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HARMONIC UNDULATOR RADIATION WITH DUAL NON PERIODIC MAGNETIC COMPONENTS

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Abstract

Undulator radiation at third harmonics generated by harmonic undulator in the presence dual non periodic constant magnetic field has been analyzed. Electron trajectories along the 'x' and 'y' direction has been determined analytical and numerical methods. Generalized Bessel function is used to determine the intensity of radiation and Simpson's numerical method of integration is used to find the effect of constant magnetic fields. Comparison with previous analysis has also been presented.

INTRODUCTION

Free electron Lasers (FEL) generation is a state of art technology and has large numbers of applications in cutting edge technologies [1]. Tunability and brilliance at lasing wave length in FEL are the key parameters for number of research applications. Lasing wavelength of FEL depends upon the values of undulator parameter, undulator wavelength and relativistic parameter of electron beam. Recent works in FEL theory has emphasised the effect on non periodic magnetic field i.e. constant magnetic field component along or perpendicular or in both directions of the periodic magnetic field of planar undulator on the out coming undulator radiation(UR)[2-3]. Partial compensation on the divergence of UR has been demonstrated by imposing weak constant magnetic component in the analytical form and all the major sources of homogeneous and inhomogeneous broadening have been accounted for the characteristics of the electrons beam by K. Zhukovsky [4]. The constant non-periodic magnetic constituents are studied to compensate the divergence of the electronic beam [5]. Dattoli *et al* has initially reported the effect on UR from planar undulator with constant magnetic field component [6]. The later studies focuses on higher harmonics generation by addition of additional harmonic field [2-5].

Higher harmonic generation has been studied by using Harmonic undulator (HU) consists additional harmonic field along with sinusoidal planar magnetic field[7-13]. HU uses modest electron beam energy and lasing at third harmonic is reported by N.Sei *etal* [14]. The harmonic field can be generated by the addition of shims in the planar undulator structure[15-17]. Constant magnetic field may present due to errors in Undulator design and horizontal component of earth's magnetic field and modify the UR Magnetic field. H Jeevakhan *et al* have presented semi analytical results for the effect of perpendicular constant magnetic field on the gain of HU at higher harmonics [3,18]. In the present paper we have analysed HU with dual non periodic magnetic field. In the previous reported works the independent effect of constant magnetic field, parallel and perpendicular to planar undulator field had been analysed. The combined effect on intensity reduction due dual magnetic field has been presented. The additional harmonic field compensates the intensity loss in UR in presented model.

UNDULATOR FIELD

Planar undulator sinusoidal magnetic field encompasses with a perpendicular constant magnetic field in present analysis and is given by

$$B = \left[B_0\kappa_x, a_0 B_0(\sin k_u z + \Delta \sin k_h z) + B_0\kappa_y, 0\right] \quad (1)$$

Where, $k_u = \frac{2\pi}{\lambda_u}$ and $k_h = \frac{2\pi}{\lambda_h}$ where k_u and k_h are undulator and HU wave number respectively , λ_u is undulator wave length and $\lambda_h = h\lambda_u$, h is harmonic integer, B_0 is peak magnetic field, $\Delta = \frac{a_1}{a_0}$, a_0 and a_1 controls the amplitude of main undulator field and additional harmonic field, $\kappa_y$and κ_x are the magnitudes of constant non periodic magnetic field parallel and perpendicular to main undulator field.

The velocity of electron passing through undulator is derived by using Lorentz force equation:

$$\frac{dv}{dt} = -\frac{e}{\gamma mc}\left(\bar{v} \times \bar{B}\right) \quad (2)$$

This gives

$$\beta_x = -\frac{K}{\gamma}\left[\cos(\Omega_u)t + \Delta\frac{\cos(h\Omega_u)t}{h} - \kappa_y\Omega_u t\right]$$

$$\beta_y = -\frac{K}{\gamma}\kappa_x\Omega_u t \quad (3)$$

$$\beta_z = \beta^* - \frac{K^2}{2\gamma^2}\left[\left\{\frac{1}{2}\cos(2\Omega_u)t + \frac{1}{2}\left(\frac{\Delta}{h}\right)^2\cos(2h\Omega_p)t + \left(\frac{\Delta}{h}\right)\cos(1 \pm h)\Omega_u t - 2\kappa_y\Omega_u t\cos(\Omega_u t) - 2\kappa_y\Omega_u t\cos(h\Omega_u t)\right\} + (\kappa_x^2 + \kappa_y^2)\Omega_u^2 t^2\right] \quad (4)$$

Where $m\ and\ m_0$ are relativistic and rest mass of electron respectively and value of m is governed by the relativistic parameter γ , $K = \frac{a_0 e B_0}{\Omega_u m_0 c}$ is the undulator parameter and $\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2+K_1^2}{2}\right]$ with $K_1 = \frac{\Delta K}{h}$ and $\Omega_u = k_u c$.

The solution of Eq. 4 gives the electron trajectory along z direction,

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$$\frac{z}{c} = \beta^* t - \frac{K^2}{8\gamma^2\Omega_u}\sin(2\Omega_u t) - \frac{K_1^2}{8\gamma^2 h\Omega_u}\sin(2h\Omega_u t) - \frac{KK_1}{2\gamma^2(1\pm h)\Omega_u}\sin(1\pm h)\,\Omega_u t + \frac{K^2\kappa_y t\sin(\Omega_u t)}{\gamma^2} + \frac{K^2\kappa_y\cos(\Omega_u t)}{\gamma^2\Omega_u} + \frac{KK_1\kappa_y t\sin(h\Omega_u t)}{h\gamma^2} + \frac{KK_1\kappa_y\cos(h\Omega_u t)}{h^2\gamma^2\Omega_u} - \frac{K^2(\kappa_x^2+\kappa_y^2)\ \ {\Omega_u}^2 t^3}{6\gamma^2} \tag{5}$$

The spectral properties of radiation can be evaluated from Lienard - Wiechart integral [19-20],

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\{\hat{n}\times(\hat{n}\times\hat{\beta}\}exp\left[i\omega\left(t-\frac{z}{c}\right)\right]dt\right| \tag{6}$$

when integrated over undulator length, $T = \frac{2N\pi}{\Omega_u}$ and $'\omega'$ is the emission frequency with variables as

$$\xi_1 = -\frac{\omega K^2}{8\gamma^2\Omega_u},\ \xi_2 = -\frac{\omega K_1^2}{8\gamma^2 h\Omega_u},$$
$$\xi_{3,4} = -\frac{\omega KK_1}{2\gamma^2(1\pm h)\Omega_u}$$
$$\xi_5 = -\frac{\omega K^2\kappa_y}{\gamma^2\Omega_u},\ \text{and}\ \xi_6 = -\frac{\omega KK_1\kappa_y}{h^2\gamma^2\Omega_u}$$

The brightness expression read as

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left(\frac{K}{\gamma}\right)^2\left[\left|\hat{\imath}\int_{-\infty}^{\infty}dt\left\{cos(\Omega_u t)+\frac{\Delta}{h}cos(h\Omega_u t)-\kappa_y\Omega_u t\right\}expi(\vartheta t + \varphi t^3)J_m(0,\xi_1)J_n(0,\xi_2)J_p(\xi_3)J_q(\xi_4)J_r(\xi_5)J_s(\xi_5)J_u(\xi_6)J_v(\xi_6)\right|^2 + \left|\hat{\jmath}\int_0^T dt\{\kappa_x\Omega_u t\}\,expi(\vartheta t + \varphi t^3)J_m(0,\xi_1)J_n(0,\xi_2)J_p(\xi_3)J_q(\xi_4)J_r(\xi_5)J_s(\xi_5)J_u(\xi_6)J_v(\xi_6)\right|^2\right] \tag{7}$$

$$\vartheta = \frac{\omega}{\omega_1} - \{m + nh + p(1+h) + q(1-h) + r + s + uh + vh\}\Omega_u$$

$$\varphi = \frac{\omega K^2(\kappa_x^2+\kappa_y^2){\Omega_u}^2}{6\gamma^2}$$

And Eq. (7) can be further reduced to

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2 T^2}{4\pi^2 c}\left\{|I_{x1}+I_{x2}+I_{x3}|^2 + \left|I_y\right|^2\right\} \tag{8}$$

With

$$I_{x1} = \frac{K}{2\gamma}\left[\begin{matrix}J_{m+1}(0,\xi_1)+J_{m-1}(0,\xi_1)\}J_n(0,\xi_2)J_p(\xi_3)\\ J_q(\xi_4)J_r(\xi_5)J_s(\xi_5)J_u(\xi_6)J_v(\xi_6)\end{matrix}\right]S(\vartheta,\varphi)$$

$$I_{x2} = \frac{K}{2\gamma}\left[\frac{\Delta}{h}\{J_{n+1}(0,\xi_2) + J_{n-1}(0,\xi_2)\}J_m(0,\xi_1)J_p(\xi_3)J_q(\xi_4)J_r(\xi_5)J_s(\xi_5)J_u(\xi_6)J_v(\xi_6) + \right]S(\vartheta,\varphi)$$

$$I_{x3} = \frac{2i\pi K\kappa_y N}{\gamma}S'(\vartheta,\varphi)\ \ I_y = \frac{2i\pi K\kappa_x N}{\gamma}S'(\vartheta,\varphi)$$
$$S(\vartheta,\varphi) = \left|\int_0^1 e^{(\vartheta'\tau+\varphi'\tau^3)}d\tau\right| \tag{9}$$

$$S'(\vartheta,\varphi) = \frac{\partial S(\vartheta,\varphi)}{\partial\vartheta} = \left|\int_0^1 \tau e^{(\vartheta'\tau+\varphi'\tau^3)}d\tau\right| \tag{10}$$

$\vartheta' = \vartheta T, \varphi' = \varphi T^3$ and $\tau = {t}/{T}$ is unit interaction time.

For $\kappa_x = 0$, Eq. (8) changes to

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2 T^2}{4\pi^2 c}\{|I_{x1}+I_{x2}+I_{x3}|^2\} \tag{11}$$

With altered value of φ as

$$\varphi = \frac{\omega K^2(\kappa_y^2){\Omega_u}^2}{6\gamma^2}$$

For $\kappa_y = 0$,Eq. (8) changes to

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2 T^2}{4\pi^2 c}\left\{|I_{x1}+I_{x2}|^2 + \left|I_y\right|^2\right\} \tag{12}$$

With altered value of I_{x1}, I_{x2} φ as

$$I_{x1} = \frac{K}{2\gamma}[J_{m+1}(0,\xi_1) + J_{m-1}(0,\xi_1)\}J_n(0,\xi_2)J_p(\xi_3)J_q(\xi_4)]S(\vartheta,\varphi)$$

$$I_{x2} = \frac{K}{2\gamma}\left[\frac{\Delta}{h}\{J_{n+1}(0,\xi_2) + J_{n-1}(0,\xi_2)\}J_m(0,\xi_1)J_p(\xi_3)J_q(\xi_4) + \right]S(\vartheta,\varphi)$$

$$\varphi = \frac{\omega K^2(\kappa_x^2){\Omega_u}^2}{6\gamma^2}$$

RESULT AND DISCUSSION

Equation (8) reads the intensity of spontaneous UR extracting from HU with dual non periodic magnetic field. The line shape functions $S(\vartheta,\varphi)$ and $S'(\vartheta,\varphi)$ in Eq.(8) are given by Eq.(9) and Eq.(10) respectively. In earlier reported work [3,6,18] the term in Eq.(8) consisting $S'(\vartheta,\varphi)$ has been neglected due to diminishing value of κ. In our analysis we have included this term in numerical integration and its effect on the line shape function. The parameters used for simulation are listed in Table 1.

Table 1: Parameters Used for Simulation

Parameter	**Symbol**
Undulator parameter	K=1
Electron bean relativistic parameter	$\gamma = 100$
Undulator wavelength	$\lambda_u = 5cm$
Addition periodic harmonic field number	$h = 3$
Harmonic field parameter	K_1=0 - 0.11
Number of period	N=100

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Figure 1 illustrates the intensity distribution of radiations given by Eq. (11) and (12) with symmetric electron beam at third harmonic in arbitrary units with selection of parameters given in table 1 and different values of $\kappa_y$and κ_x. There is a shift in resonance and reduction in intensity with effect of non periodic constant field contribution. The lines and the scattered points in the Fig. 1 are for Eq. (11) and Eq. (12) respectively, overlapping of scattered points over lines indicates that both equation results same output. For same values of κ_y or κ_x , the intensity distribution is same irrespective of change in values for arguments of Bessel function as given in Eq. (11) and (12). Values of κ_y at κ_x dominates the intensity distributions irrespective to the change of arguments of Bessel function. The shift in resonance at third harmonic is more or less same to previous reported works [3,18]. For a particular value of κ =0.00008 the resonance shift is around 0.0045 and the intensity reduction is nearly 12 % at third harmonic.

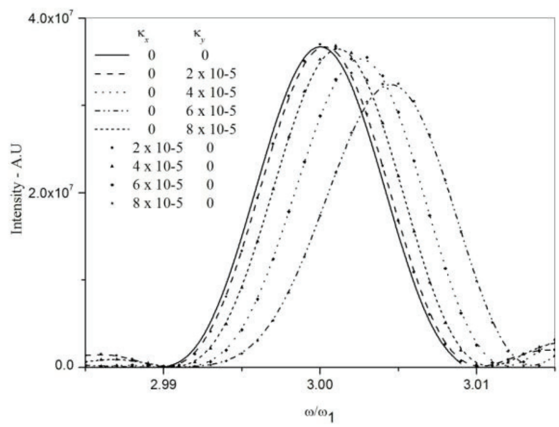

Figure 1: Frequency Spectrum at third harmonic with parameter given in table 1 varying constant magnetic field parameter κ_y and κ_x.

In Fig. 2 we have presented numerical solution for Eq. (8) and it demonstrate the combining effect of both non periodic component on intensity distribution at third harmonic. The line shape function distort from Gaussian for values κ_y and κ_x more than 0.00006. All the parameter are kept same as used in Fig. 1. The intensity degradation for the values of κ_y and κ_x at 0.00004, resembles with the value of either of κ_y and κ_x as 0.00006 as square of later nearly equals to the addition of square of former vales of κ_y and κ_x.

The intensity degradation mainly occurs due to energy spread in the electron beam, and in present case is due to additional non periodic magnetic field. It can be accommodated by additional harmonic field. The harmonic field enhances intensity and compensate the loss due to constant magnetic field. The intensity at third harmonics with constant magnetic field as $\kappa_y = \kappa_x$ = 0.00006 and variation harmonic field parameter as K1 = 0.0 to 0.11 is shown in Fig. 3 and keeping all the remaining parameter as given in Table 1. As a particular case intensity reduction by κ = 0.00006 can be compensated by additional harmonic field K1 = 0.11 as manifested in Fig. 3.

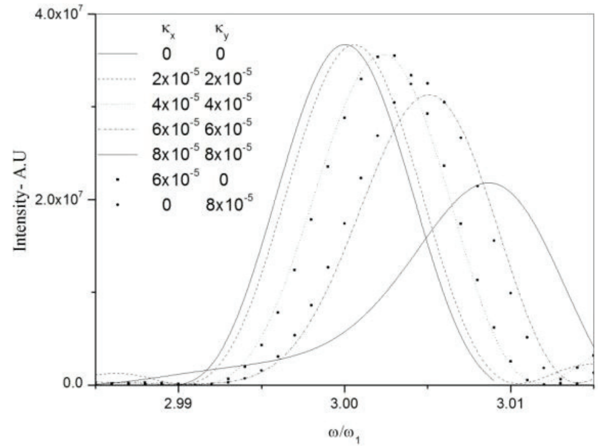

Figure 2: Frequency Spectrum at third harmonics with parameters same given in table 1, varying $\kappa_y = \kappa_x$.

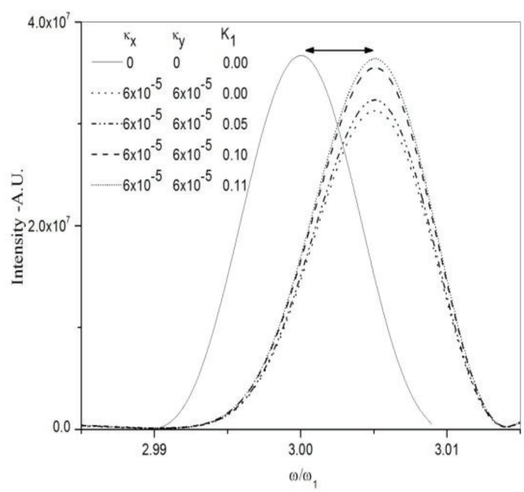

Figure 3: Frequency Spectrum at third harmonics with varying harmonic Field amplitude as K_1=0 and 0.11, $\kappa_y = \kappa_x$ =0.00006, and rest parameters same as in Fig. 1.

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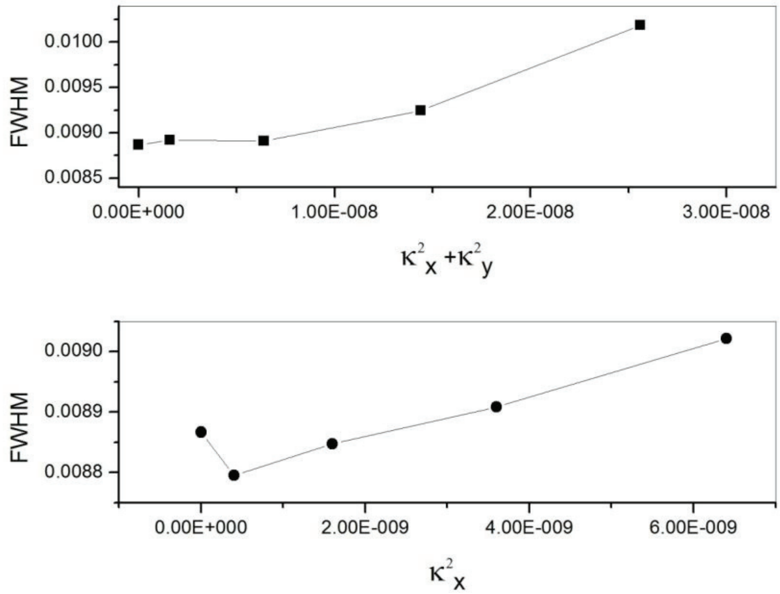

Figure 4: Variation of FWHM at third harmonics with varying κ_y and κ_x and rest parameters same as in Fig. 1.

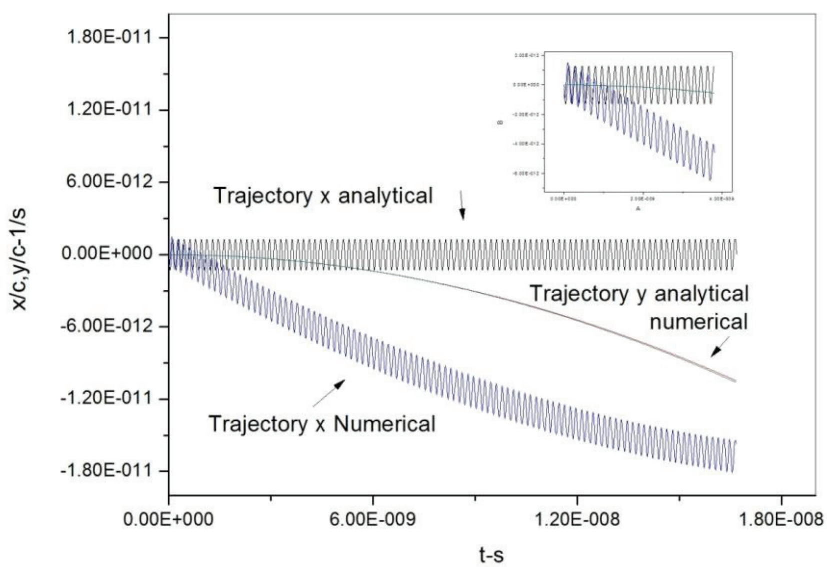

Figure 5: Trajectory of electron along x and y directions at $\kappa_y = \kappa_x = 0.00004$ by analytical and numerical method.

Figure 4 displays the variation of FWHM with κ_x^2 and $\kappa_x^2 + \kappa_y^2$. As from the figure it is clear that along with intensity reduction the FWHM also increases with increase in value of non periodic constant magnetic field. The effect will be more in the presence of dual constant magnetic components. The constant

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components of magnetic field are also possible reasons for broadening of line shape function along with energy spread in FEL systems.

Equations 3 and 4 are integrated to get trajectory along 'x' , 'y' directions and 'z ' directions. Equation 5 gives the analytical solution of Eq.4. Figure 5 gives graphical representation of the analytical and numerical solutions by Runga-Kutta method of Eq. 3 and 4.The trajectory along 'y' directions by analytical and numerical method nearly coincides, whereas there is a deviation in the values of trajectory along 'x' direction simulated by analytical and numerical method.

In conclusions, we have presented the expression for on axis spontaneous radiation by UR with harmonic and dual constant magnetic field component. There is as an intensity reduction and line shape broadening due to presence of constant magnetic field, along the main field due to error in design and perpendicular to main field due to horizontal component earth's Magnetic field. Enhancement in intensity at third harmonics can be done by additional harmonic field where as shift in resonance remains unaltered. Analytical and Numerical approach has been used to find the trajectory of electron in Multiple magnetic field. The deviation of trajectory from the axis affects out coming on axis intensity. Effect of Electron beam emittance and beam divergence in presence of constant field, properties of out coming radiation at different angle and solution to resonance shift are the future scope of studies for present work.

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