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AN ANALYSIS OF OPTIMAL INITIAL DETUNING FOR MAXIMUM ENERGY-EXTRACTION EFFICIENCY*

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Abstract

For low gain free electron laser (FEL), the phase space evolutions of trapped electrons in the phase bucket are analyzed through calculating their synchrotron oscillation periods, which vary with the initial detuning and initial phase. The optimal initial detuning for the maximum energy-extraction efficiency and the corresponding saturation length are given. The analysis demonstrated that for the low gain case the gain of the strong optical field is about a quarter of that of the weak optical field (small signal gain), and the saturation power larger than that of high gain FEL can be achieved in the resonator of oscillator FEL.

INTRODUCTION

In free-electron laser (FEL), the phase of the electron in the combined radiation optical field and the undulator magnetic field plays a crucial role in the interaction between the optical field and the electron. The evolution of electron distribution in the phase space strongly affects the FEL performance [1, 2].

Usually, the electron phase space evolution has been studied with numerical simulations (e.g. [3-5]), an analytical solution for the phase space evolution of electrons in a self-amplified spontaneous emission (SASE) FEL was given in [6]. In this paper, I present an analysis of electron phase space evolution for low gain FEL based on calculation of the synchrotron oscillation period for different electrons, and study the relations of initial detuning with the energy-extraction efficiency and saturation power.

THE EQUATIONS OF PHASE SPACE EVOLUTION

The energy exchange between the optical field and the electron beam depends on the phase of the electron in the optical field plus the undulator magnetic field: $\phi=(k_s+k_u)z-\omega t$, where k_s, k_u are the corresponding wave numbers of the optical field and the undulator magnetic field. For the resonant electron its phase is fixed, namely it has $\phi'=0$. Therefore the phase velocity of the electron ϕ' describe the resonance offset of the electron, so it is called the detuning parameter. The detuning parameter can be expressed as the relative energy deviation from the resonant energy

$$\phi' = 2k_u(\gamma-\gamma_r)/\gamma_r \tag{1}$$

where γ is the normalized energy of the electrons, γ_r is the resonant energy. From above phase equation and the energy equation of the electron, the FEL pendulum equation can be obtained (for convenience, denoting $\psi=\phi+\varphi_s+\pi/2$, φ_s is the phase of the radiation field) [2]:

$$\psi'' = -\Omega^2 \sin\psi \tag{2}$$

where $\Omega=\sqrt{2k_u k_s a_u a_s f_c}/\gamma$, $a_s=eE_s/(mc^2k_s)$ and $a_u=eB_u/(mc^2k_u)$ are dimensionless vector potential of the *rms* radiation optical field E_s and undulator magnetic field B_u, respectively; f_c is the undulator coupling factor: for circularly polarized helical undulator $f_c=1$; for linearly polarized planar undulator it is a difference of the two Bessel function $f_c=J_0(\xi)-J_1(\xi)$, $\xi=a_u^2/2(1+a_u^2)$.

The evolution of electron longitudinal dynamics can be described with the motion of electrons in the phase space (ψ, ψ'). There are two classes of electron trajectory in the phase space: the bounded and the unbounded. The separatrix is the boundary separating the two of them. The region enclosed by the separatrix is called the ponderomotive bucket. The maximum value of ψ' along the separatrix gives the half-height of the bucket : $\psi'_m=2\Omega$. As the optical field intensity increase, the bucket separatrix expands, more electrons can be trapped.

For low gain case, such as amplification of the light at each pass through the undulator in oscillator FEL, the Ω can be regarded as a constant, and the change of φ_s can be neglected, (thus the bucket can be regarded as invariant), then from the pendulum equation, one can get the first integral

$$\psi'^2/2-\Omega^2\cos\psi = U \tag{3}$$

where U is a constant determined by the initial conditions.

As a trapped electron undergoes one complete orbit in the ponderomotive bucket, it travels down the undulator by a distance called synchrotron oscillation period L_{sy}. To extract the energy from the electrons to the optical field, the electrons initially are injected in the top of the bucket, i.e. the initial energy of electron beam is larger than the resonant energy. The interaction saturates after the electrons have executed approximately half of an oscillation in the ponderomotive well. At this point, the most of electrons are located in the bottom of the ponderomotive bucket.

For the phase near zero $\psi\approx 0$, the pendulum equation (Eq. (2)) can be approximately written as

$$\psi'' \approx -\Omega^2\psi$$

So the electrons near the $\psi=0$ make a simple harmonic vibration in the phase space, the corresponding period of the synchrotron oscillation is $L_{sy0}=2\pi/\Omega$. Therefore Ω is the wave number of the synchrotron oscillation for the zero

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phase electrons.

The synchrotron oscillation period for the general trapped electrons can be given from the second integral of Eq. (2)

$$L_{sy} / L_{sy0} = K(\chi^2)\pi / 2 \tag{4}$$

here K is Elliptic function of the first kind, $\chi^2 = (1 + U / \Omega^2) / 2$. The synchrotron oscillation period varies with the initial detuning and initial phase (Fig. 1).

Let x=ψ and y=ψ'/2Ω be the detuning parameter normalized with the the half-height of the bucket, Eq. (3) become as

$$y^2 + \sin^2(x/2) = \chi^2 \tag{5}$$

We consider the electrons initially been monoenergetic and uniformly distributed in phase. For the trapped electrons in the phase bucket, they rotate clockwise along the closed

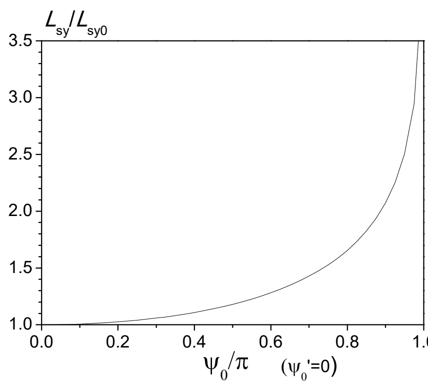

Figure 1: The synchrotron oscillation period with the initial phase (resonance case).

trajectories with different synchrotron oscillation periods determined by initial detuning and initial phase (Eq. (4)). For the ith electron, we have

$$x_i / y_i = \tan\theta_i \tag{6}$$

where

$$\theta_i = 2\pi z / L_{syi} + \theta_{i0}, \quad \theta_{i0} = \tan^{-1}(x_{i0} / y_{i0})$$

and L_{syi} is the synchrotron oscillation period of the ith electron with the initial phase x_{i0} and the initial detuning y_{i0}. It has y_{i0}= y_0 (monoenergetic), and $|x_{i0}| \le x_{0m}$, $x_{0m} = 2\arccos(y_0)$ is the maximum initial phase of the trapped electrons (see Fig. 2).

The motion of the trapped electrons in the phase bucket can be obtained by numerically solving Eqs. (4-6).

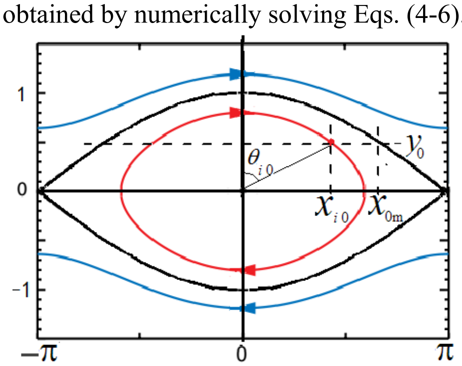

Figure 2: The phase space parameters.

THE OPTIMAL INITIAL DETUNING

In FEL process, the electrons interact with the optical field, the energy transference from the electron beam to the optical field occurred and it make the optical field be amplified. The energy-extraction efficiency can be calculated according to

$$\eta(z, y_0) = \frac{\langle \delta\gamma \rangle}{\gamma} = f_t \frac{\langle \delta\psi' \rangle_{tr}}{2k_u} = \frac{x_{0m}}{\pi}\frac{\Omega}{k_u}(y_0 - \langle y \rangle) \tag{7}$$

where $\delta\gamma$ is the electron energy losses, < > represent the average over all electrons, of which the averaged energy variation of the untrapped electrons can approximately be zero, $<>_{tr}$ represent the average over all the trapped electrons, f_t is the trapping fraction. The last step in the above equation is obtained for the initial uniformly distributed electron beam $f_t = x_{0m}/\pi$ and with mono initial detuning y_0.

To get a net gain a positive initial detuning is taken, i.e. the initial energy of the electron beam is larger than the resonant energy.

One can see that the larger initial detuning, the larger energy can be extracted from the trapped electrons, but the smaller will be the trapping fraction. Therefore there is an optimal initial detuning for the largest energy-extraction efficiency.

Using Eqs.4-7, for a given initial detuning we calculate the variation of the energy-extraction efficiency with the distance of electron traveled in the undulator, and get the maximum energy-extraction efficiency and the corresponding interaction length, i.e., the saturation length Z_s. Then for different initial detuning, we give the corresponding variation of the maximum energy-extraction efficiency and the saturation length (Fig. 3, Fig. 4).

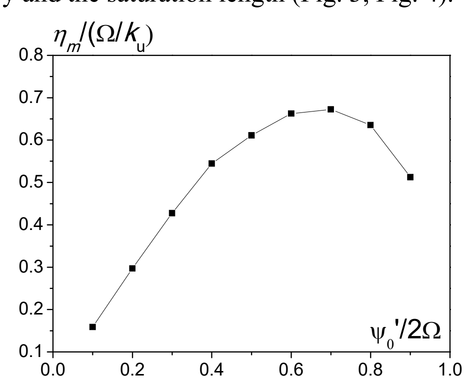

Figure 3: The maximum energy-extraction efficiency with the initial detuning.

From Figure 3 it is shown that there is an optimal initial detuning as analyzed previous, while the saturation length Z_s increases with increasing the initial detuning (Fig. 4). The optimal initial detuning is $\psi_o' / 2\Omega$=0.7, which gives the largest energy- extraction efficiency $\eta_m = 0.67 * \Omega / k_u$, the corresponding saturation length is Z_s=0.7L_{sy0} (Fig. 4), it is larger than the qualitative estimate of $L_{sy0}/2$.

For free-electron laser oscillators, the optical field

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develops from the spontaneous radiation, initially in the phase space the most of electrons are outside of the phase bucket. As the optical field intensity increase with the multi-pass gain, the phase bucket grow larger, the trapped portion of the electron bunch become larger. Then at saturation one can change the value of the electron beam detuning at entrance of the undulator to get the larger energy extraction efficiency. This has been proven already in the simulation and experiment [7], in which the energy extraction efficiency is increased by around 50% through ramping the beam energy postsaturation. The simulation results of [7] show that the energy extraction Δγ increased from 0.0552 to 0.0882, the increase by 60%, when the initial energy is changed from 0.0406 to 0.0795. The corresponding variation of the initial detuning $\psi_o'/2\Omega$ is about from 0.24 to 0.44, then from our study here (Fig. 3) it gives the change of the energy extraction efficiency about from 0.35 to 0.57, the increase by 63%, agree with the results of [7]. In addition, according to our analysis (see Fig. 3) the energy extraction efficiency can be further increased by optimizing the initial detuning.

From the relationship of the optical field gain to the energy-extraction of the electron beam, we can calculate the corresponding gain:

$$g/(2k_u\rho L)^3 = -\langle \Delta y \rangle (L_{sy0}/\pi L)^3 \tag{8}$$

where ρ is the FEL parameter, $(2\gamma\rho)^3 = 2\pi r_e n_e a_u^2 f_c^2 / k_u^2$, r_e is the classical electron radius and n_e is the peak electron density; L is the length of the undulator. With L=Z_s and the optimal parameters values previous given, Eq. (8) gives 0.063 for the gain of the strong optical field, it is about a quarter of the gain for the weak optical field: $0.27(2k_uL\rho)^3$ (the small signal gain). The numerical simulations in [2] consistent with this point.

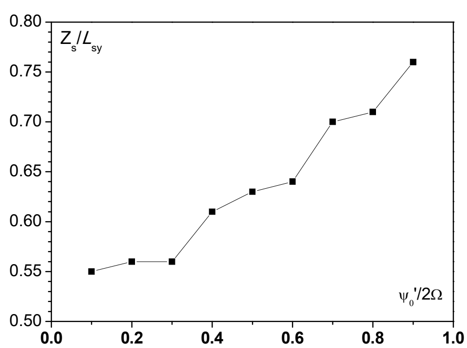

Figure 4: Varying of the saturation length with the initial detuning.

The optical field power has the relationship with the synchrotron oscillation period of the zero-phase electron as follow:

$$P = 36(\pi L_g / L_{sy0})^4 \rho P_e \tag{9}$$

where P_e is the power of the electron beam, L_g= $1/(2\sqrt{3}k_u\rho)$ is the optical power gain length. Corresponding the largest energy- extraction efficiency previous given, the saturation power is

$$P_s \approx (5L_g / L)^4 \rho P_e \tag{10}$$

For low gain FEL, it has L<3L_g [8], therefore the saturation power in the resonator of oscillator FEL can be larger than that of high gain FEL (for SASE：P_s ~ρP_e). The "low gain" refers to the gain of each single pass amplification, but the whole gain for multipass amplification is high.

For the optimal initial detuning, the evolution of the bunching factor can be exhibited (Fig. 5), which is given by

$$\tilde{b} = (\psi_{0m} \sum e^{-i\psi_i} / N_t - \sin\psi_{0m}) / \pi \tag{11}$$

where N_t is the number of the trapped electrons. We assume that no bunching occurred for the untrapped electrons. From Fig. 5, we can see that the bunching factor evolve cyclically according to the synchrotron oscillation. It firstly increase and reach the maximum at about 0.3L_{sy0}, afterward they reduce and down to the minimum at about the saturation (Z =0.7L_{sy0}). These are different from that in the high gain case, in which the bunching factor are maximum at saturation [5].

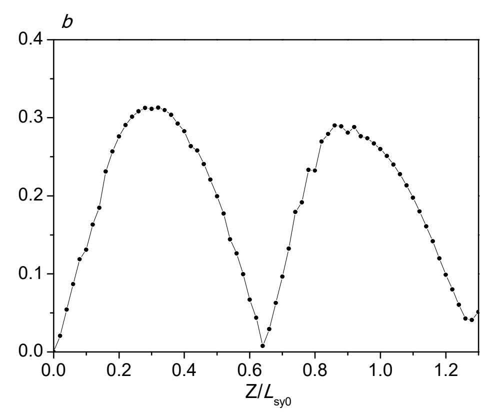

Figure 5: Varying of the bunching factor.

SUMMARY

In this paper, we analyzed the evolution of the electron distribution in the phase space. We give the phase motion of the trapped electrons in the phase bucket for low gain FEL by calculation of their synchrotron oscillation periods which are determined by initial detuning and initial phase. Then using normalized parameters, we give the optimal initial detuning for the largest energy-extraction efficiency to be $\psi_o'/2\Omega = 0.7$ and the corresponding saturation length to be L_s=0.7L_{sy0}. We exhibited that the bunching factor are minimum at about saturation. Our analysis demonstrated that for the low gain case the gain of the strong optical field is about a quarter of that of the weak optical field (i.e. the small signal gain), and for oscillator FEL the saturation power larger than that of SASE can be achieved in the resonator.

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In the analysis we consider the case with initial mono-energy and uniform phase. For the electrons initially with non-mono-energy and non-uniform phase, the cases can also be analyzed similarly.

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