

# DEVELOPMENT OF PHYSICS MODELS OF THE ISIS HEAD-TAIL INSTABILITY

R.E. Williamson, B. Jones, C.M. Warsop, ISIS, Rutherford Appleton Laboratory, STFC, UK

## Abstract

ISIS is the pulsed spallation neutron and muon source at the Rutherford Appleton Laboratory in the UK. Operation centres on a rapid cycling proton synchrotron which accelerates  $3 \times 10^{13}$  protons per pulse from 70 MeV to 800 MeV at 50 Hz, delivering a mean beam power of 0.2 MW.

As a high intensity, loss-limited machine, research and development at ISIS is focused on understanding loss mechanisms with a view to improving operational performance and guiding possible upgrade routes. The head-tail instability observed on ISIS is of particular interest as it is currently a main limitation on beam intensity.

Good models of impedance are essential for understanding instabilities and to this end, recent beam-based measurements of the effective transverse impedance of the ISIS synchrotron are presented. This paper also presents developments of a new, in-house code to simulate the head-tail instability and includes benchmarks against theory and comparisons with experimental results.

## INTRODUCTION

The transverse head-tail instability is a main concern for high intensity operation in many hadron synchrotrons including ISIS and its proposed upgrades. The instability imposes a limit on beam intensity through associated beam loss and the subsequent undesired machine activation. However classical theories, such as the model of Sacherer [1], do not include space charge and associated tune spreads which are required for accurately modelling high intensity beams.

Recent work [2 – 4] has put forward limited theoretical models to treat head-tail motion in the presence of space charge. However, currently there is no comprehensive model of head-tail with space charge. Testing of these models against observations is required to ascertain fully their usefulness and limits. As such, numerical simulations have been employed [5] to analyse collective effects and link experimental results to theory. Ultimately, understanding head-tail in high intensity beams may allow improved operations avoiding the instability, with lower beam losses and the possibility of higher beam intensities.

### *The ISIS Synchrotron*

ISIS operation centres on a rapid cycling synchrotron (RCS) with a 163 m circumference composed of 10 superperiods. It accelerates  $3 \times 10^{13}$  protons per pulse (ppp) from 70 – 800 MeV on the 10 ms rising edge of a sinusoidal main magnet field. The repetition rate of 50 Hz results in an average beam power on target of 0.2 MW.

Injection is via charge exchange of a 70 MeV, 25 mA  $H^-$  beam over  $\sim 130$  turns with painting over both transverse acceptances, collimated at  $300 \pi$  mm mrad. The unchopped, injected beam is non-adiabatically bunched and accelerated by the ring dual harmonic RF system ( $h = 2$  and 4). Nominal betatron tunes are  $(Q_x, Q_y) = (4.31, 3.83)$  with peak incoherent tune shifts exceeding  $\sim -0.5$ . The beam intensity is loss limited with the main driving mechanisms being foil losses, longitudinal trapping, transverse space charge and the head-tail instability [6].

Measurements on ISIS have consistently shown that the two proton bunches exhibit vertical head-tail motion over 1 – 2.5 ms into the 10 ms acceleration cycle [7, 8]. The instability is suppressed by ramping the vertical tune down, away from the integer ( $Q_y = 4$ ) during the time of the instability. However, with rising operating intensities, beam losses associated with head-tail increase and lowering the tune further tends to induce beam loss associated with the half integer resonance [9, 10].

Recent studies have shown that the instability is present with dual harmonic RF acceleration as well as with single harmonic RF [10, 11], and possibly worse with the second harmonic. Work is ongoing to develop a feedback system to damp the instability [12] alongside studies modelling and understanding the instability mechanism.

This study presents initial developments in building impedance and instability simulation models of the ISIS synchrotron. Beam-based measurements of the effective transverse impedance are presented. These allow for a better understanding of the driving force behind the head-tail instability.

A new in-house macro-particle simulation code, currently in development to simulate the head-tail instability as observed on ISIS, is also introduced. Convergence tests of the code are presented together with benchmarks against theory. Simulations are compared to experimental data from ISIS with single harmonic RF acceleration. Plans for future experimental studies, simulation and theory work are outlined.

### *Head-Tail Observations*

ISIS operates at the natural machine chromaticities ( $\xi_x = \xi_y = -1.4$  [13]), without sextupole correction. As mentioned above, the impedance acting on the beam leads to a coherent vertical instability early in the acceleration cycle. Measurements have been made using a vertical beam position monitor (BPM) over 0 – 5 ms during acceleration in the case of lower intensity beams ( $5 \times 10^{12}$  ppp) and single harmonic RF. Measurements at this intensity minimise the effect of space charge and allow direct comparison with Sacherer theory. Further measurements at high intensity and with dual harmonic

RF were also taken [10, 11] but are not discussed in this paper.

Figure 1 shows a typical vertical BPM sum and difference signal over several turns during the instability, with single harmonic RF, indicating clear head-tail motion with mode  $m = 1$ . Using the vertical tune at 1 ms ( $Q_y = 3.83$ ), and the chromaticity given above, the accumulated phase shift  $\chi = 11.25$  rad. From Sacherer's theory the growth rate of each head-tail mode may be calculated. However, as found in previous studies [7, 8], this value of  $\chi$  results in a higher growth rate for head-tail mode  $m = 2$  than  $m = 1$ . A modified theory that could explain this is given in the same reference.

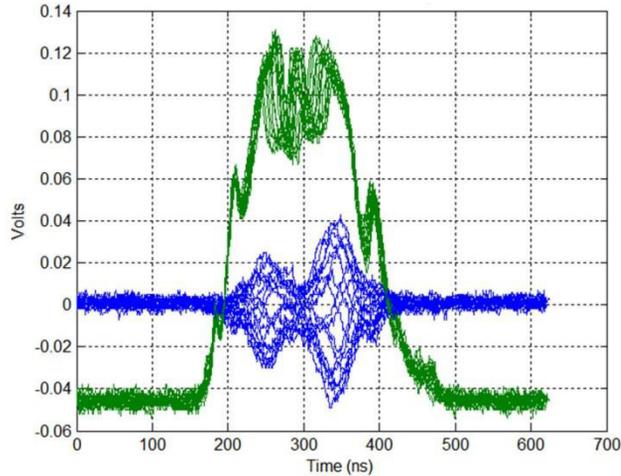


Figure 1: Sum (green) and difference (blue) vertical BPM signals over several turns around 1 ms through the acceleration cycle.

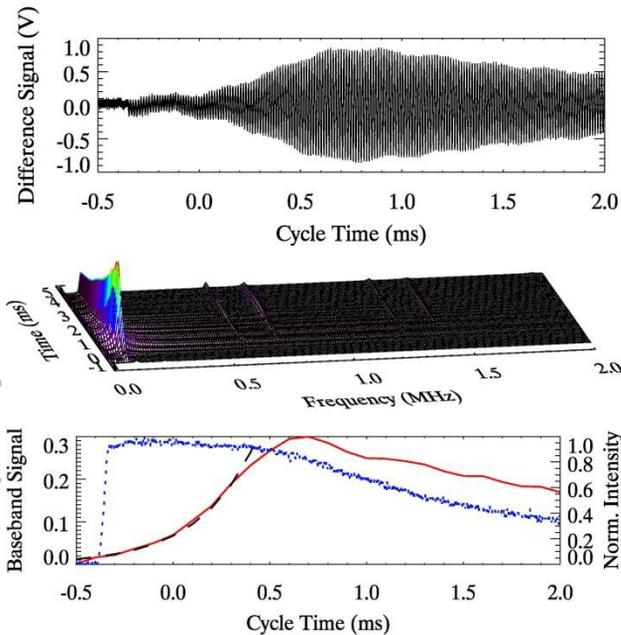


Figure 2: Vertical BPM difference signal, top; its Fourier transform as a function of time, middle; baseband (red), exponential fit (dashed black) and beam intensity (dotted blue) as a function of time, bottom. Peak intensity =  $4.6 \times 10^{12}$  ppp.

ISBN 978-3-95450-178-6

## BEAM-BASED IMPEDANCE MEASUREMENTS

Operating with the RF off and a DC main magnet field, observations of steady-state coasting beams provide important measurements related to instabilities and the effect of space charge. Central to these is an understanding of the impedance of the machine and the identification of their source(s).

Operating at nominal  $Q$  values, beam loss associated with vertical beam growth is observed at intensities  $> 3 \times 10^{12}$  ppp. Measurements of the vertical growth rate of this coasting beam instability can be made using BPMs [14]. An example, shown in figure 2, clearly exhibits strong coherent motion associated with the lowest betatron sideband frequency (baseband),  $(Q_y - 4)\omega_0 = q\omega_0$ . Growth times increase rapidly with intensity and as  $Q_y$  approaches 4.

### Growth Rate Versus Intensity

Coasting beam theory [1] predicts the transverse frequency shift due to an applied impedance  $Z_{\perp}$  to be,

$$\Delta\omega = i \frac{ec}{4\pi Q\gamma E_0} Z_{\perp} I. \quad (1)$$

Assuming a time dependence,  $e^{i\omega t}$ , the associated growth rate is  $\tau^{-1} = -\text{Im}(\Delta\omega)$  so instability corresponds to a real negative impedance. Equation 1 also implies that for a given tune ( $Q$ ) and beam energy ( $\gamma E_0$ ) the growth rate is linear with intensity ( $I$ ).

An example measurement of vertical growth rate as a function of intensity, for  $Q_y = 3.83$  is shown in figure 3, where the growth rate is given by an exponential fit to the baseband signal. Error bars on growth rate are one sigma uncertainty estimates from the exponential fit. The linear fit of growth rate against intensity may be compared to equation 1 to find an effective impedance for the frequency of that sideband. Similar results are found at different tune values.

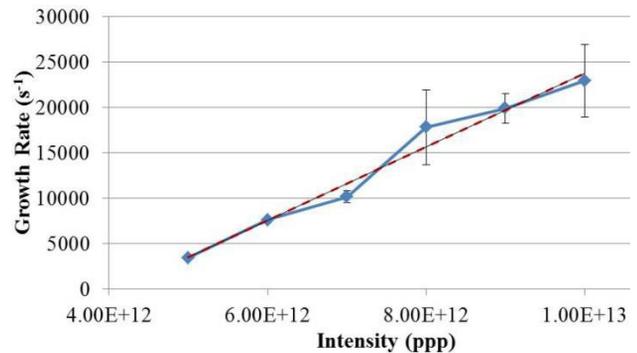


Figure 3: Growth rate versus beam intensity ( $Q_y = 3.83$ ). Effective impedance at this tune is  $5.76 \pm 0.5$  M $\Omega$ /m.

### Growth Rate Versus Tune

Changing the vertical tune moves the baseband frequency  $(Q_y - 4)\omega_0$  and, as such, probes the

impedance at a different frequency. It also alters the growth rate directly, as seen in equation 1. Figure 4 shows the growth rate as a function of vertical tune at a fixed beam intensity. The growth rate is measured as in the previous section. The error on measured  $Q_y$  ( $\pm 0.0022$ ) stems from the resolution of the Fourier transform.

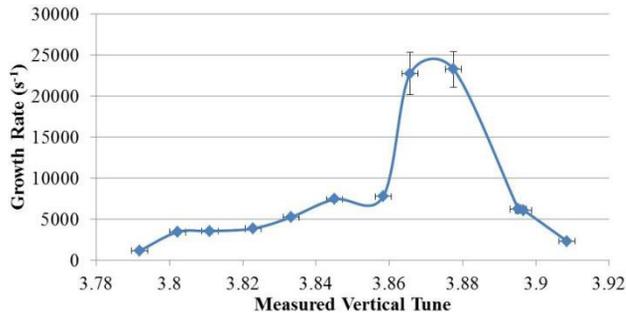


Figure 4: Growth rate versus measured vertical tune. Intensity =  $1 \times 10^{13}$  ppp.

With this data and equation 1 the effective impedance can be calculated as a function of frequency (measured baseband frequency), figure 5. The error on the measured effective impedance is combined from the exponential fit required for each data point and the error on the measured betatron tune. It is clear from figure 5 that there is a sharp narrowband impedance around 85 kHz with a full width of  $\sim 25$  kHz, peaking at  $3.71 \pm 0.35$  M $\Omega$ /m. The position of the narrowband impedance, its width and height, are consistent between measurement campaigns.

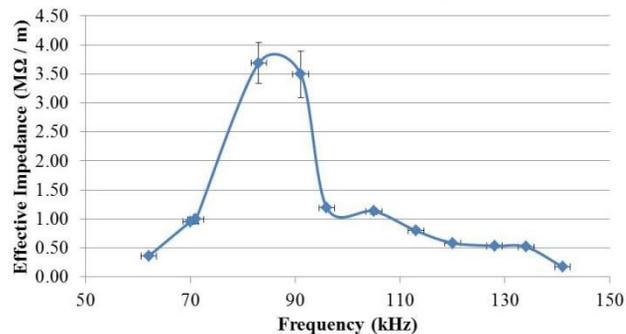


Figure 5: Effective impedance versus baseband frequency. Intensity =  $1 \times 10^{13}$  ppp.

### Measurements Summary

Measurements of coasting beam growth rates have been made and compare favourably with the theoretical model of Sacherer with growth rates increasing linearly with intensity. Data were also taken at different painted beam emittances with similar results as a function of intensity.

Experimental observations show that head-tail growth rates increase rapidly as  $Q_y$  increases toward 4 which has led to the resistive wall being cited as the driving impedance [7, 8, 11]. However, crude calculations of the resistive wall impedance, from the thick-wall formula, result in lower growth rates than those observed.

It is clear from the growth rate as a function of tune, and the interpreted effective impedance versus frequency,

that there is a vertical narrowband impedance at  $\sim 85$  kHz. This could be the driver for head-tail motion seen in ISIS RCS operation. The origin of the narrowband impedance, and its relevance to head-tail is currently under investigation.

## NEW SIMULATION MODEL

A new, stand-alone macro-particle simulation code has been written to study head-tail behaviour on ISIS. The code is based on an existing in-house longitudinal code [15] with the addition of a simple smooth focusing model for transverse motion and wakefield kicks to simulate the interaction between the beam and its environment.

### Wakefield Model

Wakefield kicks are implemented in the code on a turn-by-turn basis by default, but can be more frequent. As with similar codes [16, 17] the beam is segmented longitudinally and the wake calculated at each slice due to the effect of each upstream slice. This may be from particles within the same bunch or from preceding bunches.

Most theoretical wakefield models assume ultra-relativistic beams ( $\beta \sim 1$ ). This means that, in order not to violate the principle of causality, wakes can only propagate backwards and influence subsequent particles. However, head-tail behaviour on ISIS is observed at  $\beta$  values much lower than this ( $\sim 0.4$ ). The importance of this in instability modelling is not fully understood.

For initial convergence tests and benchmarks ISIS parameters have been assumed. A thick resistive wall wakefield has been modelled with the wake function [5],

$$W_{RW}(z) = -\frac{cL_{RW}}{b^3} \left(\frac{\beta}{\pi}\right)^{3/2} \sqrt{\frac{Z_0}{\sigma z}} \quad (2)$$

where  $L_{RW}$  is the length of the resistive wall,  $b$  is the beam pipe radius,  $\beta = v/c$  the relativistic parameter,  $Z_0$  is the impedance of free space and  $\sigma$  is the beam pipe conductivity. As noted above the  $\beta$  dependence is important in the case of ISIS beams.

### Convergence Tests

To ascertain whether the particle tracking code is accurately modelling the relevant physics a number of convergence tests have been performed. In these tests simulations were run without RF, with a DC main magnet field, resistive wall wake and a high beam pipe conductivity of  $100 \Omega^{-1}\text{m}^{-1}$ . A constant beam pipe radius was assumed with uniform impedance around the ring.

Investigations were made of the number of simulated turns, number of longitudinal slices, number of macro-particles, random number seed and the number of time points for the growth rate fit. Convergence was considered attained when the change in growth rate between parameters was smaller than the error on the calculated growth rate,  $\sim 1\%$  error. The error on the

growth rate arises from the finite length of simulation, the number of macro-particles and longitudinal slices. No space charge was included in these tests.

### Coasting Beam Benchmarks

With the converged simulation parameters a number of benchmarks were performed to evaluate the code against theory. Similar to the beam-based impedance measurements, equation 1 was tested by varying the beam intensity and tune. As in the experimental case, the growth rate was calculated from an exponential fit to the lowest betatron sideband from the Fourier transform of a simulated vertical BPM difference signal.

To simulate a BPM the average transverse displacement ( $\Delta y_i$ ) and the macro-particle population ( $I_i$ ) was calculated for each longitudinal slice ( $i$ ) and for each simulated turn of the machine. The BPM difference signal was then interpreted as the product of these ( $\Delta y_i I_i$ ) i.e. the first moment of the beam.

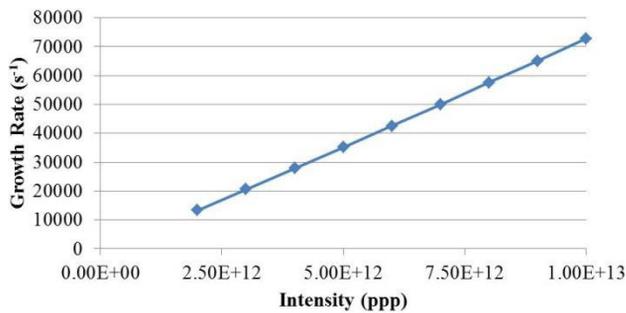


Figure 6: Simulated growth rate versus beam intensity ( $Q_y = 3.83$ ,  $\sigma = 100 \Omega^{-1} \text{m}^{-1}$ ).

Figure 6 shows the simulated growth rate as a function of beam intensity whilst keeping all other parameters constant. As previously, error bars on growth rate are one sigma uncertainty estimates from the exponential fit to the baseband signal. As expected from equation 1 the growth rate is linear with intensity and agrees reasonably well with growth rate estimates from theory.

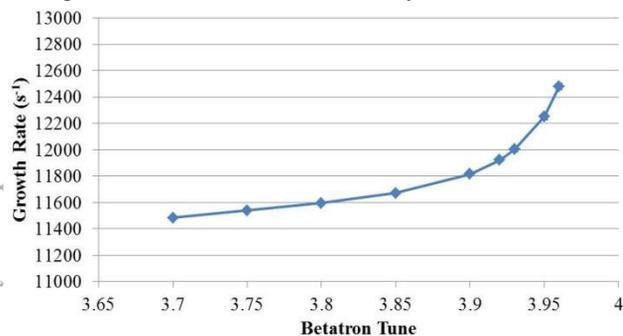


Figure 7: Simulated growth rate versus vertical tune. Intensity =  $2 \times 10^{12}$  ppp.

Figure 7 shows the simulated growth rate as a function of tune. As in the beam-based impedance measurements, equation 1 can be used to obtain the effective impedance as a function of frequency, figure 8. As expected, the

simulated effective impedance has a similar functional form to the thick resistive wall model ( $Z_{RW} \propto \omega^{-1/2}$ ).

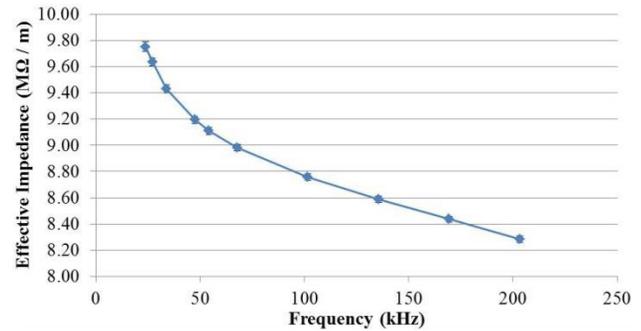


Figure 8: Simulated effective impedance versus baseband frequency. Intensity =  $2 \times 10^{12}$  ppp.

### ISIS Simulation

The thick resistive wall approximation underestimates the measured impedance at the lowest betatron sideband. While research is still ongoing into the beam-based impedance measurements, for these initial simulations the beam pipe conductivity in the resistive wall impedance model has been modified to match artificially the measured impedance at the dominant, lowest sideband.

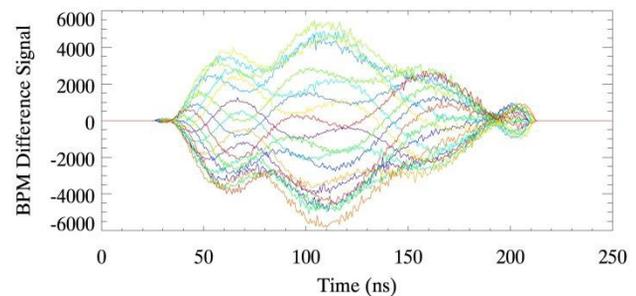


Figure 9: Simulated vertical BPM difference signal over 20 turns of the simulation around 1 ms with nominal tune.

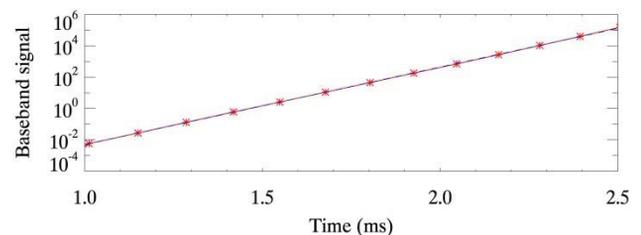


Figure 10: Simulated baseband as a function of time (red) on a logarithmic scale, exponential fit (blue dash).

Figures 9 and 10 show the simulated vertical BPM difference signal over several turns and time dependence of the baseband. The mode structure shows a mixture of  $m = 2$  and  $m = 3$  which is consistent with theory given the chromatic phase shift along the bunch. However, as with previous simulations [8], this does not match experimental observations (figure 1) where an  $m = 1$  mode is persistent over many hundreds of turns. The simulated growth rate is much faster than theory predicts ( $11,400 \text{ s}^{-1}$  compared to  $446 \text{ s}^{-1}$ ) and much closer to observed growth times of order  $10,000 \text{ s}^{-1}$ .

## SUMMARY

The head-tail instability has been identified as a key intensity limit for operation of the ISIS synchrotron. Work building impedance and instability simulation models of the synchrotron has been presented together with initial results simulating head-tail behaviour on ISIS. Combined with these studies work is underway on developing a prototype beam feedback system to damp instabilities [6, 12], due for installation in 2017.

Beam-based measurements have been made of the effective transverse impedance of the ISIS synchrotron using a coasting beam at 70 MeV (ISIS injection energy). Results indicate the presence of a narrowband impedance, the source of which is currently being investigated alongside whether it is the driver for the observed head-tail instability.

An in-house macro-particle simulation code is under development to aid understanding of the head-tail instability as observed at ISIS. Convergence tests have been performed with the code for coasting beams and zero space charge. Coasting beam code benchmarks against established theory [1] have been met with initial results looking promising. As with previous studies [7, 8] preliminary simulations of ISIS with a thick resistive wall impedance predict a head-tail mode  $m = 2$  whereas a persistent  $m = 1$  mode is observed in operations.

## FUTURE WORK

Experiments are planned to establish the source of the narrowband impedance observed in the beam-based impedance measurements. Investigations are focused on the beam extraction kicker magnets and the vertical betatron exciter as these are expected to interact strongly with the vertical motion of the beam. Tests will also be performed on whether the narrowband impedance is the driver of the observed head-tail motion early in the ISIS acceleration cycle. More precise knowledge of head-tail, the current machine impedance model and its effect on the beam will inform future machine upgrades.

Developments of the in-house simulation code are planned to benchmark fully the results against Sacherer theory. Simulation results of the ISIS synchrotron will then be compared to theory and experiment for resistive wall impedance only, and the measured impedance model as it develops. It is planned to analyse the experimental and simulation data with reference to the theory of Rees [7].

Once the observations of low intensity, single harmonic RF head-tail motion at ISIS are understood studies will progress to high intensity beams. Previous experiments have shown space charge has a strong effect on the head-tail instability [11]. Simulations at high intensity will require the additional modelling for transverse space charge. Finally, work will move to investigating head-tail motion with dual harmonic RF where there is currently no theoretical model to describe observations.

## REFERENCES

- [1] F. Sacherer, “Transverse bunched beam instabilities”, Proc. First Course of the International School of Particle Accelerators of the ‘Ettore Majorana’ Centre for Scientific Culture, Rep. CERN 77-13, Jul. 1977, pp. 198 – 218.
- [2] M. Blaskiewicz, “Fast head-tail instability with space charge”, *Phys. Rev. ST Accel. Beams*, vol. 1, p. 044201, Aug. 1998.
- [3] A. Burov, “Head-tail modes for strong space charge”, *Phys. Rev. ST Accel. Beams*, vol. 12, p. 044202, Apr. 2009.
- [4] V. Balbekov, “Transverse modes of a bunched beam with space charge dominated impedance”, *Phys. Rev. ST Accel. Beams*, vol. 12, p. 124402, Dec. 2009.
- [5] V. Kornilov, O. Boine-Frankenheim, “Head-tail instability and Landau damping in bunches with space charge”, *Phys. Rev. ST Accel. Beams*, vol. 13, p. 114201, Nov. 2010.
- [6] D.J. Adams *et al.*, “Operational experience and future plans at ISIS”, presented at HB’16, Malmö, Sweden, Jul. 2016, paper TUPM3Y01.
- [7] G.H. Rees, “Interpretation of the higher mode, head tail motion observed on ISIS”, *Particle Accelerators*, Vol. 39, pp. 159 – 167, May 1992.
- [8] R.E. Williamson, D.J. Adams, B. Jones, V. Kornilov, and C.M. Warsop, “Simulation of the head-tail instability on the ISIS Synchrotron”, in *Proc. HB’14*, East Lansing, MI, USA, Nov. 2014, pp. 113 – 117.
- [9] C.M. Warsop, D.J. Adams, B. Jones, and B.G. Pine, “Simple models for beam loss near the half integer resonance with space charge”, presented at HB’16, Malmö, Sweden, Jul. 2016, paper MOPR030.
- [10] C.M. Warsop *et al.*, “High intensity loss mechanisms on the ISIS rapid cycling synchrotron”, *Proc. HB’14*, East Lansing, MI, USA, Nov. 2014, pp. 203 – 207.
- [11] V. Kornilov *et al.*, “Thresholds of the head-tail instability in bunches with space charge”, in *Proc. HB’14*, East Lansing, MI, USA, Nov. 2014, pp. 240 – 244.
- [12] S.J. Payne, “Strip line monitor design for the ISIS proton synchrotron using the FEA program HFSS”, in *Proc. IBIC’13*, Oxford, UK, Sep. 2013, pp. 435 – 437.
- [13] C.M. Warsop, “Special diagnostic methods and beam loss control on high intensity proton synchrotrons and storage rings”, Ph.D. thesis, Phys. Dept., Sheffield University, UK, Jan. 2002, pp. 86 – 90.
- [14] C.M. Warsop *et al.*, “High intensity studies on the ISIS synchrotron, including key factors for upgrades and the effects of half integer resonance”, in *Proc. HB’10*, Morschach, Switzerland, Sep. 2010, pp. 619 – 623.
- [15] R.E. Williamson, D.J. Adams, and C.M. Warsop, “High intensity longitudinal dynamics studies for an ISIS injection upgrade”, in *Proc. HB’12*, Beijing, China, Sep. 2012, pp. 492 – 496.
- [16] G. Rumolo, F. Zimmermann, “Electron cloud simulations: beam instabilities and wakefields”, *Phys. Rev. ST Accel. Beams*, vol. 5, p. 121002, Dec. 2010.
- [17] M. Blaskiewicz, “The TRANFT User’s Manual”, BNL, Upton, NY, USA, Rep. BNL-77074-2006-IR, Aug. 2006.