ANALYTICAL APPROACH FOR ACHROMATIC STRUCTURE STUDY AND DESIGN

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Abstract

The analytical approach is proposed to study the achromatic structures. The fully kinetic self-consistent time-dependent models are implemented in the approach. The method allows to predict the beam phase portrait behavior in magnetic fields of the structure with easy scaling and wide physical generality. The preliminary results of the method application for the bending magnets and the quadrupoles are presented.

INTRODUCTION

Achromatic structures are the important elements of the modern accelerator facilities [1-3]. The choice of suitable achromatic structures for the specific accelerator facility is a significant part of the facility research and development. The beam dynamic simulation with the help of numerous program codes (as example, [4-6]) is usually applied for this aims. In the case of multi-parameter task of charged particle beam formation with high intensity and high brightness the analytical approach is an attractive tool to describe the beam dynamics because it allows the task scalability and predicts the beam behavior with the most physical generality. Such an approach becomes possible, for instance, while using the self-consistent time-dependent models [7-9]. These models are the modifications of well-known Kapchinsky-Vladimirsky model (K-V model), which describes quasistationary continuous beam. In the paper presented the 2D and 3D models are used for the analysis of the beam phase portrait behavior in the dipole and quadrupole magnets, involved into the achromatic structure. These models are fully kinetic and time-dependent and correspond to uniformly charged intense beam both continuous and bunched. The models consider the continuous beam with elliptical cross-section and the bunched beam shaped as an ellipsoid with various relations between the semiaxes.

MOTION IN BENDING MAGNET

To describe analytically the motion of the beam with elliptical cross-section in the bending magnet the model is developed, corresponding to the uniformly charged beam. Some idealization of the task geometry is applied, namely, the bunch should have the most size in the coordinate direction corresponding to the direction of the magnetic force lines (2D approximation), and the magnetic field has a sharp edge, i.e. at this step the fringe fields of the dipole magnet are not taken into account. In addition, we assume the simple beam structure when it consists of one kind of the particle with the same values of both the charge and the mass.

Let us begin from the case of non-relativistic and non-intense or emittance-dominant beam. The approximation of uniformly charged beam moving in the uniform external field allow to write the invariant I for the linear equations of the beam particle motion:

$$I = \frac{(\dot{u}x - u\dot{x})^2}{\varepsilon_x^2} + \frac{(\dot{v}y - v\dot{y})^2}{\varepsilon_y^2} + \frac{x^2}{u^2} + \frac{y^2}{v^2} + + C_0(\dot{x}y - x\dot{y}) \quad (1)$$

where x,y – the coordinate axes, connected with the beam mass center and rotating with the mass center in the laboratory coordinate system, u,v – the auxiliary time-dependent functions, C_0 – the mean angular momentum of the particle, the dot means the differentiation with respect to the time.

The kinetic distribution function f corresponding to the particle oscillations in the plane of the turn may be written as

$$f = \kappa\delta(I-1), \quad (2)$$

where κ – the normalization constant, δ – the delta function. Such function automatically satisfies to Vlasov equation and really describes the beam with elliptical cross-section in the plane of the turn:

$$n = \int f(I)dxdy = \frac{\pi\kappa}{uv}\varepsilon_1\varepsilon_2\sigma(1 - Ax^2 - By^2 - Cxy) \quad (3)$$

where

$$A = \frac{1}{u^2} - \frac{C_0^2\varepsilon_2^2}{4v^2}, B = \frac{1}{v^2} - \frac{C_0^2\varepsilon_1^2}{4u^2}, C = C_0\left(\frac{\dot{u}}{u} - \frac{\dot{v}}{v}\right).$$

Here σ is Heaviside function, ε_1 , ε_2 – the values, which characterize the partial emittances of the beam in the cross-section, corresponding to the plane of the turn.

Using the equations (1) and (2), one can obtain for the beam rms values:

$$\frac{1}{R_x^2} = \left(A + B + ((A-B)^2 + C^2)^{1/2}\right)/2$$

$$\frac{1}{R_y^2} = \left(A + B - ((A-B)^2 + C^2)^{1/2}\right)/2 \quad (4)$$

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where R_x and R_y – the rms values of the semiaxes of the elliptical beam cross-section, corresponding to the coordinate plane, connected with the plane of the turn.

The equations obtained above give the possibility to estimate the effect of the emittance transformation in the plane of the turn and to determine the general factors affecting the phenomenon.

Analytically estimated maximum value of the emittance transfer is $k = \varepsilon_2 / \varepsilon_1$, which corresponds to the turn of the beam center of the mass at the angle 180º.

It is evident, that initial relations between the beam phase characteristics (at the inlet into the magnetic field area) as well as the initial beam angular momentum affect strongly the beam phase transformation followed. For the quanitative estimate of the emittance transformation during the beam turn at the arbitrary angle the system of the ODE should be solved representing the particle motion in the magnet.

Note here, that first the effect of the emittance transfer for the simple geometry of the bunch was studied in [10,11].

MOTION IN QUADRUPOLE MAGNET

Here the phase portrait behavior of intense charged particle bunch moving in the magnetic field of the stationary quadrupole is studied. We assume that the beam mass center moves in the symmetry plane of the magnet. To study this case the approximation of strong linear dependence of the forces acting on the particles is supposed for both the external field of the magnet and the own bunch fields arisen due to the own space charge. Let us consider only non-relativistic beam motion again.

In the coordinate system (x,y,z), connected with the bunch center of mass, particle motion equations may be written as

$$\begin{aligned} x^{\cdot\cdot} &= \omega' x_0 y^{\cdot} + \omega' x y_0^{\cdot} + \frac{e}{m}\frac{\partial \Phi}{\partial x}, \\ y^{\cdot\cdot} &= -\omega' x_0^{\cdot} x - \omega' x_0 x^{\cdot} + \frac{e}{m}\frac{\partial \Phi}{\partial y}, \\ z^{\cdot\cdot} &= -\omega' y_0^{\cdot} z + \frac{e}{m}\frac{\partial \Phi}{\partial z}. \end{aligned} \tag{5}$$

Here $x_0(t)$, $y_0(t)$ are the coordinates of the ellipsoidal bunch center in laboratory coordinate system, $\Phi(x,y,z)$ – the potential of the self-consistent bunch field, ω' is the gradient of the cyclotron frequency corresponding to the field of the quadrupole, e and m are the charge and the mass of the beam particle respectively.

The motion of the bunch center is described by the equations

$$x_0^{\cdot\cdot} = \omega' x_0 y_0^{\cdot},$$
$$y_0^{\cdot\cdot} = -\omega' x_0 x_0^{\cdot},$$

with initial conditions

$$x_0\big|_{t=0} = y_0\big|_{t=0} = 0, x_0^{\cdot}\big|_{t=0} = v, y_0^{\cdot}\big|_{t=0} = 0.$$

Here v is the bunch velocity. So the equation for the motion of the bunch mass center may be rewritten as

$$x_0^{\cdot} = \pm\sqrt{v^2 - \omega'^2 x_0^4 / 4}.$$

The potential of the uniformly charged ellipsoid we may represent as [12]:

$$\frac{e\Phi}{m} = \frac{e\Phi_0}{m} + \frac{3q}{2}(K_x x_1^2 + K_y y_1^2 + K_z z_1^2),$$

where Φ_0 – the potential in the center of the ellipsoid, (x_1,y_1,z_1) – the coordinate system, connected with the main axes of the bunch, K_x, K_y, and K_z are determined by the following equations:

$$\begin{aligned} K_x &= 2(K-E)/\varepsilon^2 R_x^3, \\ K_y &= 2(E - K(1-\varepsilon^2))/R_x R_y^2, \\ K_z &= 2(1 - R_z E / R_x (1-\varepsilon^2)^{1/2}) / R_x R_y R_z \end{aligned}$$

Here K and E are the full elliptical integrals of 1st and 2nd type respectively, $\varepsilon = (1 - R_y^2/R_x^2)^{1/2}$ is the argument of the integrals K and E, $q = e^2 N / m$, N – the value of the particles per the bunch, e and m are the charge and the mass of the particle respectively.

Taking into account the equations (5), one can write the following invariants for the case of the strong bunch center motion in the symmetry plane of the magnet:

$$\begin{aligned} I_{xy} &= A_1 x^{\cdot 2} + 2A_2 x x^{\cdot} + A_3 x^2 + B_1 y^{\cdot 2} + \\ &+ 2B_2 y y^{\cdot} + B_3 y^2 + C_1 x^{\cdot} y^{\cdot} + C_2 y x^{\cdot} + C_3 x y^{\cdot} + C_4 \end{aligned} \tag{6}$$

$$I_z = L z^{\cdot} - L^{\cdot} z,$$

where L(t) is the solution of the equation

$$L^{\cdot\cdot} - (\omega' y_0^{\cdot} - 3qK_z)L = 0. \tag{7}$$

From the condition $dI/dt \equiv 0$ we can obtain the system of the ODE of 1st order, which fully determines the bunch phase portrait behavior in the magnetic field of the quadrupole:

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$$
\begin{aligned}
&A_1' = -2A_2 + \omega_H C_1 \\
&A_2' = -A_3 - A_1 M_1 + (\omega_H C_3 - C_1(M_2 - \omega' x_0'))/2 \\
&A_3' = -2A_2 M_1 - C_3(M_2 - \omega' x_0') - 2A_2 \omega' y_0' \\
&B_1' = -2B_2 + \omega_H C_1 \\
&B_2' = -B_3 - B_1 M_3 - (\omega_H C_2 + C_1 M_2)/2 \\
&B_3' = -2B_2 M_3 - C_2 M_2 \qquad (8) \\
&C_1' = -C_2 - C_3 + 2\omega_H (B_1 - A_1) \\
&C_2' = -C_4 - C_1 M_3 - 2(A_1 M_2 - \omega_H B_2) \\
&C_3' = -C_4 - C_1 M_1 - 2(A_2 \omega_H - B_1 M_2) - C_1 \omega' x_0' \\
&C_4' = -C_2 M_1 - C_3 M_3 - 2(A_2 + B_2) M_2 - \\
&- C_2 \omega' y_0' + 2B_2 \omega' x_0'
\end{aligned}
$$

Here

$$
\begin{aligned}
&\omega_H = \omega' x_0, M_1 = 3q(K_y + \cos^2\theta(K_x - K_y)), \\
&M_2 = 3q \sin 2\theta (K_x - K_y)/2, \\
&M_3 = 3q(K_x - \cos^2\theta(K_x - K_y))
\end{aligned}
$$

The angle θ characterizes the position of the coordinate system, connected with the main axes of the elliptical cross-section of the hipper-ellipsoid in 4D phase spase, corresponding to the usual space, with respect to the axes of the coordinate plane (x,y). The angle α corresponds to the angle between the axes (x,y) and the main axes of the elliptical cross-section of the hipper-ellipsoid in 4D phase space, corresponding to the velocity space.

Using the invariants, let us write the distribution function as

$$
f = \kappa \delta(I_{xy} + I_z^{(1)2} - 1)\delta(I_z^{(2)2}), \qquad (9)
$$

where $I_z^{(1)}$ and $I_z^{(2)}$ are the linear invariants, corresponding to both independent solutions of the equation (7).

The direct calculation of the density with the distribution function (9) confirms the self-consistency of the model.

The equations (8) should be solved numerically. Some results of such calculations by means of the Runge-Kutta method of the 4th order are shown in Figure 1 and Figure 2.

In Figure 1 the effect of the emmitance transfer is shown dependent on the cyclotron frequency. Figure 2 illustrates the time-dependent behavior of the specific angles of the bunch cross-section in the coordinate space (θ) and in the velocity space (α) with respect to the initial position of the main axes of the phase ellipses.

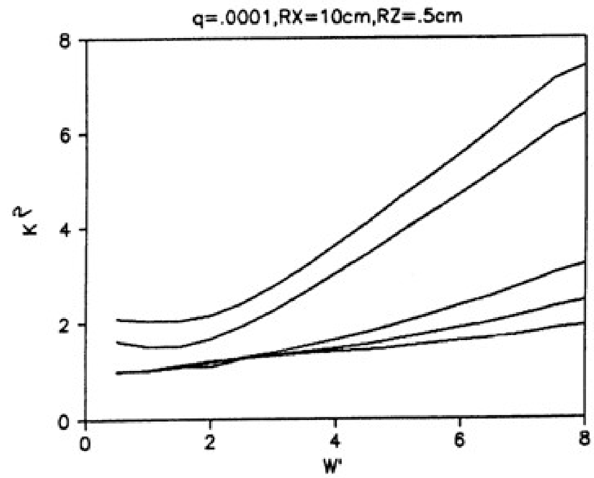

Figure 1: The dependence of the coefficient of the emittance transfer on the gradient of the cyclotron frequency.

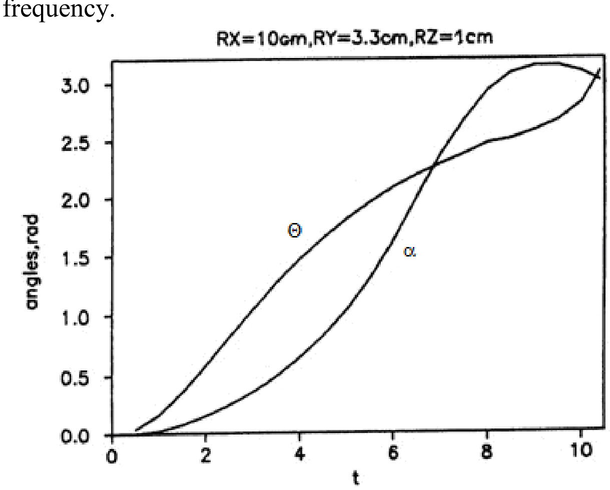

Figure 2: Time-dependence of the turn angles of the phase ellipses with respect to their initial position.

CONCLUSIONS

The self-consistent models are the basis of the proposed analytical approach to study and develop the achromatic structures involving the dipole and quadrupole magnets. The approach allows to determine the general physical factors which affect the properties of the achromatic structure, particularly its possibility to change all the beam phase characteristics, both desired and undesired. The preliminary study shows the significant influence of the initial beam phase characteristics on the phase portrait transformation during the beam motion in the dipole and quadrupole magnets. The dependence of the emittance transfer on the quadrupole field gradient, the affect of the initial beam angular momentum and other peculiarities of the task geometry on the phase portrait behavior are found.

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