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# THE EFFECT OF TRANSVERSE SPACE CHARGE ON BEAM EVOLUTION AND PHOTON COHERENCE

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*Abstract*

An electron beam experiences a transverse electric field which tests to act like a defocusing force on the beam. This defocusing force will act with different strengths at different locations in the electron beam because the current varies along the beam. A single, quasi-analytic method is presented to calculate the impact of this force on beam projected emittance.


## INTRODUCTION

The effect of transverse space charge on the transverse emittance of the beam in the MaRIE [1] accelerator is estimated. The dominant effect of transverse space charge is that it causes an extra defocusing term in the electron beam transverse evolution. This defocusing is different at different locations along the electron beam, because electron current is different at different locations of the beam. This will cause the beam to go through different betatron oscillations along the accelerator, which will increase the projected emittance.

## THE MaRIE ACCELERATOR

Figure 1 shows a schematic of the MaRIE accelerator [2]. The accelerator consists of three accelerator sections, L1, L2, and L3, with two bunch compressors, BC1 and BC2. The initial current after the photocathode is 15 A. After BC1, the current is compressed to 150 A, and after BC2, the current is compressed to 3 kA. The final energy of the electron beam is 12 GeV. The accelerating gradients of the three sections are all slightly different, with the accelerating gradient in L2 being much lower than the accelerating gradient in the other two accelerator sections. The gradient in L2 is lower because the beam is accelerating off crest in order to provide a chirp for the BC2 bunch compressor.

The effect of the transverse space charge decreases as the beam accelerates, but increases with current. Because of this, the effect of the transverse space charge is strongest right after the bunch compressors.

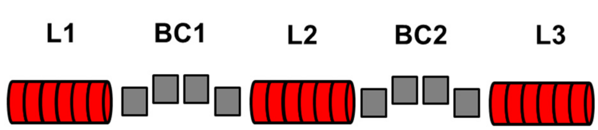


Figure 1: A simple schematic of the MaRIE accelerator.

## EQUATIONS OF MOTIONS

### *Beam Evolution*

In order to examine the effect of transverse space charge, we must first look at the transverse dynamics of a single electron that is being accelerated at a constant rate. The time derivative of the radial component of an electron under the force of a radial electric field is given by $d/dt\,(\gamma m\dot{r}) = \left(eE_{r,l}\right)/\gamma^2$. Here $\dot{r}$ is the time derivative of the electron's radial coordinate, $E_{r,l}$ is the radial field, as described in the lab frame of reference, and $\gamma$ is the relativistic factor.

For a long bunch ($\gamma l \gg a$) with a constant radial current density, given in the lab frame by $\rho_l(z)$ ($z$ is the location along the electron bunch) we can estimate the radial electric field using Gauss's law. Then we have

$$d/dt\,(\gamma m\dot{r}) = er\rho_l(z)/(2\epsilon_0\gamma^2).$$

Next, we expand out the derivative, replace $\dot{r}$ with $\beta c r'$, substitute in the Alven current $I_A = (4\pi\epsilon_0 mc^3)/e$, replace the current density with the total line current ( $\rho_l(z) = I(z)/(\pi R^2\beta c)$ ), and rearrange to get:

$$r'' = \frac{2\,I(z)}{\beta^3\gamma^3R^2I_A}r - \frac{\gamma'}{\gamma}r'. \quad (1)$$

Equation (1) describes the motion of a single electron under acceleration, with no external focusing. We next want to solve for the evolution of the rms value of the radius of the electron beam: $R^2 = \langle r^2\rangle$, when the electron beam is under constant acceleration. We use the radial envelope equation (3):

$$R'' = \frac{\langle rr''\rangle}{R} + \frac{\epsilon_r^2}{R^3}. \quad (2)$$

Next, we solve for $\langle rr''\rangle$. Using equation 1, we get:

$$\langle rr''\rangle = \frac{2\,I(z)}{\beta^3\gamma^3I_A} - \frac{\gamma'}{\gamma}RR'. \quad (3)$$

Then we can use the approximation that the normalized slice emittance remains constant in an accelerator: $\epsilon_n = \gamma\epsilon$. Plugging this and equation (3) into equation (2) gives:

$$R'' = \frac{2\,I(z)}{I_A}\frac{1}{R\beta(s)^3\gamma(s)^3} - \frac{\gamma'(s)}{\gamma(s)}R' + \frac{\epsilon_{nr}^2}{\gamma(s)^2R^3}. \quad (4)$$

Here $s$ represents the distance along the accelerator. Equation (4) can be used to calculate the evolution of electron beams with different values of electron current $I(z)$.

### *Beam Evolution*

In this paper, we analyse the evolution of the transverse size of the electron beam at different locations of the electron beam. We assume that the electron current profile is a Gaussian, given by $I(z) = I_0\exp(-z^2/(2\sigma^2))$ .

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A MATLAB [4] code was written to solve equation (4) in order to calculate the evolution of the electron beams rms size for different values of $I(z)$. Figures 2-4 show various solutions of equation (4) with different starting and accelerator parameters (see IV).

By solving $R[I(z)]$ up to the point at the end of an accelerator section, the growth in electron beam projected emittance can be calculated in terms of $R(z)$ and $R'(z)$. Figures 5-7 show plots that include both the beam current, and the final values of $R(z)$ and $R'(z)$ at the end of one of the accelerator sections, calculated using equation (4).

The projected emittance is defined as:

$$\epsilon_p = \sqrt{\langle r^2\rangle_p \langle (r')^2\rangle_p - \langle rr'\rangle_p^2} \tag{5}$$

The projected values $\langle r^2\rangle_p$ and $\langle rr'\rangle_p^2$ can easily be calculated using once $R(z)$ and $R'(z)$ are known:

$$\langle r^2\rangle_p = \frac{\int I(z)R^2(z)dz}{\int I(z)dz} \tag{6}$$

$$\langle rr'\rangle_p^2 = \frac{\int I(z)R(z)R'(z)dz}{\int I(z)dz} \tag{7}$$

In order to calculate $\langle (r')^2\rangle_p$, the slice value $\langle (r')^2\rangle_s$ must first be calculated:

$$\langle (r')^2\rangle_s = \frac{\epsilon_r^2 + (RR')^2}{R^2} \tag{8}$$

Then the projected $\langle (r')^2\rangle_p$ is just:

$$\langle (r')^2\rangle_p = \frac{\int I(z)\langle (r')^2\rangle_s dz}{\int I(z)dz} \tag{9}$$

In order to calculate emittance, equations (7-9) are integrated down to the point where the electron current has decreased to 1/3 of the peak current. This is done in order to make the emittance calculate relevant for free electron lasers [5], where there is little photon generation at low electron currents.

## RESULTS

We analyse the evolution of the electron beam after BC1 and BC2, to look for an increase in the projected emittance due to transverse space charge. The electron beam is assumed to emerge from each chicane at a waist, with a projected and a slice emittance of $\epsilon_x = \epsilon_y = 0.2$ µm.

### *Emittance Growth in L2*

Figures 2 and 3 show the evolution of $R$ and $R'$ for an electron beam slice with different values of current in L2, for an electron beam with a starting $R$ of 100 µm. Figure 4 shows the current profile of the electron beam in and the final values of $R$ at different locations of the beam. Figure 5 shows the current profile and the final values of $R'$.

The final projected emittance at the end of L2 is $\epsilon_x = \epsilon_y = 0.220$ µm when the initial rms size coming out of BC1 is 100 µm, and is 0.285 µm when the initial rms size is 1 mm. The emittance increase can be decreased by making the beam small coming out of the bunch compressor.

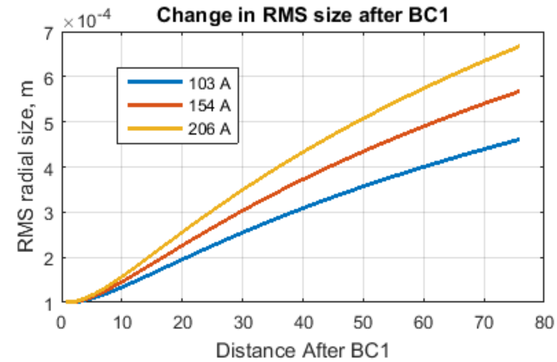


Figure 2: The evolution of rms beam size $R$ in L2 for different slice current values.

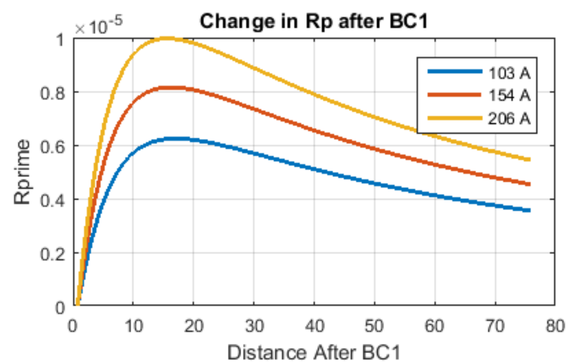


Figure 3: The evolution of $R'$ in L2 for different slice current values.

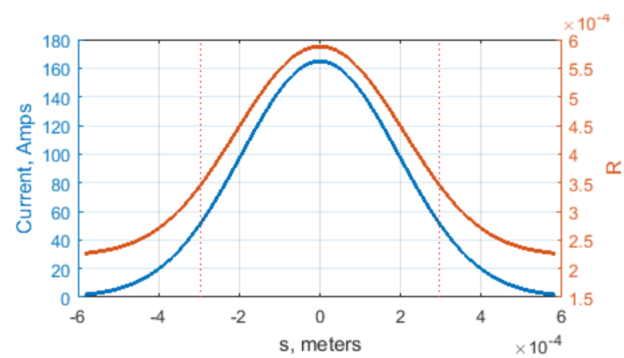


Figure 4: The beam current profile (blue, left), and the final values of R at the end of L2 (red, right), for a beam with initial rms size of 100 µm.

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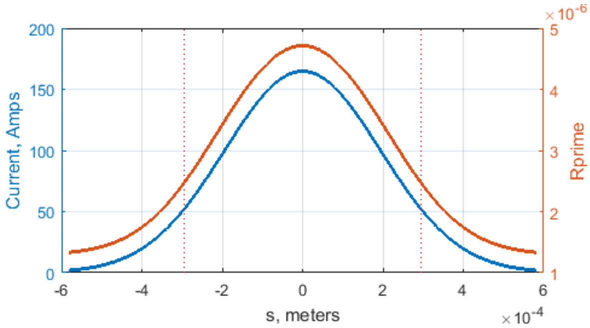

Figure 5: The beam current profile (blue, left), and the final values of R' at the end of L2 (red, right), for a beam with initial rms size of 100 µm.

### *Emittance Growth in L3*

Figures 6 and 7 show the evolution of $R$ and $R'$ for an electron beam slice with different values of current in L3, for an electron beam with a starting $R$ of 100 µm. Figure 8 shows the current profile of the electron beam in L3, and the final values of $R$ at different locations of the beam. Figure 9 shows the current profile and the final values of $R'$.

The final projected emittance at the end of L2 is $\epsilon_x = \epsilon_y = 0.255$ µm when the initial rms size coming out of BC1 is 100 µm, and is 0.405 µm when the initial rms size is 1 mm. The emittance increase is more dramatic in L3 than in L2.

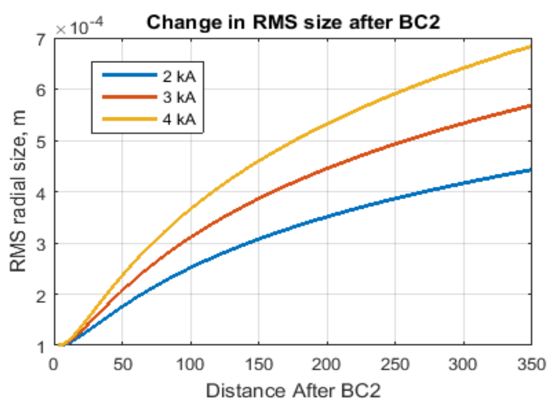

Figure 6: The evolution of rms beam size R in L3 for different slice current values.

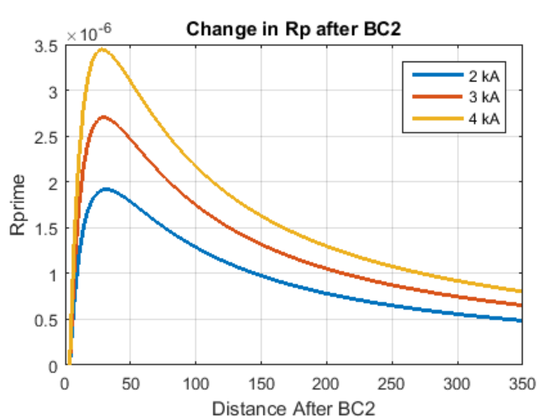

Figure 7: The evolution of R' in L3 for different slice current values.

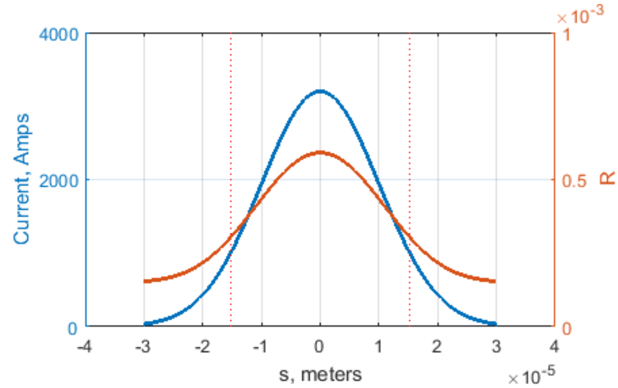

Figure 8: The beam current profile (blue, left), and the final values of R at the end of L3 (red, right), for a beam with initial rms size of 100 µm.

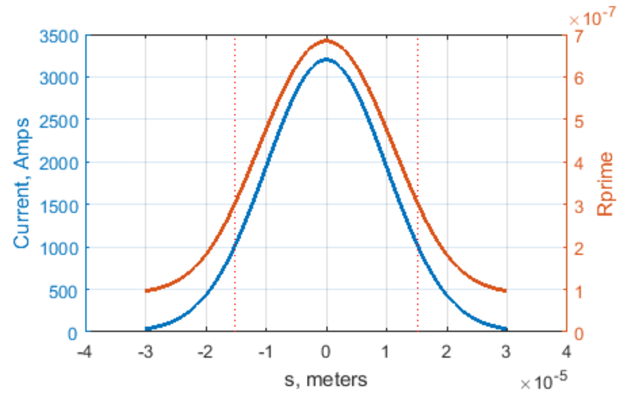

Figure 9: The beam current profile (blue, left), and the final values of R' at the end of L2 (red, right), for a beam with initial rms size of 100 µm.

## CONCLUSION

Modest increases in the projected emittance were found in both L2 and L3 as a results of transverse space charge. This effect can be reduced by making the electron beam have a smaller transverse size at the locations of the chicanes. The increase in projected emittance was greater in the final stage of the accelerator, L3, than in the central stage.

## ACKNOWLEDGMENT

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