}}_w|^2}{\pi^2\beta_z^2}\frac{L_w^2}{A_{em\,q}}\,\text{sinc}^2(\theta L_w/2),$$

and the ST-SR radiated power

$$P_{ST-SR} = \frac{1}{4}|\tilde{C}_q(0)|\frac{Ne\omega_0|\tilde{\boldsymbol{\beta}}_w|}{\pi\beta_z}\sqrt{\frac{2Z_q\mathcal{P}_q}{A_{em\,q}}}L_w\times \cos(\varphi_{qb0} - \theta L_w/2)\,\text{sinc}(\theta L_w/2)\,.$$

DYNAMICS OF A PERIODICALLY BUNCHED BEAM INTERACTING WITH RADIATION FIELD

In this section we include the influence of the radiated field on the charged bunches, and include this influence in the calculation of the radiated power.

The power of the electron bunches

$$N_b mc^2\frac{d\gamma}{dt} = Q_b\mathbf{v}\cdot\mathbf{E}(\mathbf{r},t),$$

combined with the excitation equation

$$\frac{d\tilde{C}_q}{dz} = \frac{-1}{4\mathcal{P}_q}\int \tilde{\mathbf{J}}\cdot \tilde{E}^*_q d^2\mathbf{r}_\perp,$$

using the definition

$$\psi \equiv -[\varphi_b(z) - \varphi_q(z) - \pi/2] = -\int_0^z \theta(z')dz' + \psi(0),$$

results in a Shifted-Pendulum equation:

$$\frac{d|\tilde{C}_q|}{dz} = B\sin\psi$$

$$\frac{d\delta\gamma}{dz} = -\frac{\beta_{zr}^3\gamma_{zr}^2\gamma_r}{k_0}K_s^2(z)[\sin\psi - \sin\psi_r]$$

$$\frac{d\psi}{dz} = \frac{k_0}{\beta_{zr}^3\gamma_{zr}^2\gamma_r}\delta\gamma + \frac{B}{|\tilde{C}_q|}\cos\psi\,,$$

where $0 < \psi_r < \pi/2$. We use the parameters of the NOCIBUR experiment [6] assuming idealized tight bunching and moderate tapering. In the following figures we show phase-space results for different cases of uniform or tapered wiggler. In all following figures panel (a) shows the phase-space diagram $\psi - \theta$, where the black line shows the separatrix at the end of the trajectory and panel (b) shows the radiation power change, the electron beam power change, and their sum which remains at zero.

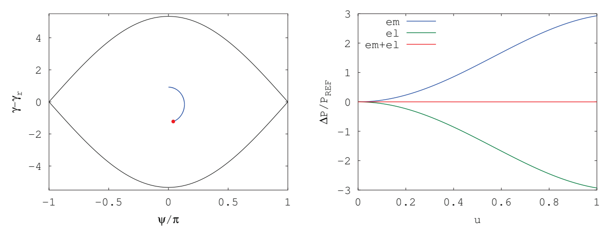

Figure 1: Uniform wiggler supperadiance.

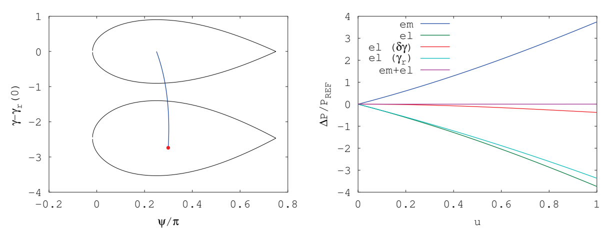

Figure 2: Tapered wiggler with initial phase ψ_r. Contributions of the tapering $\gamma_r(u)$ (cyan), synchrotron oscillation dynamics $\delta\gamma(u)$ (red, and the total beam power drop ΔP_{el} (green).

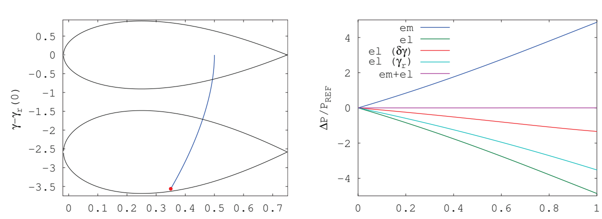

Figure 3: Same as Figure 2 with initial phase $\pi/2$.

CONCLUSION

We showed in this work a simplified approach for including the force on the charged bunches in the calculation of the

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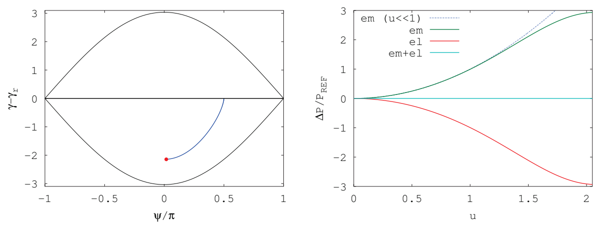

Figure 4: Uniform wiggler self interaction.

radiation. The simplification is in assuming perfectly tight bunches, and infinite train of bunches, in the single frequency approach. In spite of those simplifications, the method is useful for better understanding the tapering mechanism and improve it.

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