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STOCHASTIC EFFECTS FROM CLASSICAL 3D SYNCHROTRON RADIATION

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Abstract

In most cases, the one-dimensional coherent synchrotron radiation wakefield gives an excellent approximation to the total coherent effect due to classical synchrotron radiation in bend magnets. However, full particle Liénard-Wiechert simulations have revealed that there is non-numerical, stochastic noise which generates fluctuations about the approximate 1D solution. We present a model for this stochastic term in which this noise is due to long-range interaction with a discrete number of synchrotron radiation cones. The nature of this noise and how it depends on the 3D dimensions of the beam are explored.

INTRODUCTION

Much study has focused on the so-called Coherent Synchrotron Radiation (CSR) effect in bend magnets [1] [2] [3] [4]. Radiation from the tail of the electron bunch catches up with the head electrons where it exchanges energy with them. This leads to a well-known, one-dimensional wakefield which is a deterministic function of the longitudinal distance along the bunch.

While this wakefield is often undesirable and can lead to emittance growth in bend magnets [5], it is a deterministic function and can therefore in principle always be removed. Other effects which can cause emittance degradation are stochastic in nature, and thus provide an irreversible heating of the beam. Two effects are especially important in the context of bending magnet systems: Incoherent Synchrotron Radiation (ISR) [6] and Intra-Beam Scattering (IBS) [7]. ISR is caused by the quantum nature of the synchrotron radiation emission process, and its effect on the beam grows strongly with electron beam energy. By contrast, IBS is caused by multiple small-angle Coulomb scattering events and increases with electron beam density.

Computational results in the past few years have suggested that the CSR wakefield result also contains a stochastic component [8] [9]. Recently, an analytical model has been developed which explains this stochastic noise term [10]. The goal of this paper is to compare the analytical theory with computational results obtained from full 3D Liénard-Wiechert simulations.

THEORY OVERVIEW

The full theory on the stochastic origin of the CSR noise is presented in [10], so we first briefly present the main results. The analytical theory is based on a 3D extension of the steady state two-dimensional CSR model developed by Huang, Kwan, and Carlsten [11]. In this work they noticed a long-ranged, narrow cone of longitudinal synchrotron radiation trailing behind the electron. It is this feature of the radiation profile which is ultimately responsible for the noise.

We describe the electron bunch and radiation via the scaled coordinates $\alpha = s/R$, $x = \chi/R$, and $y = \Upsilon/R$, where s is the arclength along a circular trajectory of radius R, χ is the physical radial displacement, and Υ the physical vertical displacement. It is found that this long-ranged longitudinal radiation component has magnitude $E_s^{\mathrm{T}} \approx \frac{-q\beta^2\gamma^4}{\pi\epsilon_0 R^2}$ for a beam with relativistic factor γ. The net effect of this region integrated over a uniform electron beam is found to be zero. However, counting statistics on the number of particles contained within this region leads to a stochastic variation in the total field.

This field region does not decay in the radial dimension but opens up in the vertical plane. This leads to two distinct regimes characterized by the parameter $\Xi \equiv \frac{\gamma^4\sigma_y^4}{\sigma_x^2}$, where $\sigma_{x,y}$ are the (scaled) rms beam sizes for a Gaussian electron distribution. The case of $\Xi \ll 1$ is essentially a 2-D beam, while for $\Xi \gg 1$ the beam's vertical size is much larger than the radiation extent.

The probability f that an electron in a group of N_p electrons will be contained within the trough can be computed analytically. There will therefore be a variance in the total (longitudinal) field due to this finite number of contained electrons which can be expressed as,

$$\sigma_{E_s} = gE_s^{\mathrm{T}}\sqrt{fN_p}\,, \tag{1}$$

where g is an $O(1)$ geometric factor related to the non-constant value of the field across the trough ($g = 4/9$ for a parabolic profile, for example). While the exact expression is complicated, the scaling of this field variance with energy can be written down in the two Ξ regimes as,

$$\sigma_{E_s} \sim \begin{cases} \gamma^2 & \Xi \gg 1 \\ \gamma^{2.5} & \Xi \ll 1 \end{cases}. \tag{2}$$

The above result is derived for the variations in the field at the center of the electron bunch. However, one can easily generalize the electron fraction f for an off-radial electron with displacement $a = x/\sigma_x$. The resulting ratio of f factors

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is given by the more complicated expression,

$$\frac{f(a)}{f(a=0)} = \begin{cases} e^{-\frac{a^2}{2}}\left({}_1F_1\left[\frac{3}{4};\frac{1}{2};\frac{a^2}{2}\right] + \frac{\sqrt{2}a\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}\,{}_1F_1\left[\frac{5}{4};\frac{3}{2};\frac{a^2}{2}\right]\right) & \Xi \gg 1 \\ \mathrm{erf}\left(\frac{a}{\sqrt{2}}\right)+1 & \Xi \ll 1 \end{cases}, \quad (3)$$

where ${}_1F_1$ is the generalized hypergeometric function.

These results are true for an electron bunch in a bend magnet at a single instance. We must also understand how this field variation evolves in time, or distance, through the bend of length z.

In a simple (zero-emittance) model, off-center electrons simply travel on displaced circular trajectories. As a result, they drift slightly in relative position with respect to the central particle and traverse the trough region of radiation. After a short evolution distance $\bar{z}$, all electrons which were previously contained in this region have left and a new cohort has entered. This distance $\bar{z}$ can be considered as the step size in a fixed-time random walk. This geometric motion gives,

$$\bar{z} = \frac{4R}{\sqrt{2\pi}3\gamma^3\sigma_x}\left(-\gamma_E + \log 2 - \log\frac{b}{\sigma_x}\right), \quad (4)$$

where $b = 4/(3\gamma^3\Theta)$ for a total evolution angle of Θ, and $\gamma_E \approx 0.577$ is the Euler-Mascheroni constant. These variations in longitudinal field lead to a diffusion in energy according to,

$$\sigma_{\mathcal{E}} = q\sigma_{E_s}\sqrt{z\bar{z}}. \quad (5)$$

This diffusion of energy is therefore proportional to $\gamma^{0.5\sim1}$, depending on Ξ, and inversely proportional to the electron beam density (through both the factors f and $\bar{z}$). These are the essential scaling results we wish to compare with numerical simulations.

LIÉNARD-WIECHERT SOLVER

The Liénard-Wiechert code used in this study is a massively parallelized Liénard-Wiechert solver designed to simulate a realistic ($> 10^9$) number of electrons [8]. By simple superposition, then, one computes the total electric field by summing the individual Liénard-Wiechert contributions. This allows the modeling of realistic discrete particle effects, one manifestation of which is the above-derived noise about the deterministic solution.

This model is steady state in that it assumes the trajectories of the particles to be perfect circles which can be traced arbitrarily far back in time. Practically, this means that it is only a good model once the electron bunch is sufficiently deep within the bend magnet ($z > \sqrt[3]{24\sigma_z R^2}$) [4].

As an evolution code which can step the particle distribution through a finite bend angle, the simulation is not self consistent. Although the fields are computed at each time step, they do not interact back on the electrons. This is equivalent to maintaining the electron distribution shape which distorts only geometrically. Future study may illuminate whether or not this restriction turns out to be overly limiting.

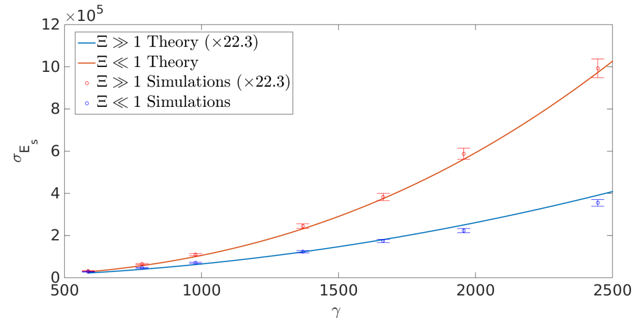

Figure 1: Comparison between theory and simulation for the variation in the longitudinal CSR field with γ for both Ξ limits.

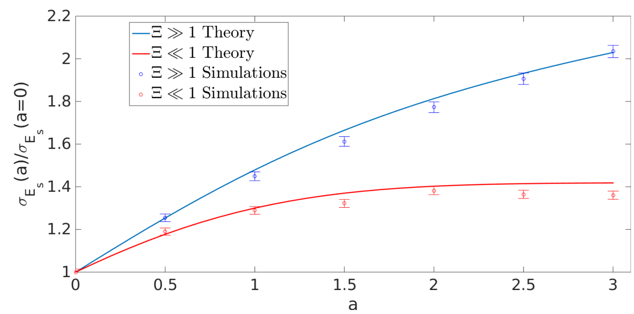

Figure 2: A comparison of the off-axis field variation $\sigma_{E_s}(a)$, normalized to its central value $\sigma_{E_s}(a=0)$ for both $\Xi \gg 1$ and $\Xi \ll 1$ bunches.

COMPARISON WITH LIÉNARD-WIECHERT SIMULATIONS

The first result to compare is the γ dependence of the variation in the longitudinal field. The electron beam used has a charge of 10 pC, and is therefore composed of 6.25×10^7 electron macroparticles. The bend radius is taken to be one meter in all studies. We compare the results from the analytical theory for the field at bunch center (Eqn. 1) and 500 separate simulations in Fig. 1.

The simulation with $\Xi \ll 1$ has $\sigma_x = 1\times10^{-5}$, $\sigma_y = 5\times10^{-7}$, and $\sigma_\alpha = 1\times10^{-5}$, while the $\Xi \gg 1$ has $\sigma_x = 5\times10^{-7}$, $\sigma_y = 1\times10^{-4}$, and $\sigma_\alpha = 1\times10^{-4}$. The error bars in the simulation data points represent the finite simulation sample size. A value of $g = 0.33$ is assumed for the each analytical calculation.The agreement between simulation and theory is remarkably good with the adjusted value of g, confirming the γ dependence of Eqn. 2.

We also simulate the field in off-axis locations to test the radial dependence of Eqn. 3. The results are shown in Fig. 2 for 5000 simulations of both a $\Xi \gg 1$ and $\Xi \ll 1$ beam. Both simulations have $E = 500$ MeV with 6.24×10^6 particles and $\sigma_x = \sigma_r = 10^{-5}$. The $\Xi \ll 1$ simulation has

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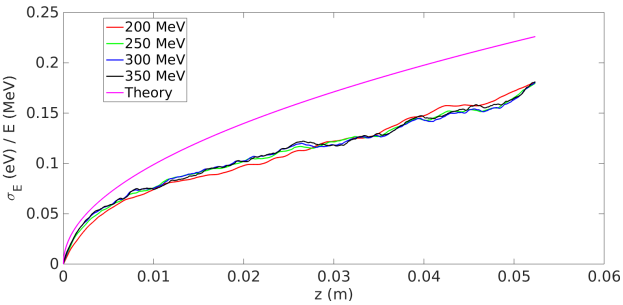

Figure 3: The cumulative diffusion in energy, normalized to the mean electron energy, due to the longitudinal field through the bend. The four averaged simulation curves are shown for each energy are each computed from 100 separate realizations of the electron beam. The 200 MeV analytic curve is shown in magenta.

$\sigma_y = 5 \times 10^{-7}$ while the $\Xi \gg 1$ case has $\sigma_y = 4 \times 10^{-5}$. The agreement between simulation and theory is quite good here as the theoretical uncertainty of the g factor of Eqn. 1 is normalized out.

Having established agreement between theory and simulation for the static field result, we turn to the evolution of this field and resulting diffusion. Again, we are primarily interested in two aspects of this diffusion: its scaling with energy and beam density.

For these simulations, we take the electron beam to have an energy 200-350 MeV and low charge 1 pC, which then evolves through three degrees of a 1 m bend magnet. The evolution proceeds in discrete steps of size $\delta z \approx 25$ μm, which is sufficient to resolve the noise structure at these low energies. The electron beam for this study has $\sigma_\alpha = \sigma_x = 10^{-5}$, and $\sigma_y = 10^{-6}$. For each beam energy, 100 separate runs are performed and compared with the analytical results in Fig. 3.

While the normalization of the analytical result is in slight disagreement, this is not entirely unexpected. The definition of $\bar{z}$ as computed in Eqn. 4, and the $\bar{z}$ which represents the fixed-time random walk step size are not necessarily identical. Nevertheless, Fig. 3 supports both the γ^{-1} dependence (in this $\Xi \ll 1$ regime), as well as the $\sqrt{z}$ random walk character of the cumulative energy change.

In order to investigate the dependence on the radial beam size σ_x, we fix the energy and simulate beams with several different values for σ_x. Evolution curves for various values of σ_x are plotted in Fig. 4. These simulations have E = 200MeV, $\sigma_\alpha = 5 \times 10^{-5}$, and $\sigma_y = 10^{-6}$, and 6.24×10^6 electrons. The scaled curves represent the $\sigma_x = 5 \times 10^{-5}$ curve scaled according to the theoretical prediction for $\bar{z}$ of Eqn. 4. As before, the normalization is off due to the ambiguity in converting between the differing values for $\bar{z}$. However, the scaled curves accurately reproduce the σ_x dependence, supporting the logarithmic dependence of Eqn. 4.

Given the myriad of assumptions that enter into ultimately obtaining Eqn. 5, the agreement between simulation and theory, especially in the scaling with various parameters, is remarkable. This suggests that the essential physics encapsulated in Eqn. 5 is more or less correct.

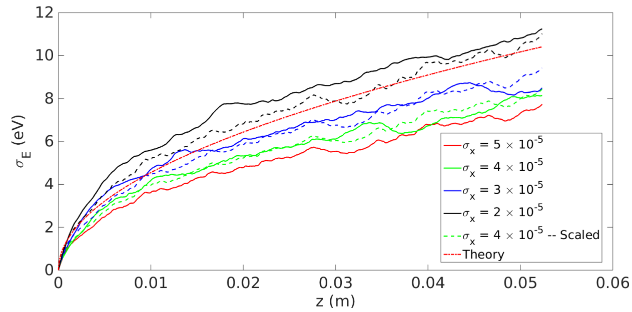

Figure 4: The cumulative diffusion in energy for various values of σ_x. Each computational curve is the result of 100 separate simulations. The dashed scaled curves are taken from the $\sigma_x = 5 \times 10^{-5}$ and scaled according to Eqn. 4 and 5. The theory curve is the pure result from Eqn. 5.

DISCUSSION

We have provided a comparison of the analytical theory of stochastic synchrotron radiation effects to full particle simulations. In all the various scaling parameters compared, the agreement is quite good, although there is an $O(1)$ ambiguity in the absolute normalization of the theory. There are however two main limitations on the results of this study.

The first is that only the longitudinal field E_s has been computed in detail and compared with simulations. The full theory predicts that there should be a similar noise term in both the radial and vertical synchrotron radiation fields [10].It may be the case that the direct contribution to emittance growth via radial and vertical fields is more important than the dispersion-induced growth from the noisy longitudinal field. Further work should therefore explore the other dimensions of the field in simulation and theory.

The second main limitation is that neither the analytical theory nor the simulation model the reaction of the electron distribution to the radiation. Further work should develop, if not a self consistent model, at least an analytically guided result for how this bunch distortion would take place. This might begin with, for example, a z-dependent bunch profile which distorts under the influence only of the 1D-CSR mean field result. Such a study will illuminate the effect, if any, to which a self-consistent model would differ from the one derived and simulation in this work.

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