USE OF RF QUADRUPOLE STRUCTURES TO ENHANCE STABILITY IN ACCELERATOR RINGS

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Abstract

The beams required for the high luminosity upgrade of the Large Hadron Collider (HL-LHC) at CERN call for efficient mechanisms to suppress transverse collective instabilities. In addition to octupole magnets installed for the purpose of Landau damping, we propose to use radio frequency (rf) quadrupole structures to considerably enhance the aforementioned stabilising effect. By means of the PyHEADTAIL macroparticle tracking code, the stabilising mechanism introduced by an rf quadrupole is studied and discussed. As a specific example, the performance of an rf quadrupole system in presence of magnetic octupoles is demonstrated for HL-LHC. Furthermore, potential performance limitations such as the excitation of synchro-betatron resonances are pointed out. Finally, efforts towards possible measurements with the CERN Super Proton Synchrotron (SPS) are discussed aiming at studying the underlying stabilising mechanisms experimentally.

INTRODUCTION

To push the limits of the Large Hadron Collider (LHC) [1] and its future upgrade HL-LHC [2] towards higher luminosities, the machine must be able to handle beams with significantly increased brightnesses and intensities. Amongst others, these parameters are limited by the presence of transverse collective instabilities induced by the transverse beam coupling impedance of the accelerator structure. A successful stabilising mechanism for the so-called slow head-tail instabilities is the effect of Landau damping [3]. It is present when there is a spread in the betatron frequency, or tune, of the particles in the beam as discussed in detail e.g. in [4] and references therein. This so-called incoherent tune spread is a result of non-linearities in the machine. Partially, they are of parasitical nature, i.e. originating from non-linear space-charge forces, non-linearities in the magnetic focusing systems, beam-beam interactions at collision, etc. In addition, they are often introduced by design through dedicated non-linear elements for better control and efficiency. Magnetic octupoles are commonly used for the latter purpose. They change the betatron tunes of a particle depending on its transverse actions. In LHC for instance, families of 84 focusing and 84 defocusing, 0.32 m long superconducting magnetic octupoles are installed and successfully used to suppress a variety of instabilities [5,6]. Nevertheless, LHC operation in 2012 at an energy of 4 TeV and a bunch spacing of 50 ns, as well as in 2015 at 6.5 TeV and 25 ns has shown that these Landau octupoles (LO) need to be powered close to their maximum strength in order to guarantee stable beams [7]. With the HL-LHC upgrade, the bunch intensity will be increased from $1.15 \cdot 10^{11}$ p$^+$/b to $2.2 \cdot 10^{11}$ p$^+$/b which will potentially lead to more violent instabilities. At the same time, the normalised transverse emittances will be reduced from 3 μm (LHC operational) to 2.5 μm, and the beam energy will be increased from 6.5 TeV to 7 TeV, thus rendering the LO less effective.

An rf quadrupole operating in a transverse magnetic quadrupolar mode TM_{210} was proposed as a more efficient device for introducing a non-vanishing betatron tune spread in the beams of a high energy particle accelerator like (HL-)LHC [8]. A first numerical proof-of-principle study was presented in [9]. The quadrupolar focusing strength of such a device has a harmonic dependence on the longitudinal position of the particles in the bunch. As a result, the betatron tunes of a particle are changed as a function of its longitudinal position. Since in high energy hadron colliders, the longitudinal emittance is much larger than the transverse ones, the rf quadrupole is able to generate a betatron tune spread more efficiently than magnetic octupoles.

The publication is structured as follows. Firstly, the working principle of an rf quadrupole as well as the PyHEADTAIL macroparticle tracking code are introduced. Thereafter, the applicability of an rf quadrupole for HL-LHC as well as the excitation of synchro-betatron resonances are discussed. Finally, experimental studies of the concerned stabilising mechanism are proposed for the SPS.

THEORY

Radio Frequency Quadrupole

A bunch of electrically charged particles of momentum p traverses an rf quadrupole along the z-axis. Given that the thin-lens approximation holds, a particle i located at position z_i measured with respect to the zero crossing of the main rf wave ($z = 0$) experiences both transverse and longitudinal kicks

$$\Delta \mathbf{p}_\perp^i = p k_2 \left(y_i \mathbf{u}_y - x_i \mathbf{u}_x \right) \cos \left[\frac{\omega z_i}{\beta c} + \varphi_0 \right], \tag{1}$$

$$\Delta p_\parallel^i = \frac{\omega p k_2}{2 \beta c} \left(x_i^2 - y_i^2 \right) \sin \left[\frac{\omega z_i}{\beta c} + \varphi_0 \right]. \tag{2}$$

x_i and y_i are the transverse coordinates of the particle, ω denotes the rf quadrupole angular frequency, $\mathbf{u}_{x,y}$ are the unit vectors along the x and y coordinates respectively, and φ_0 is a constant phase offset of the rf quadrupole wave with respect to $z = 0$. It has been added for reasons of generality. β and c denote the relativistic beta and the speed of light respectively. k_2 is the amplitude of the normalised integrated

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quadrupolar strength of the cavity

$$k_2 = \frac{q}{\pi r p c} \int_0^{2\pi} \left\| \int_0^L \left(E_x - c B_y \right) e^{j\omega z/\beta c} dz \right\| \cos\varphi \, d\varphi, \tag{3}$$

with q the electric charge of the particle, L the length of the cavity and $[r, \varphi, z]$ the cylindrical coordinates. E_x and B_y denote respectively the horizontal component of the electric and the vertical component of the magnetic field vectors. Throughout this article, k_2 is expressed in magnetic units [Tm/m] using the conversion $b^{(2)} = B_0 \rho k_2$, where the product $B_0\rho$ denotes the magnetic rigidity of the particle beam.

A particle that is subject to the transverse quadrupole kicks given in Eq. 1 experiences a change of its betatron tunes $Q^i_{x,y}$ [10]

$$\Delta Q^i_{x,y} = \pm \beta_{x,y} \frac{b^{(2)}}{4\pi B_0 \rho} \cos\left[\frac{\omega z_i}{\beta c} + \varphi_0 \right], \tag{4}$$

where $\beta_{x,y}$ are the transverse beta functions of the accelerator at the location of the kicks. To obtain an incoherent betatron tune spread that is as large as possible and does not average out over one synchrotron period, the rf quadrupole is operated with a phase of $\varphi_0 = 0$ or $\varphi_0 = \pi$. Choosing $\varphi_0 = 0$ means that the device is focusing (defocusing) in the horizontal (vertical) plane for particles at $z_i = 0$. The situation is reversed for $\varphi_0 = \pi$. Equivalently, operating the cavity in one of these two modes means that particles located at $z_i = 0$ enter the device (anti-)on-crest of the rf wave. It is worth mentioning that this approach is different from the one described in [11, 12] where an rf quadrupole is proposed to increase the threshold of the transverse mode coupling instability (TMCI), also known as the strong head-tail instability [4]. For the latter, the cavity is operated at zero-crossing of the rf wave for particles at $z_i = 0$.

In the following, $\varphi_0 = 0$. Equation 4 can be expanded into a Taylor series in z_i

$$\Delta Q^i_{x,y} = \pm \beta_{x,y} \frac{b^{(2)}}{4\pi B_0 \rho} \left[1 - \frac{1}{2} \left(\frac{\omega z_i}{\beta c} \right)^2 + O(z_i^3) \right]. \tag{5}$$

Given that the wavelength of the rf wave is much larger than the bunch length σ_z, i.e. $\omega\sigma_z/\beta c \ll 1$, the higher order terms $O(z_i^3)$ can be neglected. Assuming linear synchrotron motion, averaging over one synchrotron period T_s reveals that the rf quadrupole changes the betatron tune as a function of the longitudinal action J^i_z of the particle

$$\left\langle \Delta Q^i_{x,y} \right\rangle_{T_s} \approx \pm \beta_{x,y} \frac{b^{(2)}}{4\pi B_0 \rho} \left[1 - \frac{1}{2} \left(\frac{\omega}{\beta c} \right)^2 \beta_z J^i_z \right], \tag{6}$$

where β_z is the longitudinal beta function.

Equation 5 shows the z_i^2 dependence of the betatron tune change caused by an rf quadrupole (first order). For comparison, the betatron tune change introduced by a non-zero second order chromaticity $Q''_{x,y}$ is given by

$$\Delta Q^i_{x,y} = \frac{Q''_{x,y}}{2} \delta_i^2, \tag{7}$$

where δ_i is the relative momentum deviation $\Delta p_i/p$ of the particle. Hence, similarly to the rf quadrupole, the average betatron tune change over one synchrotron period is dependent on the longitudinal action J^i_z of the particle. In the following this is referred to as betatron detuning with *longitudinal* amplitude which stands in contrast to betatron detuning with *transverse* amplitude introduced for instance by magnetic octupoles.

PyHEADTAIL

PyHEADTAIL is a macroparticle tracking software under development at CERN [13]. It is the successor of the well-established HEADTAIL code [14] and allows to study the formation of collective instabilities in circular accelerators and to evaluate appropriate methods for their mitigation. The model assumes linear periodic transport in the transverse planes from one section to the next along the accelerator ring. In the longitudinal plane, the code supports linear synchrotron motion or multi-harmonic rf systems. All the impedance contributions of the accelerator structure are lumped in one point where the wake kicks are computed and applied on a longitudinal slice-by-slice basis. The effect of chromaticity, including higher orders, as well as detuning with transverse amplitude originating from magnetic octupoles are modelled as a change in the individual phase advance of every macroparticle in the bunch (incoherent detuning). The rf quadrupole effect, on the other hand, is modelled as localised kicks computed according to Eq. 1 and 2. PyHEADTAIL has been successfully benchmarked against LHC measurements for a head-tail instability that can be suppressed by LO [9].

APPLICABILITY FOR HL-LHC

PyHEADTAIL is used to assess the potential gain of an rf quadrupole for HL-LHC. The goal is to evaluate the performance of an ensemble of 800 MHz superconducting rf quadrupole cavities [8] in presence of the LHC LO. The beta functions at the location of the rf quadrupole kicks are set to a conservative $\beta_{x,y} = 200$ m according to [8]. To begin with, only single-bunch instabilities are studied for operational flat-top beam and machine parameters [2], the major

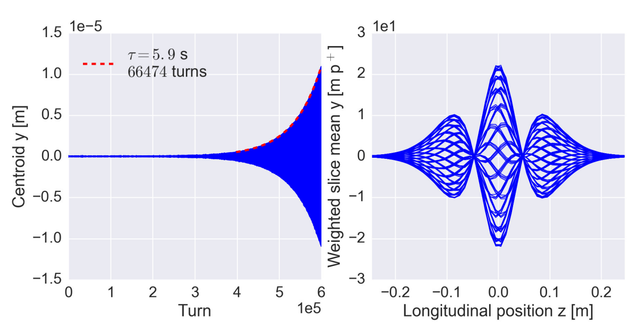

Figure 1: PyHEADTAIL output for HL-LHC at a chromaticity of $Q'_{x,y} = 10$. *Left:* Vertical bunch centroid motion with an exponential fit (red line) and the corresponding instability rise time τ. *Right:* Acquired bunch position monitor data.

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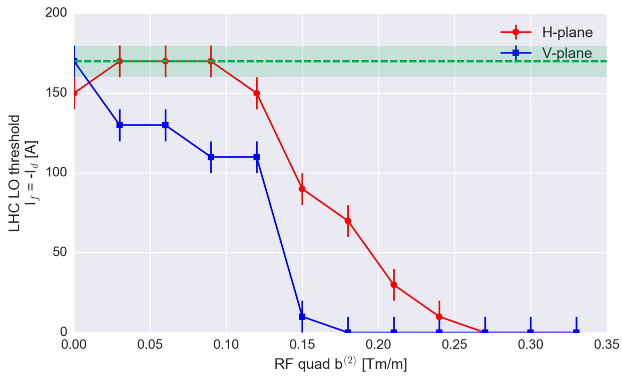

Figure 2: LO current required for HL-LHC operation at a chromaticity of $Q'_{x,y} = 10$ vs. rf quadrupole strength. The green dashed line marks the stabilising current when using LO alone.

ones being summarised in Tab. 1. A first order chromaticity of $Q'_{x,y} = 10$ is chosen for the study as it is a likely working point judging from past LHC operation. Furthermore, an ideal bunch-by-bunch transverse feedback system with a damping time of 50 turns is included in the machine setup. Throughout the study, the latest HL-LHC dipolar impedance model is used.

In a first step, the instability expected for the given machine configuration is characterised. The simulation setup is hence chosen such that both the rf quadrupole and the LO are switched off. The results are summarised in Fig. 1. The vertical centroid position acquired over $6 \cdot 10^5$ turns is shown on the left and the bunch position monitor data is given on the right. The latter is an overlay of 200 consecutive traces and clearly illustrates the head-tail pattern. After carrying out a frequency analysis on the centroid motion, the instability can be characterised as a head-tail mode with azimuthal and radial mode numbers of 0 and 2 respectively. This result is consistent with experimental observations made in the LHC at 6.5 TeV [5].

PyHEADTAIL indicates that beam stabilisation with LO alone is achieved when they are powered at focusing and defocusing currents of $I_f = -I_d = 170 \pm 10$ A respectively. Figure 2 summarises the effect of an rf quadrupole system on the required stabilising current in the LO for the instability shown in Fig. 1. The green dashed line represents the required LO current in absence of an rf quadrupole. The gain in margin for stabilisation with the additional rf quadrupole system is significant in both planes. Although there is only a small effect on the threshold current $I_{f,d}$ up to a cavity strength of about $b^{(2)} = 0.1$ Tm/m, the numerical model suggests that for $b^{(2)} \approx 0.25$ Tm/m the LO would no longer be required to suppress the instability. Given the preliminary cavity design, the latter strength can already be provided by two rf cavities [8]. Further optimisations of the cavity geometry in terms of kick strength and impedance are currently ongoing. Apart from the strong decrease of the required LO current with rf quadrupole strength, the plot illustrates that there is an asymmetry in the behaviour for stabilisation in the two planes. The vertical plane behaves more favourably even if the required stabilising LO current without rf quadrupole is slightly larger than in the horizontal plane (higher vertical impedance). This is a consequence of the asymmetric nature of the incoherent tune spreads (compare Eq. 6) combined with the fact that the rf quadrupole is focusing in one plane and defocusing in the other for a given longitudinal position. Indeed, it has been verified that the asymmetry can be inverted by operating the rf quadrupole with $\varphi_0 = \pi$ instead of $\varphi_0 = 0$ (Eq. 1 and 2). These effects are currently under study and are addressed in more detail below in the section on experimental studies in the SPS.

Table 1: Main Machine and Simulation Parameters used for the HL-LHC Studies with PyHEADTAIL.

Parameter	Symbol	Value
Bunch intensity	N_b	$2.2 \cdot 10^{11}$ p^+/b
Beam energy	E	7 TeV
Chromaticity	$Q'_{x,y}$	10
Transverse normalised emittance	$\epsilon_{x,y}$	2.5 μm
Longitudinal emittance	ϵ_z	2.5 eVs
Number of macroparticles		$8 \cdot 10^5$ p
Number of turns		$6 \cdot 10^5$ turns

SYNCHRO-BETATRON RESONANCES

The numerical simulations that are carried out to define possible scenarios for an rf quadrupole proof-of-principle experiment in the SPS at CERN revealed that this type of cavity can excite synchro-betatron resonances (SBR). Such resonances have also been studied and reported by [12] for an rf quadrupole aiming at increasing the TMCI threshold. Figure 3 shows the growth rates (top) and the fraction of losses

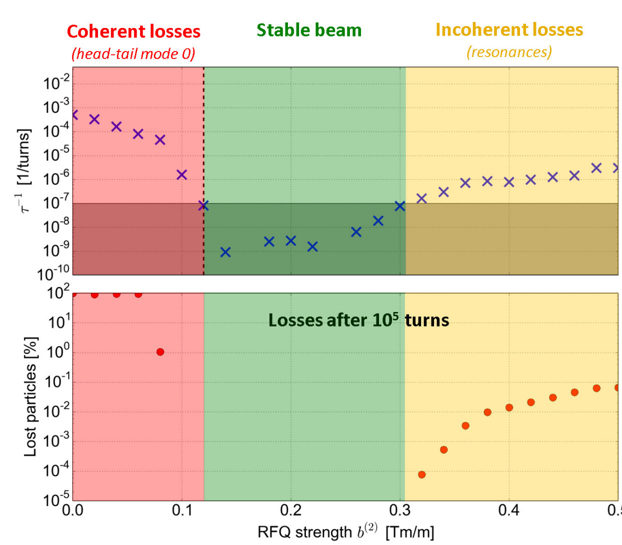

Figure 3: PyHEADTAIL results for a mode 0 head-tail instability in the SPS. *Top:* Instability growth rate vs. rf quadrupole strength. *Bottom:* Fraction of lost particles over 10^5 turns.

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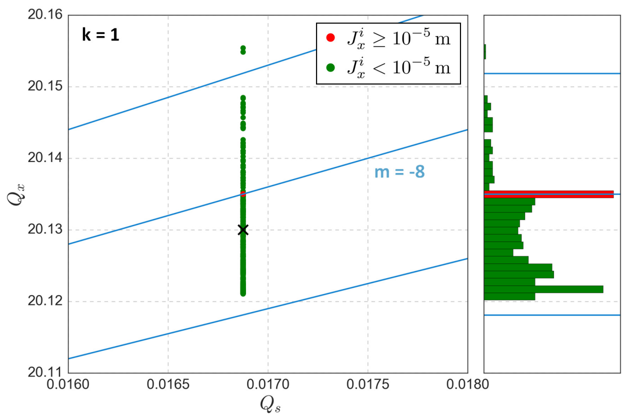

Figure 4: Subset of the tune parameter space (Q_s, Q_x) around the SPS working point (black cross). Shown are the SBR lines (blue) expected from Eq. 8 as well as the incoherent tune spread induced by an rf quadrupole for particles of high (red) and low (green) horizontal action.

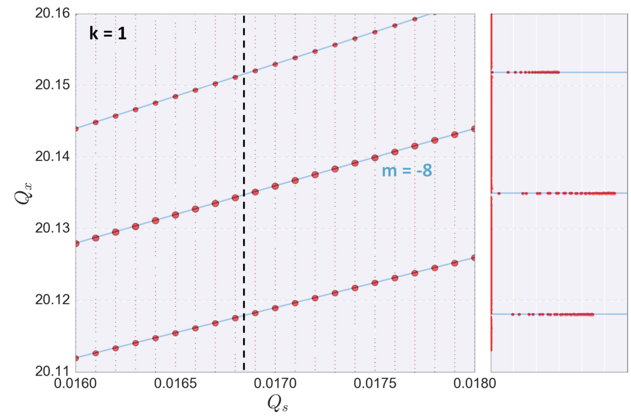

Figure 5: Subset of the tune parameter space (Q_s, Q_x) as in Fig. 4. Shown are tracking results from a 2D scan in (Q_s, Q_x) (red) overlaid with the expected SBR lines (blue). The plot on the right shows a detailed view of the results for a fixed value of Q_s (black dashed line).

over 10^5 turns (bottom) as a function of the rf quadrupole strength $b^{(2)}$ for a vertical mode 0 head-tail instability in the SPS. Three regimes can be distinguished: (i) at low $b^{(2)}$ the coherent instability can still develop and even if the growth rate is reduced almost the whole beam is lost after 10^5 turns (red), (ii) at intermediate $b^{(2)}$ the beam is stabilised and no losses are observed over the given number of turns (green), and (iii) at high $b^{(2)}$ only a small fraction of the beam is lost (yellow). The focus lies on the latter regime where the observed losses can be explained by SBR excited by the rf quadrupole. The condition for SBR for a particle i oscillating at tunes Q_x^i, Q_y^i, and Q_s^i is given by [15]

$$k \cdot Q_x^i + l \cdot Q_y^i + m \cdot Q_s^i = n, \quad k, l, m, n \in \mathbb{Z}. \tag{8}$$

In a first approximation, the rf quadrupole does not couple the horizontal and vertical planes (see Eq. 1 and 2). Hence, without loss of generality one can set $l = 0$ and treat the transverse planes independently. Equation 1 shows that the rf quadrupole gives a horizontal kick dependent on the coordinates $x_i(t)$ and $z_i(t)$, which describe respectively the betatron and synchrotron oscillations in time. Given that the particle tunes Q_x^i and Q_s^i fulfil the resonance condition in Eq. 8, the resulting kick $\Delta p_x^i(t)$ leads to an amplitude growth of the particle motion over time. The same arguments are valid for the vertical plane.

To prove the presence of SBR in a first step, a tracking simulation was performed over 200 synchrotron periods using linear synchrotron motion, a fixed rf quadrupole strength and a set of 1000 macroparticles. Particles with a high horizontal action J_x^i at the end of the simulation were separated from the rest and their individual tunes were analysed by means of the SUSSIX code [16]. The distribution of the tunes is shown in Fig. 4 in a subset of the tune space (Q_s, Q_x) around the SPS working point (black cross). The incoherent tune distribution is introduced by the rf quadrupole. The green dots mark particles maintaining a constant horizontal action, while those marked red exhibit a strong amplitude growth over time. The SBR lines (blue) expected from Eq. 8 are overlaid and show excellent agreement with tracking results. Even though this method is good enough to prove the excitation of SBR through an rf quadrupole, it is very time-consuming due to the SUSSIX tune analysis over the number of particles. In addition, the limited number of macroparticles cannot give a complete picture of all the potential resonance lines. To study and scan the tune space (Q_s, Q_x) more systematically, which is important especially in view of evaluating the presence of SBR for HL-LHC, the tool has been modified such that only one particle is tracked at a time, but for many different working points (Q_s^i, Q_x^i). For every tune setting, the amplitude of the particle motion is analysed and fitted with an exponential function to obtain the growth rate. The results are shown in Fig. 5. Red markers represent tracking results and their size is proportional to the measured amplitude growth rate of the betatron motion. The plot illustrates again the excellent agreement between the tracking code and the analytical formula (blue) and serves as a successful benchmark of the developed tool.

EXPERIMENTAL STUDIES IN THE SPS

As described in the theory section of this article, the betatron detuning introduced by a non-zero second order chromaticity $Q''_{x,y}$ is dependent on the longitudinal amplitude of the particles, similarly to the rf quadrupole (Eq. 6). Hence, studying the effect of a non-zero $Q''_{x,y}$ on beam stability will lead to a better understanding of the underlying mechanisms driven by betatron detuning with longitudinal amplitude. Other than the rf quadrupole, where a prototype must first be designed, built and installed in an accelerator, $Q''_{x,y}$ can in principle be controlled immediately by changing the optics of the machine. It thus makes experimental studies possible on a shorter time scale and allows to validate the numerical model on which the rf quadrupole results are based.

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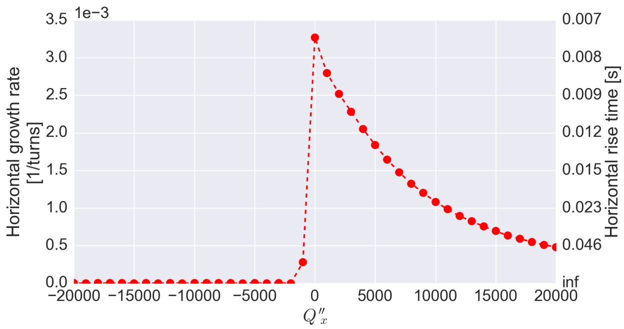

Figure 6: PyHEADTAIL results showing growth rate vs. Q''_x for a mode 0 head-tail instability in the horizontal plane of the SPS.

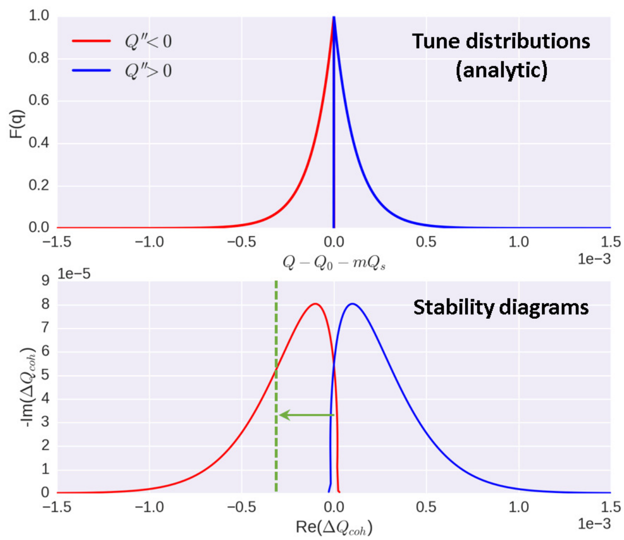

Figure 7: *Top:* Analytical incoherent tune distribution for positive and negative Q'' respectively. *Bottom:* Corresponding stability diagrams. The green dashed line represents $\mathrm{Re}\,(\Delta Q_{coh}) < 0$ caused by a dipolar impedance.

At CERN, the SPS is the most suitable machine for this kind of experiments. At present, there are ongoing studies towards improving the non-linear optics model of the accelerator in order to be able to control the horizontal second order chromaticity Q''_x reliably [17]. The idea is to vary Q''_x by using the magnetic octupoles installed in the high dispersion regions of the accelerator. Powering these elements means, however, that detuning with transverse amplitude is introduced in the machine as well. Naturally, the latter needs to be compensated for to disentangle beam stabilisation through detuning with transverse and longitudinal amplitude respectively. MAD [18] simulations of the SPS optics model show that such a compensation can indeed be achieved by means of the magnetic octupoles located in the low dispersion regions.

In parallel, PyHEADTAIL studies are under way to define possible scenarios for experiments as well as the required beam and machine parameter values. The accurate impedance model of the SPS [19] allows to obtain a clear idea of the expected instability growth rates and the amount of Q''_x required for beam stabilisation. Figure 6 shows an example of the dependence of the growth rate of a horizontal mode 0 head-tail instability in the SPS as obtained from PyHEADTAIL simulations. The operating point is a bunch intensity of $5 \cdot 10^{10}$ p$^+$/b and a first order chromaticity of $Q'_x = -2$. The plot illustrates that already a small amount of negative Q''_x leads to beam stabilisation. On the other hand, for positive Q''_x the instability cannot be suppressed within the scanned range of values. This strong asymmetry can be understood better when considering the shape of the incoherent tune spread introduced by second order chromaticity as well as the stability diagram (SD) for Landau damping. The latter can be obtained by solving the dispersion integral for betatron detuning with longitudinal amplitude. It has been derived by Scott Berg and Ruggiero [20] and can be integrated numerically. Figure 7 illustrates the analytic tune distribution (top) as well as the SD for a mode 0 head-tail instability in the plane of complex coherent tune shifts ΔQ_{coh} (bottom), both for positive and negative Q''_x respectively. The interpretation of an SD is such that head-tail instabilities characterised by a ΔQ_{coh} situated below the curve are suppressed by the presence of the corresponding incoherent tune spread. The real part of the coherent tune shift caused by a dipolar impedance is negative as indicated by the green arrow. This explains not only why much better stabilisation behaviour is expected for negative Q''_x, but also why stabilisation with positive Q''_x is much more difficult. Both observations are in qualitative agreement with tracking results. Nevertheless, the quantitative comparison with PyHEADTAIL simulations still remains to be done.

CONCLUSIONS

The potential advantages of an rf quadrupole for HL-LHC have been evaluated numerically by means of PyHEADTAIL simulations for a likely flat-top machine working point. It has been shown that a few rf cavities would be sufficient to reduce the stabilising Landau octupole current to zero. Synchro-betatron resonances can, however, be a possible performance limitation and need to be understood in detail in particular for the HL-LHC tune working point. The necessary tools for systematic studies are now available and benchmarked. The first steps towards experimental studies of the underlying stabilising mechanism in the SPS have been discussed. The second order chromaticity serves as a convenient means to study the effect of betatron detuning with longitudinal amplitude experimentally, making a validation of the numerical models of beam stabilisation through an rf quadrupole possible.

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